PHYS 1442 – Section 001
Lecture #5

Monday, June 10, 2013

Dr. Jaehoon Yu

- Chapter 17
  - Electric Potential and Electric Field
  - Equi-potential Lines
  - The Electron Volt, a Unit of Energy
  - Capacitor and Capacitance
  - Di-electrics
  - Storage of Electric Energy
- Chapter 18
  - The Electric Battery
  - Ohm's Law: Resisters
  - Resistivity

Today's homework is homework #3, due 11pm, Thursday, June 13!!

Announcements

- Quiz results
  - Class average: 21.2/41
    - Equivalent to 51.7/100
  - Top score: 40/41
- Reading assignments
  - CH17 – 6, 17 – 10, and 17 – 11
- First term exam tomorrow, Tuesday, June 11
  - Covers CH1 – Ch7 plus A1 – A8
  - Mixture of multiple choice and free response problems
- Bring extra credit special project #1 during the break

Special Project #2 – Angels & Demons

- Compute the total possible energy released from an annihilation of x-grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the last two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class Thursday, June 13.

Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, what are the connections between these two quantities?

Electric Potential and Electric Field

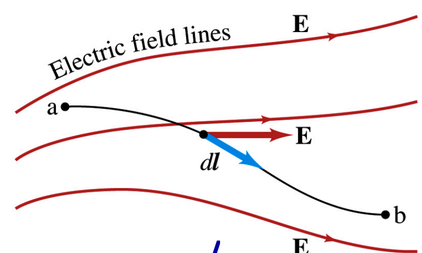
- Potential energy change is expressed in terms of a conservative force (point ***a*** at a higher potential)

$$U_b - U_a = -\vec{F}\cdot\vec{D} = -W_C$$

- For the electrical case, we are more interested in the potential difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q}\cdot\vec{D} = -\vec{E}\cdot\vec{D} = -ED\cos\theta$$

  – This formula can be used to determine V_{ba} when the electric field is given.



- When the field is uniform

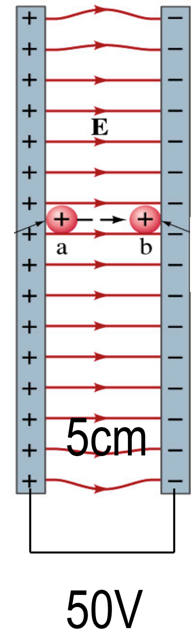
$$V_b - V_a = -\vec{E}\cdot\vec{D} = -ED\cos\theta = -Ed \quad \text{so} \quad E = -V_{ba}/d$$

What does "-"sign mean? The direction of E is along that of decreasing potential.

Unit of the electric field in terms of potential? V/m Can you derive this from N/C?

Example 17 – 3

Uniform electric field obtained from voltage: Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.



What is the relationship between electric field and the potential for a uniform field?

$$V = Ed$$

Solving for E

$$E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5\times10^{-2}m} = 1000V/m$$

Which direction is the field? Direction of decreasing potential!

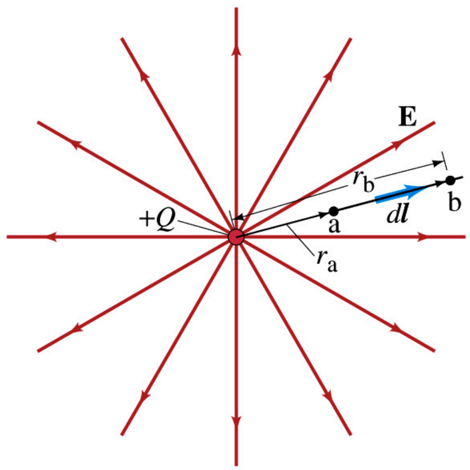
Electric Potential due to Point Charges

- What is the electric field by a single point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in radial direction away from the charge Q is

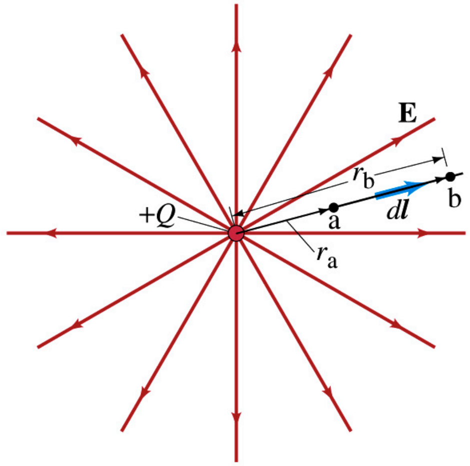
$$V_b - V_a = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$



Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electric potential V at a distance r from a single point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of as **the potential difference by a single point charge between r and infinity**

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?
  - Electric potential

    $$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the source charges to obtain the total potential from multiple charges, since potential is a scalar quantity
  - Electric field

    $$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

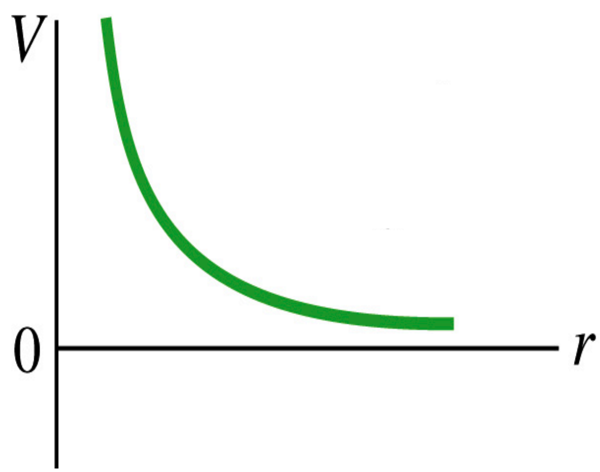
    - Electric force per unit charge
    - Inversely proportional to the square of the distance
    - Need vector sums to obtain the total field from multiple source charges
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at the large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at a large distance.

Shape of the Electric Potential

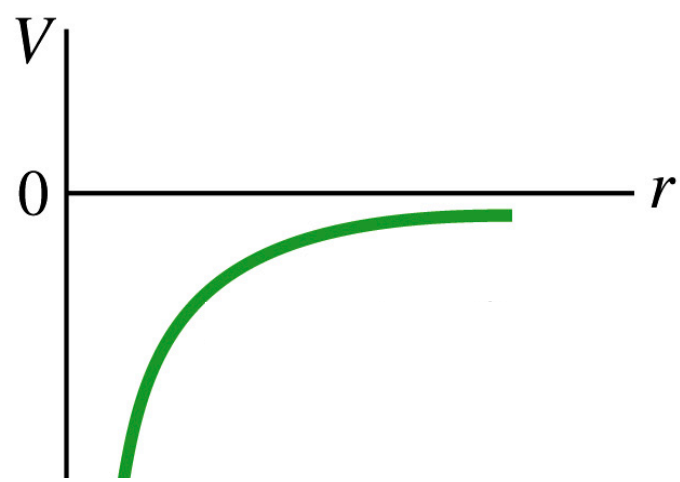
- So, how does the electric potential look like as a function of distance from the source charge?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same shape as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 17 – 4

Potential due to a positive or negative charge: Determine the potential at a point 0.50m (a) from a +20μC point charge and (b) from a -20μC point charge.

The formula for absolute potential at a point r away from the charge Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(a) For +20μC charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 9.0\times10^9 \cdot \frac{\left(+20\times10^{-6}\right)}{0.50} = 3.6\times10^5 V$

(b) For -20μC charge: $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 9.0\times10^9 \cdot \frac{\left(-20\times10^{-6}\right)}{0.50} = -3.6\times10^5 V$

It is important to express electric potential with the proper sign!!

Example 17 – 5

Work to bring two positive charges close together: What minimum work is required by an external force to bring a point charge q=+3.00μC from a great distance away (r=infinity) to a point 0.500m from a charge Q=+20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(3.00\times10^{-6}C\right)\left(20.00\times10^{-6}C\right)}{0.500m} = -1.08J$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge 3.00μC from infinity to 0.500m to the charge 20.0μC.

Electric Potential by Charge Distributions

- Let's consider that there are n individual point charges in a given space and V=0 at r=infinity.
- Then the potential due to the charge Q_i at a point a, distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

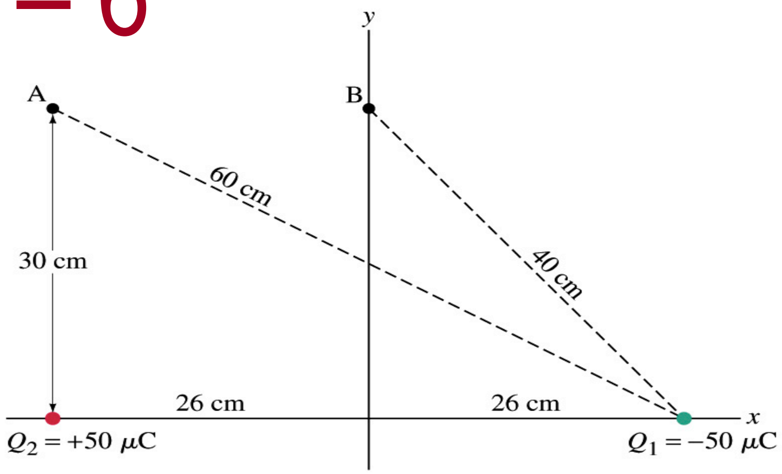
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Example 17 – 6

- **Potential due to two charges**: Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



- Potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.

(a) potential at A is $V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{iA}} =$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_{2A}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^9\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^5 V$$

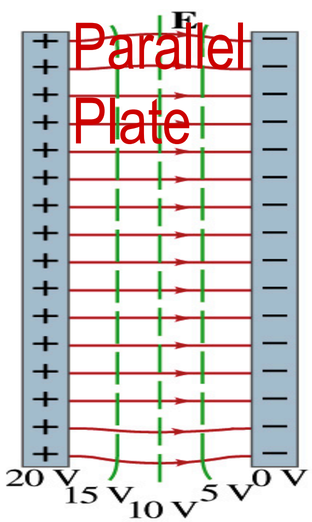
(b) How about potential at B?

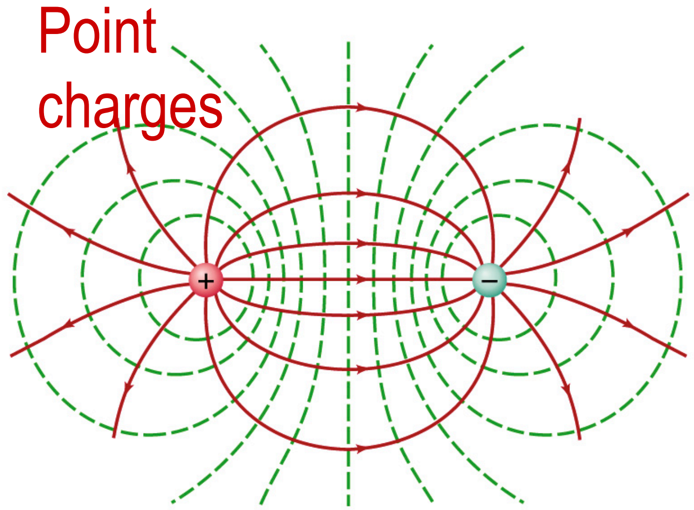
Equi-potential Surfaces

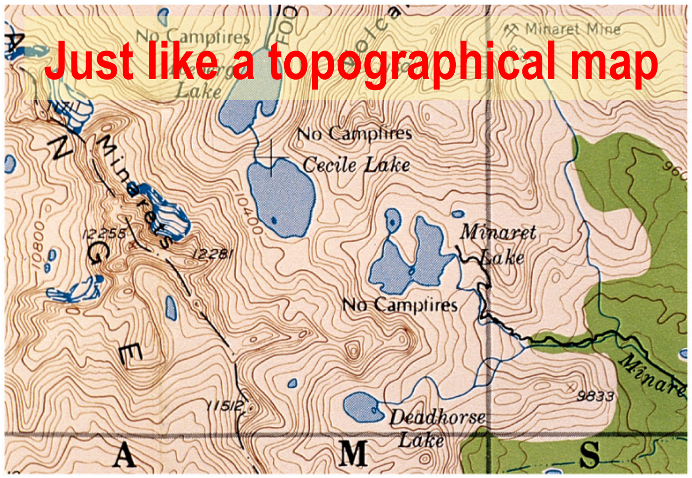
- Electric potential can be visualized using equipotential lines in 2-D or equipotential surfaces in 3-D
- Any two points on an equipotential surface (line) are on the same potential
- What does this mean in terms of the potential difference?
  – Thus, the potential difference between the two points on an equipotential surface is 0.
- How about the potential energy difference?
  – Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  – No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface, and thus a potential difference exists.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- There can be no electric field inside a conductor in static case, thus the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.







Electrostatic Potential Energy

- Consider a point charge q is moved between points a and b where the electrostatic potentials due to other charges are V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=infinity
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is exerted on with no electric force

Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at the distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r=infinity is $U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - This is the work that needs to be done by an external force to bring Q_2 from infinity to a distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain place without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - Work need to bring Q_3 to a distance to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_3}{r_{23}}$$

- So the total electrostatic potential of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right) \quad [V = 0 \text{ at } r = \infty]$$

  - What about a four charge system?

Electrostatic Potential Energy: electron Volt

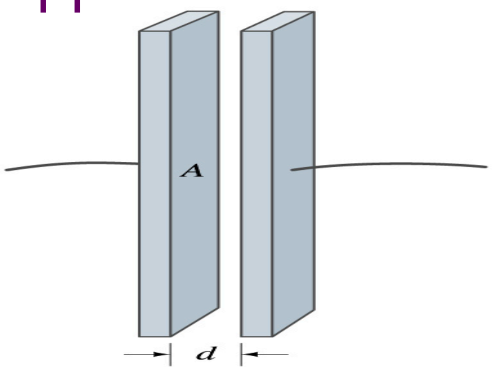
- What is the unit of electrostatic potential energy?
  – Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules in atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  – 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  – How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot 1V = 1.6\times10^{-19}J$
- eV however is **not a standard SI unit**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

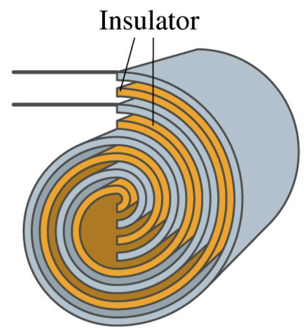
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, etc…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

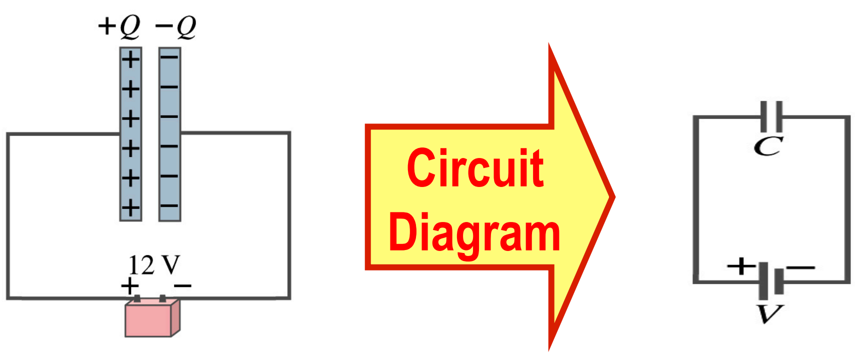
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



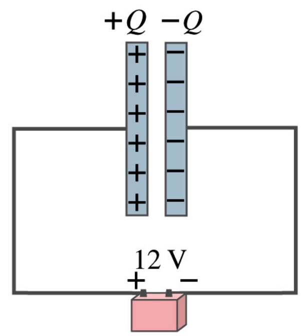


- How would you draw symbols for a capacitor and a battery?
  - Capacitor -||-
  - Battery (+) -|i- (-)



Capacitors

- What do you think will happen if a battery is connected (or a voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount.
- Each battery terminal, the wires and the plates are conductors. What does this mean?



  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

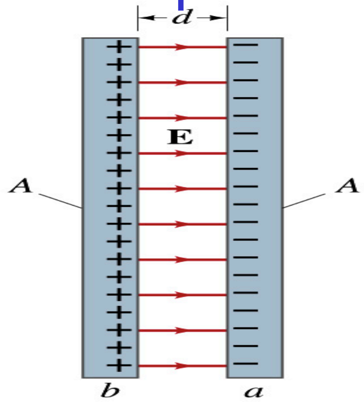
$$Q = CV_{ba}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use μF, nF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.
- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is $E=\sigma/\varepsilon_0$, $\sigma=Q/A$ is the surface charge density.



- E and V are related
- $$V_{ba} = Ed = \frac{Q}{A\varepsilon_0}d$$
- Since Q=CV, we obtain:

$$C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 17 – 8

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}m} = 53\times10^{-12}\, C^2/N\cdot m = 53pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times10^{-12}\, C^2/N\cdot m\right)\left(12V\right) = 6.4\times10^{-10}C = 640pC$$

Example 17 – 8

(C) Using the formula for the electric field in two parallel plates

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{6.4\times10^{-10}\,C}{6.0\times10^{-3}\,m^2 \times 8.85\times10^{-12}\,C^2/N\cdot m^2} = 1.2\times10^4\,N/C = 1.2\times10^4\,V/m$$

Or, since $V = Ed$ we can obtain $E = \frac{V}{d} = \frac{12V}{1.0\times10^{-3}\,m} = 1.2\times10^4\,V/m$

(d) Solving the capacitance formula for A, we obtain

$$C = \frac{\varepsilon_0 A}{d}$$

Solve for A

$$A = \frac{Cd}{\varepsilon_0} = \frac{1F\cdot 1\times10^{-3}\,m}{\left(9\times10^{-12}\,C^2/N\cdot m^2\right)} \approx 10^8\,m^2 \approx 100km^2$$

About 40% the area of Arlington ($256km^2$).