PHYS 1442 – Section 001 Lecture #10

Monday, June 24, 2013
Dr. Jaehoon Yu

- ## Chapter 20
    - ### Electric Current and Magnetism
    - ### Magnetic Forces on Electric Current
    - ### Magnetic Forces on a Moving Charge
    - ### Charged Particle Path in a Magnetic Field

Today's homework is #6, due 11pm, Monday, July 1!!

Dr. Jaehoon Yu

Announcements

- Exam results
  - Class average: 77.1/110
    - Equivalent to 70.1/100
      - Previous result 51.7/100
  - Top score: 109/110

- 2nd term exam
  - In class this Thursday, June 27
  - Non-comprehensive exam
    - Covers CH20.1 – what we finish this Wednesday

- Reading assignments
  - CH20.8 – 20.11

- Will give you a chance to submit your special project #3
  - Deadline by the beginning of the class tomorrow, Tuesday, June 25
 - You will get 75% of the original maximum

Earth's Magnetic Field

- What magnetic pole does the geographic north pole has to have?
  - W. Gilbert realized in 1600s that the Earth is a giant magnet
  - Magnetic south pole. What? How do you know that?
  - Since the magnetic north pole points to the geographic north, the geographic north must have magnetic south pole
    - The pole in the north is still called geomagnetic north pole just because it is in the north
  - Similarly, south pole has magnetic north pole

- The Earth's magnetic poles do not coincide with the geographic poles ➔ magnetic declination (0 – 20° in the US)
  - Geomagnetic north pole is in northern Canada, some 1300km off the true north pole

- Earth's magnetic field line is not tangent to the Earth's surface at all points
  - The angle the Earth's field makes to the horizontal line is called the angle dip

North geographic pole

Magnetic pole

Compass

Magnetic pole

South geographic pole

3
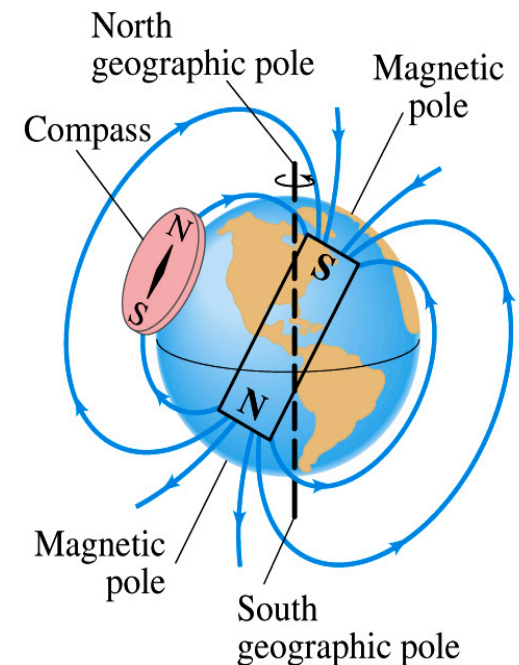
Electric Current and Magnetism

- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
  - Electric current produces a magnetic field
    - The first indication that electricity and magnetism are the same thing
  - What about a stationary electric charge and magnet?
    - They don't affect each other.
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the "right-hand" rule
  - The field lines follow right-hand's fingers wrapped around the wire when the thumb points to the direction of the electric current
 - The arrow head points from North to South

I

(a)
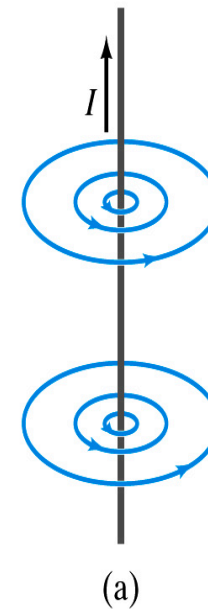
Directions in a Circular Wire?

- OK, then what are the directions of the magnetic fields generated by the current flowing through circular loops?
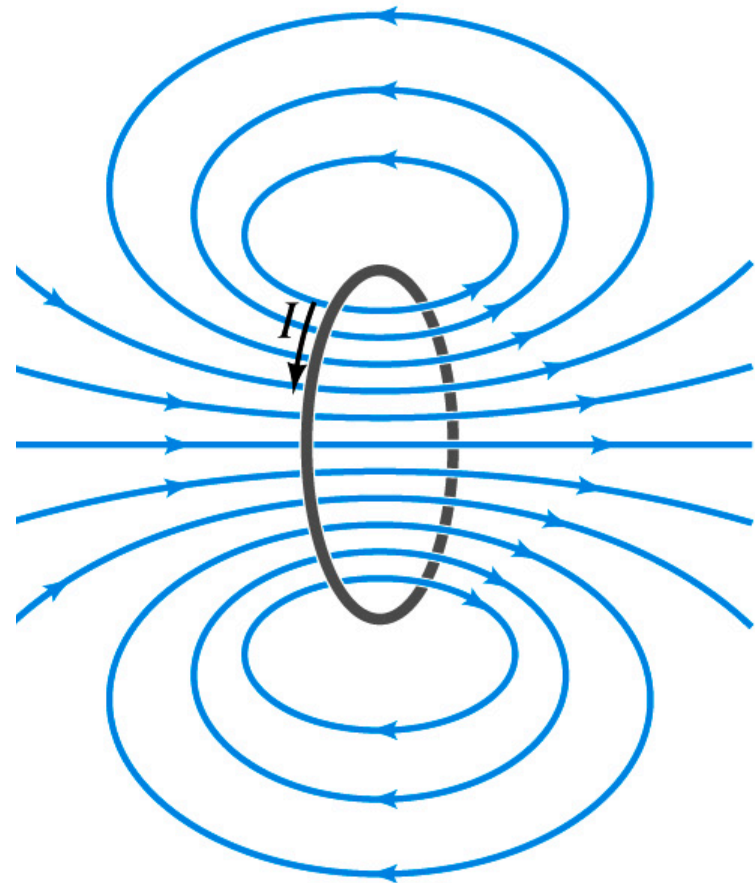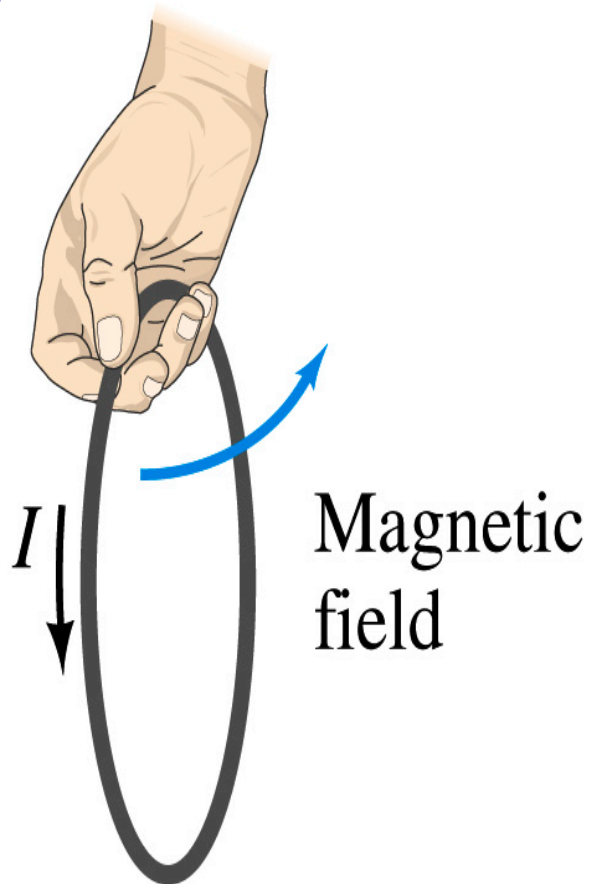
Magnetic field

Magnetic Forces on Electric Current

- Since the electric current exerts force on a magnet, the magnet should also exert force on the electric current
  - Which law justifies this?
    - Newton's 3rd law
  - This was also discovered by Oersted
- Direction of the force is always
  - perpendicular to the direction of the current
  - And also perpendicular to the direction of the magnetic field, **B**
- Experimentally the direction of the force is given by another right-hand rule ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to the direction of the force

F

I

B

Right-hand rule

Force is up

N   S

B

I

I

−   +

B

N   S

Force is down

I

I

+   −
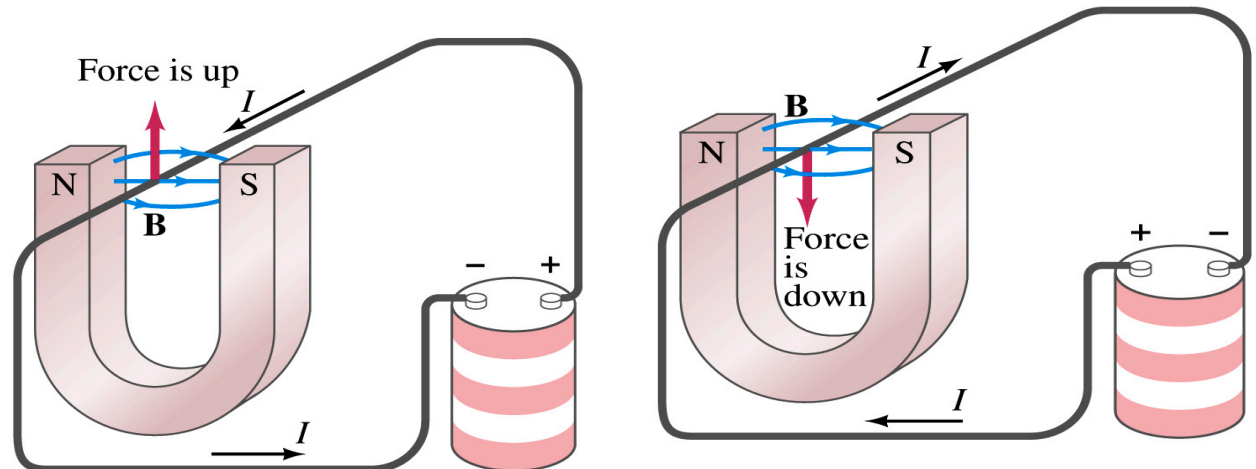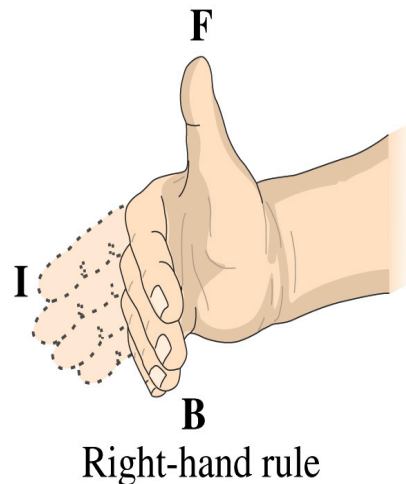
Magnetic Forces on Electric Current

- OK, we are set for the direction but what about the magnitude?
- It is found that the magnitude of the force is directly proportional to
  - The current flowing in the wire
  - The length of the wire in the magnetic field (if the field is uniform)
  - The strength of the magnetic field
- The force also depends on the angle θ between the directions of the current and the magnetic field
  - When the wire is perpendicular to the field, the force is the strongest
  - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in the wire with the length l in a uniform field $\mathscr{B}$ is

$$F \propto IlB \sin \theta$$

Magnetic Forces on Electric Current

- Magnetic field strength B can be defined using the previous proportionality relationship w/ the constant 1: $F = IlB \sin \theta$

- If θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$

- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of wire carrying a current I when the wire is perpendicular to **B**

- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - l is the vector whose magnitude is the length of the wire and its direction is along the wire in the direction of the conventional current
  - This formula works if B is uniform.

- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

About the Magnetic Field, B

- The magnetic field is a vector quantity

- The SI unit for B is tesla (T)
  - What is the definition of 1 Tesla in terms of other known units?
  - 1T=1N/Am
  - In older names, tesla is the same as weber per meter-squared
    - $1Wb/m^2 = 1T$

- The cgs unit for B is gauss (G)
  - How many T is one G?
    - $1G = 10^{-4}$ T
  - For computation, one MUST convert G to T at all times

- Magnetic field on the Earth's surface is about $0.5G = 0.5 \times 10^{-4}T$

- On a diagram, $\odot$ for field coming out and $\otimes$ for going in.

Properties of Vector Product

<u>Vector Product is Non-commutative</u> What does this mean?

If the order of operation changes the result changes

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Following the right-hand rule, the direction changes

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

<u>Vector Product of two parallel vectors is 0.</u>

$$\left|\vec{C}\right| = \left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta = \left|\vec{A}\right|\left|\vec{B}\right| \sin 0 = 0 \quad \text{Thus,}$$

$$\vec{A} \times \vec{A} = 0$$

<u>If two vectors are perpendicular to each other</u>

$$\left|\vec{A} \times \vec{B}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta = \left|\vec{A}\right|\left|\vec{B}\right| \sin 90° = \left|\vec{A}\right|\left|\vec{B}\right| = AB$$

<u>Vector product follows distribution law</u>

$$\vec{A} \times \left(\vec{B} + \vec{C}\right) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

<u>The derivative of a Vector product with respect to a scalar variable is</u>

$$\frac{d\left(\vec{A} \times \vec{B}\right)}{dt} = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

More Properties of Vector Product

The relationship between unit vectors, $\vec{i}, \vec{j}$ and $\vec{k}$

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k}$$

$$\vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}$$

$$\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

Vector product of two vectors can be expressed in the following determinant form

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= \left(A_y B_z - A_z B_y \right) \vec{i} - \left(A_x B_z - A_z B_x \right) \vec{j} + \left(A_x B_y - A_y B_x \right) \vec{k}$$

Example 20 – 1

Magnetic force on a current carrying wire. A wire carrying a 30 A current I, has a length ℓ = 12cm between the pole faces of a magnet at an angle θ=60° as in the figure. The magnetic field is approximately uniform at 0.9T. We ignore the field beyond the pole pieces. What is the magnitude of the force on the wire?
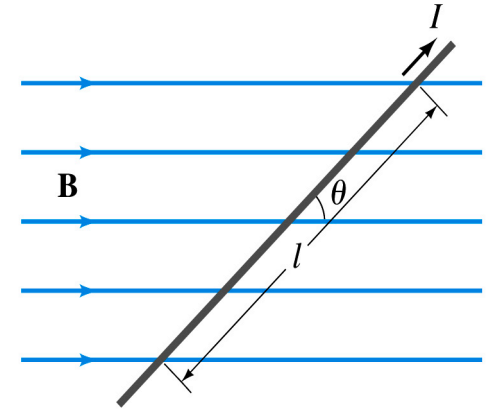
Which formula should we use for this problem?
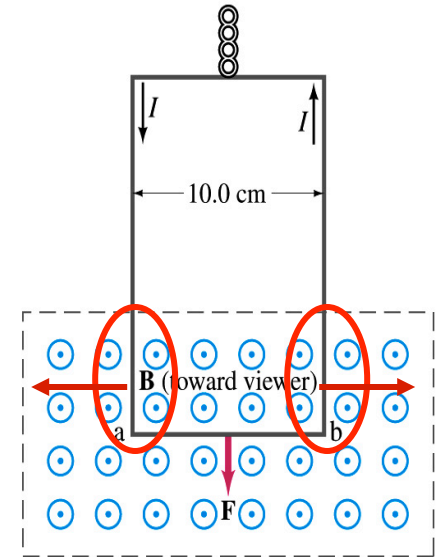
$$\vec{F} = I\left(\vec{l} \times \vec{B}\right)$$

The magnitude of the magnitude of the magnetic force is

$$\left|\vec{F}\right| = I\left|\left(\vec{l} \times \vec{B}\right)\right| = IlB\sin\theta_{IB}$$

$$\left|\vec{F}\right| = IlB\sin\theta_{IB} = 30\cdot 0.12\cdot 0.9\cdot\sin 60° = 2.8N$$

Example 20 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire ab (length l=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of F=3.48x10^{-2}N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?

Magnetic force exerted on the wire due to the uniform field is

$$\vec{F} = I\vec{l} \times \vec{B}$$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

Solving for B

$$B = \frac{F}{Il} = \frac{3.48 \times 10^{-2}\,N}{0.245A \cdot 0.10m} = 1.42T$$

Something is not right! What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since the wire carrying a current (moving charge) experience force in a magnetic field, a free moving charge must feel the same kind of force…☺

- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by the given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for a charge q to travel a distance ℓ in a magnetic field **B**
    - Then, the displacement vector ℓ becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the incident particle with charge q

- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$

- The force on one particle with charge q, $\boxed{\vec{F} = q\vec{v} \times \vec{B}}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity **v** in the field B is
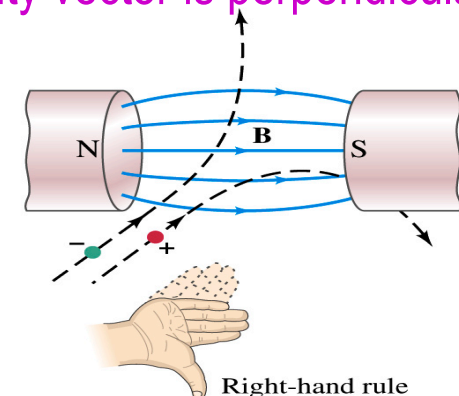   - $F = qvB \sin \theta$
    - What is θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the angle between the field and the velocity vector is perpendicular.
    - $F_{max} = qvB$ ➜ $B = \dfrac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field

Right-hand rule

Example 20 – 4

Magnetic force on a proton. A proton having a speed of 5×10^6m/s in a magnetic field feels a force of F=8.0×10^{-14}N toward the west when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6\times10^{-19}\,C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction.   Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward the west, the field should be pointing to .... North

Using the formula for the magnitude of the field B, we obtain

$$ B = \frac{F}{qv} = \frac{8.0\times10^{-14}\,N}{1.6\times10^{-19}\,C \cdot 5.0\times10^6\,m/s} = 0.10T $$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field

- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?

  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward

  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.
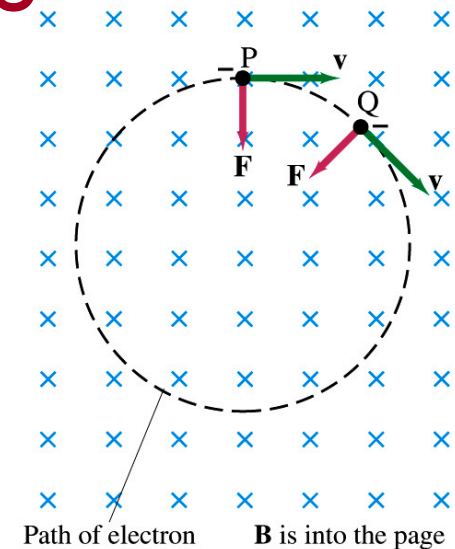
Example 20 – 5

Electron's path in a uniform magnetic field. An electron travels at a speed of 2.0×10^7 m/s in a plane perpendicular to a 0.010-T magnetic field. Describe its path.

What is formula for the centripetal force? $F = ma = m\dfrac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

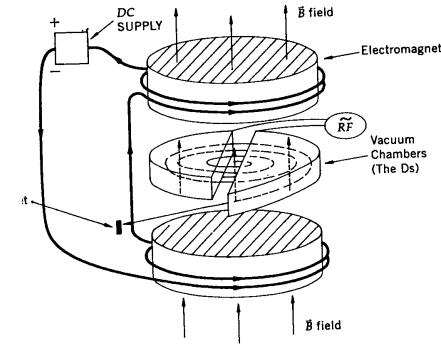Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\dfrac{v^2}{r}$

Solving for r $r = \dfrac{mv}{eB} = \dfrac{\left(9.1 \times 10^{-31}\, kg\right) \cdot \left(2.0 \times 10^7\, m/s\right)}{\left(1.6 \times 10^{-19}\, C\right) \cdot \left(0.010T\right)} = 1.1 \times 10^{-2}\, m$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator

  - While r depends on v, the frequency is independent of v and r.