PHYS 1442 – Section 001
Lecture #11

Tuesday, June 25, 2013
Dr. Jaehoon Yu

- Chapter 20
  - Sources of Magnetic Field
  - Magnetic Field Due to Straight Wire
  - Forces Between Two Parallel Wires
  - Ampére's Law and Its Verification
  - Solenoid and Toroidal Magnetic Field
- Chapter 21
  - Induced EMF and Electromagnetic Induction
  - Faraday's Law of Induction

Announcements

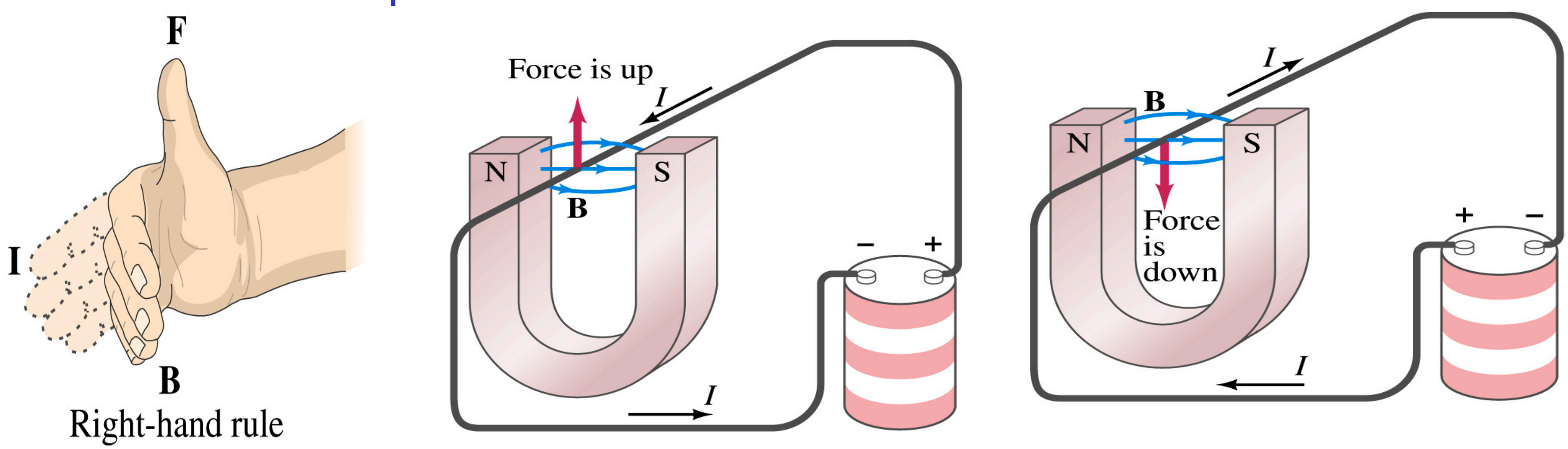
- 2[nd] term exam
  - In class this Thursday, June 27
  - Non-comprehensive exam
    - Covers CH20.1 – what we finish tomorrow, Wednesday (CH21.x?)
  - BYOF (Bring your own formula sheet)
  - This exam can replace your first term exam if better
  - If you miss the exam you will get an F no matter how well you have been doing in the class!
- Homework #5 deadline moved to 11pm tonight!
- Reading assignments
  - CH20.8 – 20.11

Refresher: Magnetic Forces on Electric Current

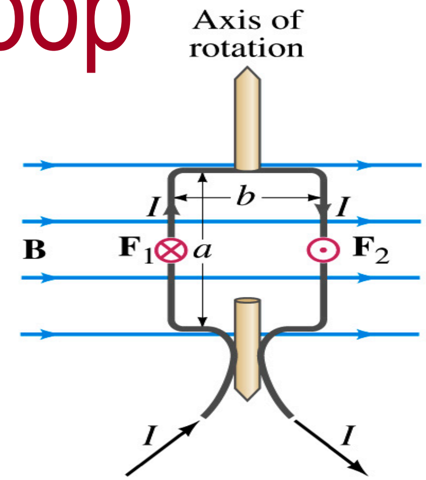
- Since the electric current exerts force on a magnet, the magnet should also exert force on the electric current
  - Which law justifies this?
    - Newton's 3rd law
  - This was also discovered by Oersted

$$F = IlB \sin\theta$$

- Direction of the force is always
  - Perpendicular to the direction of the current
  - And also perpendicular to the direction of the magnetic field, **B**
- Experimentally the direction of the force is given by another **right-hand rule** ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to the direction of the force

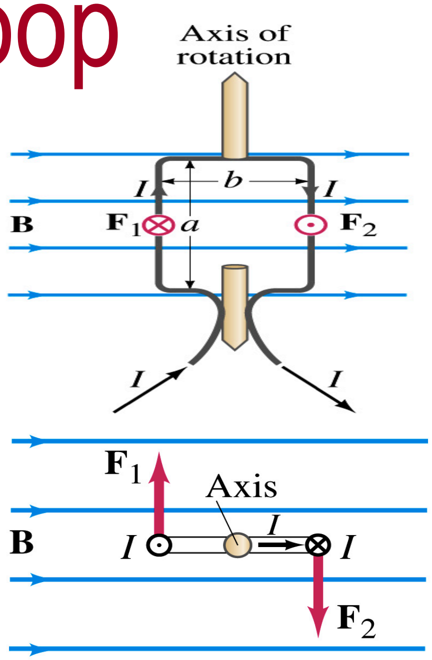


Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A=ab$ is the area of the coil
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Sources of Magnetic Field

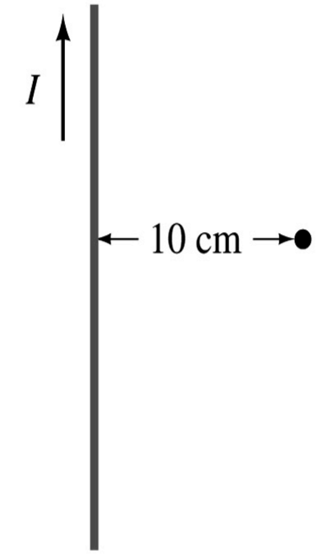
- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
  - How do we determine magnetic field strengths in certain situations?
  - How do two wires with electric current interact?
  - What is the general approach to finding the connection between current and magnetic field?

Magnetic Field due to a Straight Wire

- The magnetic field (B) due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of the current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, and thus the field strength becomes $B = \frac{\mu_0 I}{2\pi r}$
  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$
    - Permeability: the degree in which magnetizable material modifies the magnetic flux in the region occupied by it in a magnetic field

Example 20 – 7

Calculation of B near a wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due north of this wire?



Using the formula for the magnetic field near a straight wire

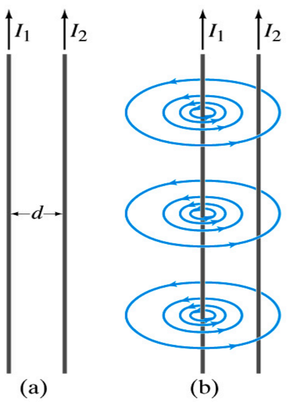
$$B = \frac{\mu_0 I}{2\pi r}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\, T \cdot m/A\right) \cdot \left(25A\right)}{\left(2\pi\right) \cdot \left(0.01m\right)} = 5.0 \times 10^{-5}\, T$$

Force Between Two Parallel Wires

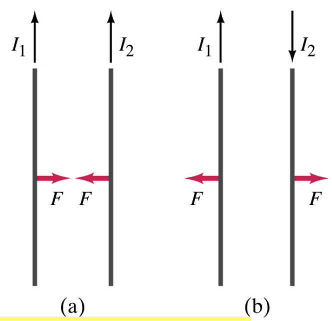
- We have learned that a wire carrying the current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or attract?
  - Depending on the direction of the currents



- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance d, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

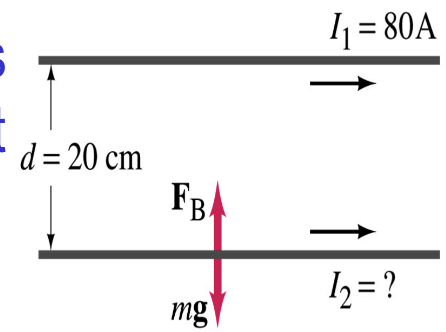
- The force F by a magnetic field B_1 on a wire of length l, carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi} \frac{I_1}{d}$
  - This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in the opposite direction
- So the force per unit length is $\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 20 – 11

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} = \frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times10^{-3}\,kg/m\right)\cdot\left(0.20m\right)}{\left(4\pi\times10^{-7}\,T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

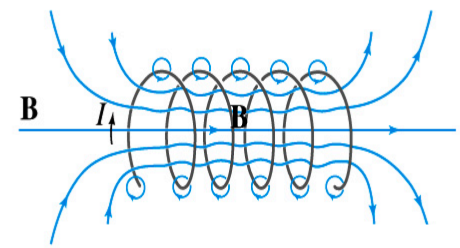
$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- The unit of current, ampere, is defined using the force between two wires each carrying 1A of current and separated by 1m

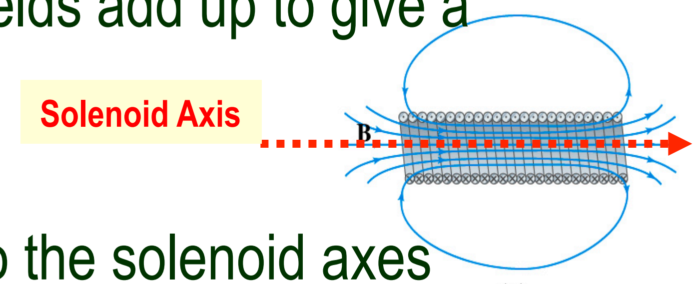
$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

  - So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2\text{x}10^{-7}$N/m.
- Coulomb is then defined as exactly 1C=1A.s.
- We do it this way since current is measured more accurately and controlled more easily than charge.

Solenoid and Its Magnetic Field



- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform



  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space

Solenoid Magnetic Field

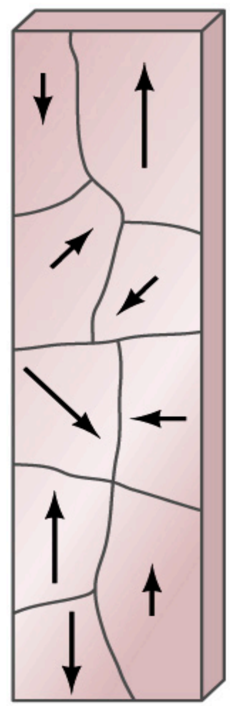
- If the current I flows in the wire of the solenoid, the total current enclosed is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- The magnetic field due to a solenoid of N turns is $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the **magnetic field within the solenoid** becomes
-

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI$$

  - B depends on the number of loops per unit length, n, and the current I
    - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called **ferromagnetic** material
- In a microscopic sense, ferromagnetic materials consists of many tiny regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged
- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature



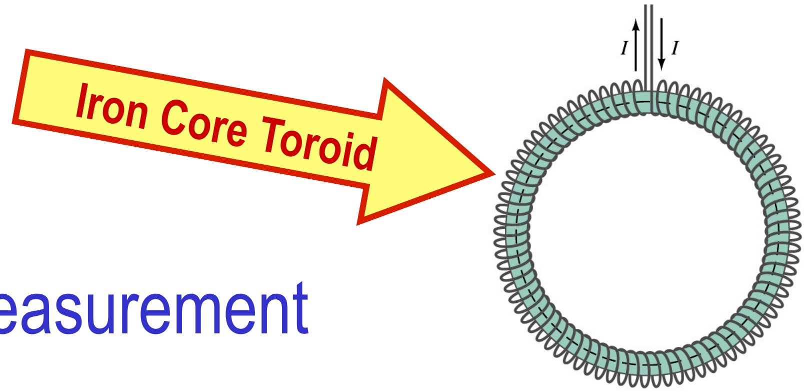
B in Magnetic Materials

- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

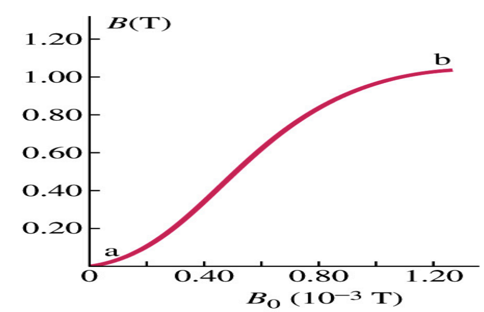
B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $\vec{B} = \vec{B}_0 + \vec{B}_M$
  - **B_0** is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - **B_M** is the additional field due to the ferromagnetic material itself; often **B_M**>>**B_0**
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $B = \mu n I$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

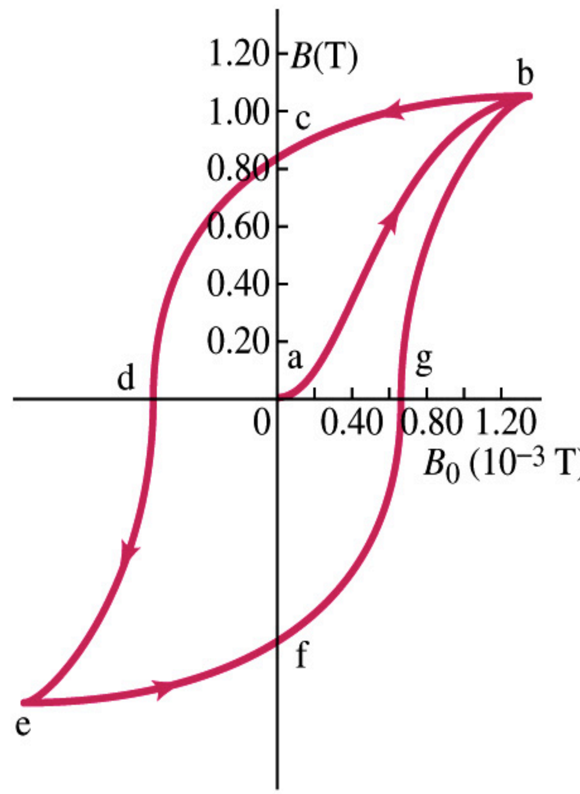


- What is a toroid?
  - A solenoid bent into a shape
- Toroid is used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



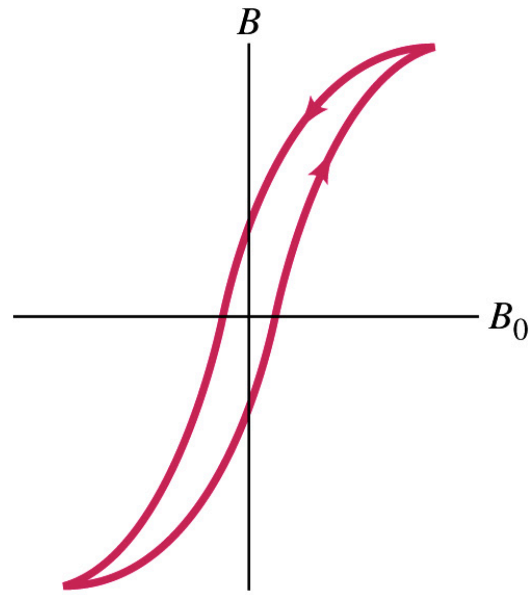
Hysteresis

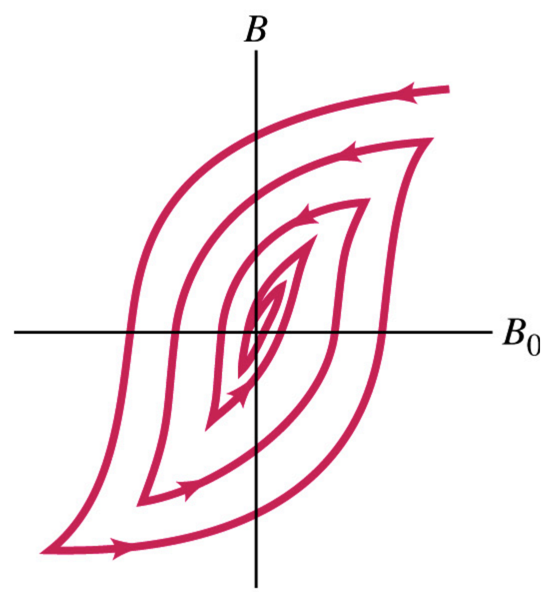
- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
  - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
  - Which ones do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.





Summary on Solenoid and Toroid

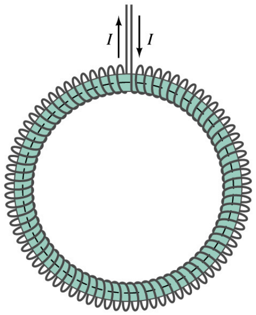
- The magnitude of the solenoid magnetic field without any material inside of the loop

$$B = \mu_0 n I$$

  - n is the number of loops per unit length
  - I is the current flowing through the loop
- If the loop has some material inside of it:

$$B = \mu n I$$

- The magnitude of the Toroid magnetic field with radius r:



$$B = \frac{\mu_0 N I}{2\pi r}$$

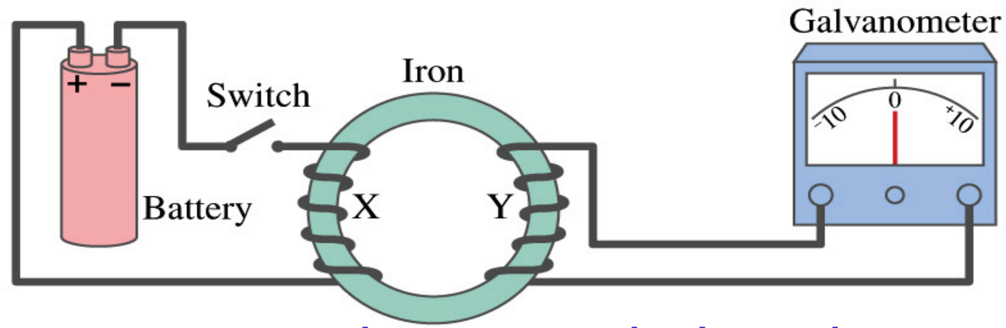
$$B = \frac{\mu N I}{2\pi r}$$

Induced EMF

- It has been discovered by Oersted and company in early 19th century that
  - Magnetic field can be produced by the electric current
  - Magnetic field can exert force on electric charge
- So if you were scientists at that time, what would you wonder?
  - Yes, you are absolutely right. You would wonder if the magnetic field can create the electric current.
  - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
    - Though, Faraday was given the credit since he published his work before Henry did
      - He also did a lot of detailed studies on magnetic induction

Electromagnetic Induction

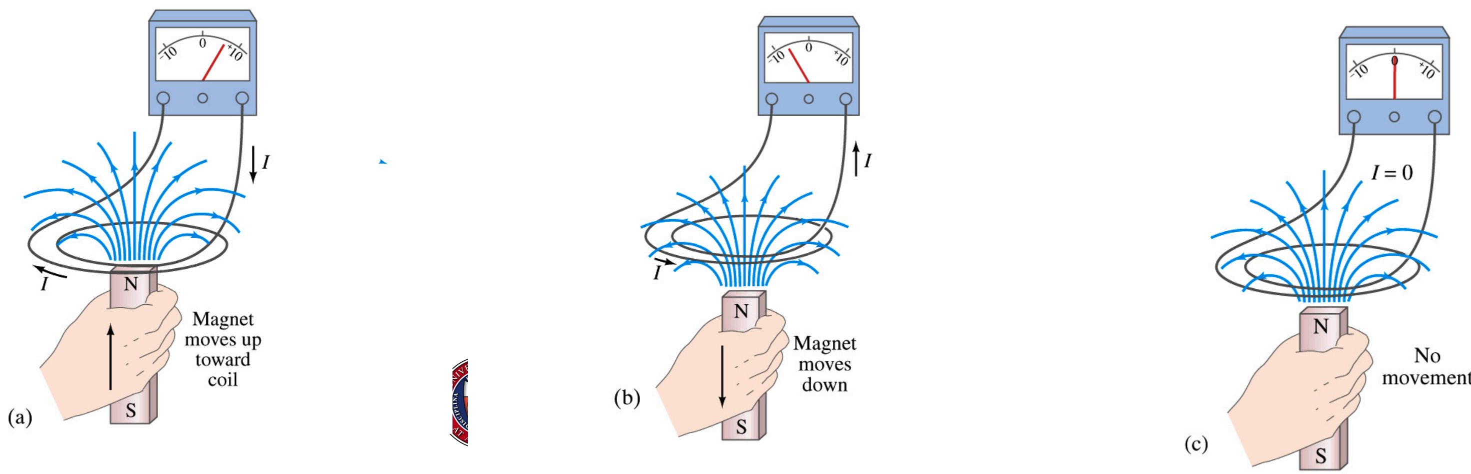
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
  - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

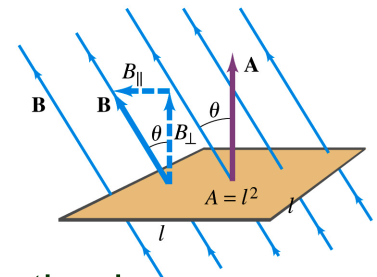
Electromagnetic Induction

- Further studies on electromagnetic induction taught
  - If magnet is moved quickly into a coil of wire, a current is induced in the wire.
  - If the magnet is removed from the coil, a current is induced in the wire in the opposite direction
  - By the same token, current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
  - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
  - The rate of changes of the magnetic field?
    - the higher the changes the higher the induction
  - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B.
  - Magnetic flux is defined as (just like the electric flux)



  - $$\Phi_B = B_{\perp} A = BA\cos\theta = \vec{B} \cdot \vec{A}$$
    - θ is the angle between **B** and the area vector **A** whose direction is perpendicular to the face of the loop based on the right-hand rule
  - What kind of quantity is the magnetic flux?
    - Scalar. Unit?
    - $T \cdot m^2$ or weber $$1Wb = 1T \cdot m^2$$
- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as $$\Phi_B = \sum \vec{B} \cdot \Delta\vec{A}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}$$

Faraday's Law of Induction

- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf…