PHYS 1442 – Section 001
Lecture #13

Monday, July 1, 2013

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- Chapter 21
  - Mutual and Self Inductance
  - Energy Stored in Magnetic Field
  - Alternating Current and AC Circuits
- Chapter 22
  - Gauss' Law of Magnetism

Today's homework is #7, due 11pm, Saturday, July 6!!

Announcements

- Quiz Wednesday, July 3
  - At the beginning of the class
  - Will cover what we've learned today and tomorrow
- Final exam
  - Comprehensive exam
    - Covers CH16.1 – what we finish this Wednesday plus Appendices A1 – A8
  - Monday, July 8
  - Please do not miss this exam!
    - You will get an F if you miss it!
  - BYOF with the same rules as before
- Your planetarium extra credit
  - Please bring your planetarium extra credit sheet by the beginning of the exam Monday, July 8
  - Be sure to tape one edge of the ticket stub with the title of the show on top and your name on the sheet

Reminder: Special Project #4

1. Derive the unit of speed from the product of the permitivity and permeability, starting from their original units. (5 points)
2. Derive and compute the speed of light in free space from the four Maxwell’s equations. (20 points for derivation and 3 points for computation.)
3. Compute the speed of the EM waves in copper, water and one other material which is different from other students. (3 points each)

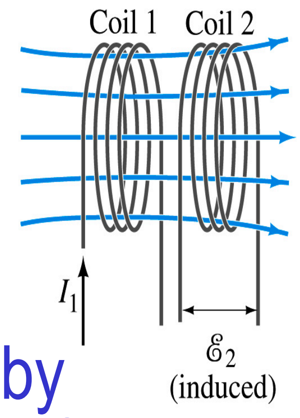
- Due of these projects are the start of the class this Wednesday, July 3!

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21}I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $M_{21} = N_2\Phi_{21}/I_1$. $\Rightarrow \Phi_{21} = M_{21}I_1/N_2$
- The emf induced in coil2 due to the changing current in coil1 is



$$\varepsilon_2 = -N_2\frac{\Delta\Phi_{21}}{\Delta t} = -\frac{\Delta(N_2\Phi_{21})}{\Delta t} = -\frac{\Delta(N_2 M_{21}I_1/N_2)}{\Delta t} = -M_{21}\frac{\Delta I_1}{\Delta t}$$

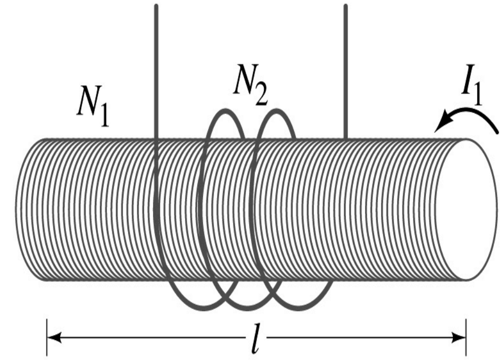
Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil #2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$
  - SI unit for mutual inductance is Henry (H) $$1H = 1V\cdot s/A = 1\Omega\cdot s$$

Example 21 – 14



Solenoid and coil. A long thin solenoid of length l and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.

First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $$B = \frac{\mu_0 N_1 I_1}{l}$$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What happens
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L\,I$
- We define self-inductance, $\mathcal{L}$: $L = \frac{N\Phi_B}{I} = \mu_0 N^2 A/l$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t} = -L\frac{\Delta I}{\Delta t}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 21 – 14

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B=\mu_0 nI=\mu_0 NI/l$

The flux is, therefore, $\Phi_B=BA=\mu_0 NIA/l$

Using the formula for self inductance: $L=\dfrac{N\Phi_B}{I}=\dfrac{N\cdot\mu_0 NIA/l}{I}=\dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B=\mu NI/l$

$$L=\frac{\mu N^2 A}{l}=\frac{4000\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=0.030H=30mH$$

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes a constant change of flux
- But it also provides means to store energy, just like the capacitance

Energy Stored in a Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate $\Delta I/\Delta t$, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\dfrac{\Delta I}{\Delta t}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, ΔW, done in time Δt is $\Delta W = P\Delta t = LI\Delta I$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \sum \Delta W = \sum_I LI\Delta I = \frac{1}{2}LI^2$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $U = \frac{1}{2}LI^2$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V, $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered as that stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect
    $$L = \mu_0 N^2 A / l$$
  - The magnetic field in a solenoid is $B = \mu_0 NI/l \Rightarrow I = Bl/\mu_0 N$
  - Thus the energy stored in an inductor is
    $$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$
  - Thus the energy density is
    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$
  - This formula is valid in any regions of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What is this?

Volume V

$$U = \frac{1}{2}\frac{B^2}{\mu_0}Al$$ **E**

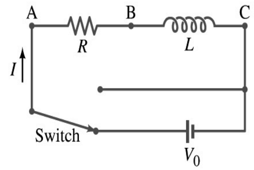
$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$ **E density**

What volume does Al represent?

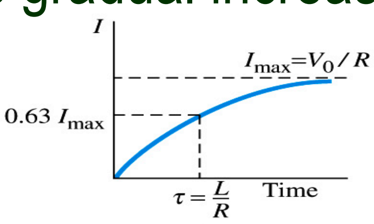
The volume inside a solenoid!!

LR Circuits

- What happens when an emf is applied to an inductor?
  - An inductor has some resistance, however negligible



    - So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**
  - What happens at the instance when the switch is thrown to apply emf to the circuit?
    - The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor → the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is gradual increase, reaching to the maximum current $I_{max}=V_0/R$.



Why do we care about circuits on AC?

- The circuits we've learned so far contain resistors, capacitors and inductors and have been connected to a DC source or a fully charged capacitor
  - What? This does not make sense.
  - The inductor does not work as an impedance unless the current is changing. So an inductor in a circuit with DC source does not make sense.
  - Well, actually it does. When does it impede?
    - Immediately after the circuit is connected to the source so the current is still changing. So?
      - It causes the change of magnetic flux.
  - Now does it make sense?
- Anyhow, learning the responses of resistors, capacitors and inductors in a circuit connected to an AC emf source is important. Why is this?
  - Since most the generators produce sinusoidal current
  - Any voltage that varies over time can be expressed as a superposition of sine and cosine functions

AC Circuits – the preamble

- Do you remember how the rms and peak values for current and voltage in AC are related?

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

- The symbol for an AC power source is

- We assume that the voltage gives rise to current

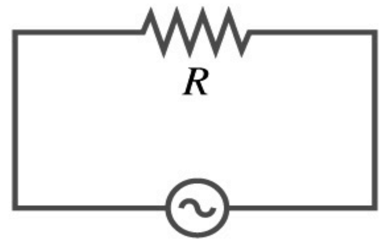
$$I = I_0 \sin \omega t = I_0 \sin 2\pi ft$$

  – where $\omega = 2\pi f$

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by a changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any regions in space

AC Circuit w/ Resistance only



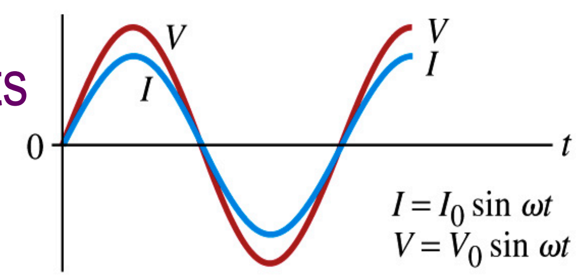
- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \omega t = V_0 \sin \omega t$$

  - where $V_0 = I_0 R$
- What does this mean?
  - Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
  - Current and voltage are "in phase"



- Energy is lost via the transformation into heat at an average rate $\overline{P} = \overline{I}\,\overline{V} = I_{rms}^2 R = V_{rms}^2 / R$
-

Reminder: Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\varepsilon = -\frac{\Delta \Phi_B}{\Delta t}$$

Faraday's Law of Induction

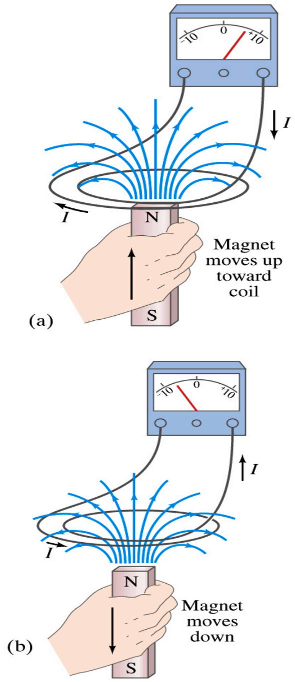
- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\varepsilon = -N\frac{\Delta \Phi_B}{\Delta t}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf…

Reminder: Lenz's Law

- It is experimentally found that
  - An induced emf gives rise to a current whose magnetic field opposes the original change in flux ➔ This is known as **Lenz's Law**
  - In other words, an induced emf is always in the direction that opposes the original change in flux that caused it.
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise



- Which law is Lenz's law result of?
  - Energy conservation. Why?

Displacement Current

- Maxwell predicted the second term in the generalized Ampere's law and interpreted it as equivalent to an electric current
  - If a change of magnetic field induces an electric current, a change of electric field must do the same to magnetic field…
  - He called this term as the displacement current, I_D
  - While the original term is called the conduction current, I
- Ampere's law then can be written as

$$\sum \vec{B} \cdot \Delta \vec{l} = \mu_0 \left(I + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow of electric charge itself

Gauss' Law for Magnetism

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\sum_{\substack{Close \\ surface}} \vec{E} \cdot \vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\sum_{\substack{Closed \\ surface}} \vec{B} \cdot \vec{A} = 0$$

Gauss' Law for magnetism

- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\sum_{\substack{Closed \\ surface}} \vec{B} \cdot \vec{A} = 0$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
    - Electricity do have the source and the sink
  - Magnetic field lines must be continuous
  - Even for bar magnets, the field lines exist both insides and outside of the magnet