PHYS 1442 – Section 001
Lecture #14

Tuesday, July 2, 2013

Dr. Jaehoon Yu

- Chapter 22
  - Maxwell's Equations
  - Production of Electromagnetic Waves
  - EM Waves from Maxwell's Equations
  - Speed of EM Waves
  - Light as an EM Wave
  - EM Spectrum
  - EM Waves in Transmission lines

Announcements

- Term exam results:
  - Class average: 60.4/100
    - Previous exams: 52.8/100 and 73.1/100
  - Top score: 87
- Quiz tomorrow, Wednesday, July 3
  - At the beginning of the class
  - Will cover what we've learned today and tomorrow
- Final exam
  - Comprehensive exam
    - Covers CH16.1 – what we finish this Wednesday plus Appendices A1 – A8
  - Monday, July 8
  - Please do not miss this exam!
    - You will get an F if you miss it!
  - BYOF with the same rules as before
- Your planetarium extra credit
  - Please bring your planetarium extra credit sheet by the beginning of the exam Monday, July 8
  - Be sure to tape one edge of the ticket stub with the title of the show on top and your name on the sheet

Reminder: Special Project #4

1. Derive the unit of speed from the product of the permitivity and permeability, starting from their original units. (5 points)
2. Derive and compute the speed of light in free space from the four Maxwell's equations. (20 points for derivation and 3 points for computation.)
3. Compute the speed of the EM waves in copper, water and one other material which is different from other students. (3 points each)

- Due of these projects are the start of the class tomorrow, Wednesday, July 3!

The Electromagnetic Theory

- The ramification of the EM theory in late 19th century is the prediction and the experimental verification of the fact that

Waves of electromagnetic fields can travel through space!!

- Opened up a new means of communications
  - Wireless telegraphs, radio, TV, cell phones, cell phones, wireless charging device, etc
- A spectacular prediction of light being an EM wave
- This theory of EM forces was the work of a Scottish physicist, James Clerk Maxwell (1831 – 1879)

Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\sum_{\substack{Closed\\surface}} \vec{E}\cdot\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\sum_{\substack{Closed\\surface}} \vec{B}\cdot\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law, relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\sum_{\substack{Closed\\loop}} \vec{E}\cdot\vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\sum_{\substack{Closed\\loop}} \vec{B}\cdot\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{\Delta\Phi_E}{\Delta t}$$

Ampére's Law

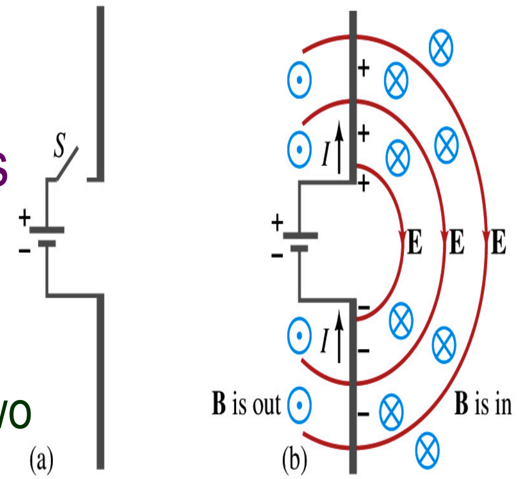
A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes
    - This changing magnetic field then in turn produces the electric field that changes
    - This process continues
  - With the manipulation of his equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

Production of EM Waves

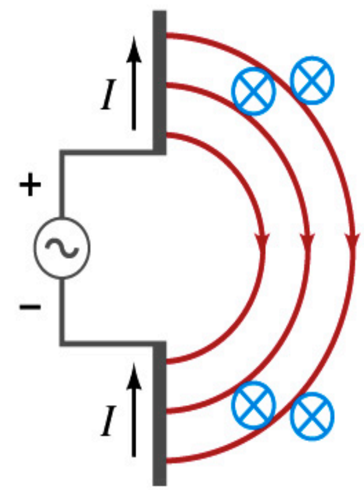
- Consider two conducting rods that will serve as an antenna are connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal is charged positive and the other negative
    - Then an electric field will be generated between the two rods
    - Since there is current that flows through the rods, a magnetic field around them will be generated



- How far would the electric and magnetic fields extend?
  - In static cases, the field extends indefinitely
  - When the switch is closed, the fields are formed near the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

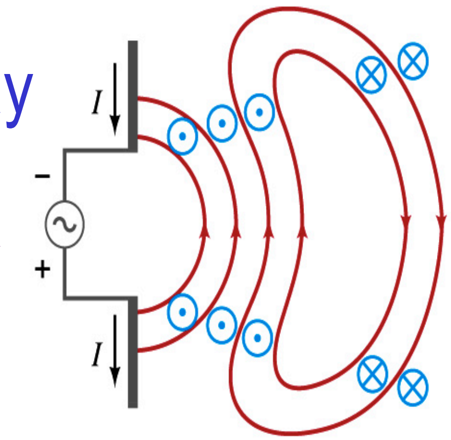
Production of EM Waves, cnt'd

- What happens if the antenna is connected to an AC power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the DC case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses



Production of EM Waves, cnt'd

- New field lines in the opposite direction forms
- While the original field lines still propagates away from the rod reaching out far
  - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
  - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge



– The fields far from the antenna is called the **radiation field**

– Both electric and magnetic fields form closed loops perpendicular to each other

Properties of Radiation Fields

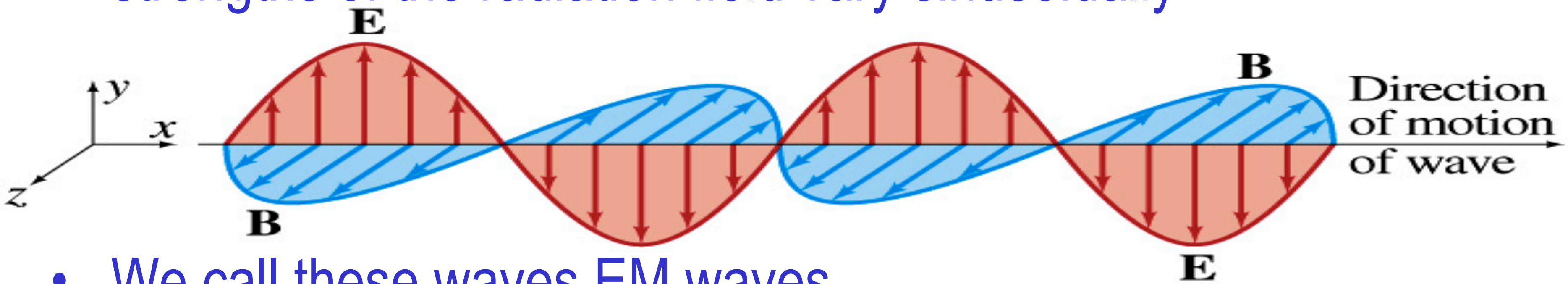
- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the parallel direction is 0
- The magnitude of **E** and **B** in the radiation field decrease with distance from the source as 1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  - So the intensity of wave decreases from the source as $1/r^2$

Properties of Radiation Fields, cnt'd

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in directions
  - The field strengths vary from maximum to 0 in one direction to 0 to maximum in the opposite direction
    - out of phase
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
  - Called plane waves

EM Waves

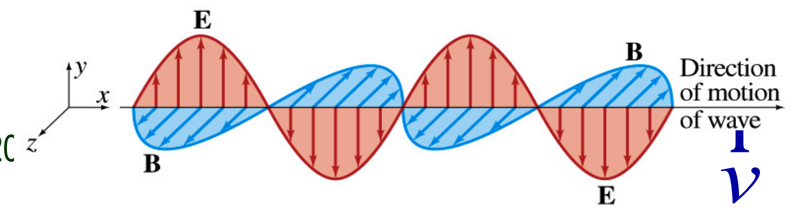
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
  - Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally proven by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - Plane waves
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis



Maxwell's Equations w/ Q=I=0

- In free space where Q=0 and I=0, the four Maxwell's equations become

$$\sum_{\substack{Closed\\surface}} \vec{E}\cdot\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \xrightarrow{Q_{encl}=0} \quad \sum_{\substack{Closed\\surface}} \vec{E}\cdot\vec{A} = 0$$

$$\sum_{\substack{Closed\\surface}} \vec{B}\cdot\vec{A} = 0 \quad \xrightarrow{\text{No Changes}} \quad \sum_{\substack{Closed\\surface}} \vec{B}\cdot\vec{A} = 0$$

$$\sum_{\substack{Closed\\loop}} \vec{E}\cdot\vec{l} = -\frac{\Delta\Phi_B}{\Delta t} \quad \xrightarrow{\text{No Changes}} \quad \sum_{\substack{Closed\\loop}} \vec{E}\cdot\vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\sum_{\substack{Closed\\loop}} \vec{B}\cdot\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{\Delta\Phi_E}{\Delta t} \quad \xrightarrow{I_{encl}=0} \quad \sum_{\substack{Closed\\loop}} \vec{B}\cdot\vec{l} = \mu_0\varepsilon_0\frac{\Delta\Phi_E}{\Delta t}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, such traveling wave can be written as

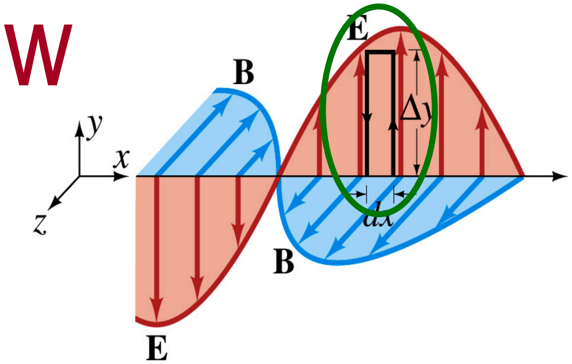
$$E = E_y = E_0 \sin\left(kx - \omega t\right)$$
$$B = B_z = B_0 \sin\left(kx - \omega t\right)$$

  – Where

$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f \quad \text{Thus} \quad f\lambda = \frac{\omega}{k} = \mathrm{v}$$

  – What is v?
    - The speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\sum_{loop} \vec{E}\cdot\Delta\vec{l} = -\frac{\Delta\Phi_B}{\Delta t}$$

  – to the rectangular loop of height Δy and width dx
- $\vec{E}\cdot\Delta\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since **E** is perpendicular to Δl.
  – So the result of the sum through the loop counterclockwise becomes $\sum_{loop} \vec{E}\cdot\Delta\vec{l} = \vec{E}\cdot d\vec{x} + \left(\vec{E}+\Delta\vec{E}\right)\cdot\Delta\vec{y} + \vec{E}\cdot d\vec{x}' + \vec{E}\cdot\Delta\vec{y}' =$

$$= 0 + \left(E+\Delta E\right)\Delta y - 0 - E\Delta y = \Delta E\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{\Delta\Phi_B}{\Delta t} = \frac{\Delta B}{\Delta t}\Delta x\Delta y$$

Thus

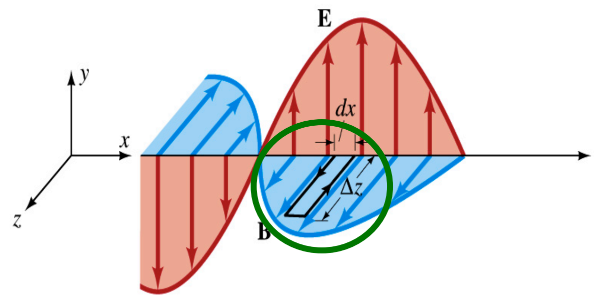
$$\Delta E\Delta y = -\frac{\Delta B}{\Delta t}\Delta x\Delta y$$

$$\frac{\Delta E}{\Delta x} = -\frac{\Delta B}{\Delta t}$$

From Modified Ampére's Law

- Let's apply Maxwell's 4th equation

$$\sum_{loop} \vec{B}\cdot\Delta\vec{l} = \mu_0\varepsilon_0 \frac{\Delta\Phi_E}{\Delta t}$$



  – to the rectangular loop of length Δz and width Δx

- $\vec{B}\cdot d\vec{l}$ along the x-axis of the loop is 0
  – Since **B** is perpendicular to d*l*.
  – So the result of the sum through the loop counterclockwise becomes $\sum_{loop} \vec{B}\cdot\Delta\vec{l} = B\Delta Z - \left(B+\Delta B\right)\Delta Z = -\Delta B\Delta Z$
  – For the right-hand side of the equation is

$$\mu_0\varepsilon_0 \frac{\Delta\Phi_E}{\Delta t} = \mu_0\varepsilon_0 \frac{\Delta E}{\Delta t}\Delta x\Delta z$$

Thus

$$-\Delta B\Delta z = \mu_0\varepsilon_0 \frac{\Delta E}{\Delta t}\Delta x\Delta z$$

  – $$\frac{\Delta B}{\Delta x} = -\mu_0\varepsilon_0 \frac{\Delta E}{\Delta t}$$

Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\Delta E}{\Delta x}=-\frac{\Delta B}{\Delta t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\Delta E}{\Delta x}=kE_0\cos(kx-\omega t)$$

$$\frac{\Delta B}{\Delta t}=-\omega B_0\cos(kx-\omega t)$$

Since $\frac{\Delta E}{\Delta x}=-\frac{\Delta B}{\Delta t}$ **We obtain** $kE_0\cancel{\cos(kx-\omega t)}=\omega B_0\cancel{\cos(kx-\omega t)}$

Thus $\frac{E_0}{B_0}=\frac{\omega}{k}=\mathrm{v}$

- Since E and B are in phase, we can write $E/B=\mathrm{v}$
  - This is valid at any point and time in space. What is v?
    - The speed of the wave

Speed of EM Waves

- Let's now use the relationship from Ampere's law $\frac{\Delta B}{\Delta x} = -\varepsilon_0 \mu_0 \frac{\Delta E}{\Delta t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\Delta B}{\Delta x} = kB_0 \cos(kx - \omega t)$$

$$\frac{\Delta E}{\Delta t} = -\omega E_0 \cos(kx - \omega t)$$

Since $\frac{\Delta B}{\Delta x} = -\varepsilon_0 \mu_0 \frac{\Delta E}{\Delta t}$ **We obtain** $kB_0 \cancel{\cos(kx - \omega t)} = \varepsilon_0 \mu_0 \omega E_0 \cancel{\cos(kx - \omega t)}$

Thus $\frac{B_0}{E_0} = \frac{\varepsilon_0 \mu_0 \omega}{k} = \varepsilon_0 \mu_0 \mathrm{v}$

- However, from the previous page we obtain $E_0 / B_0 = \mathrm{v} = \frac{1}{\varepsilon_0 \mu_0 \mathrm{v}}$
- Thus $\mathrm{v}^2 = \frac{1}{\varepsilon_0 \mu_0}$

$$\boxed{\mathrm{v} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\left(8.85 \times 10^{-12}\, C^2 / N \cdot m^2\right) \cdot \left(4\pi \times 10^{-7}\, T \cdot m / A\right)}} = 3.00 \times 10^8\, m/s}$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.