PHYS 1441 – Section 001
Lecture #5

Tuesday, June 10, 2014

Dr. Jaehoon Yu

- Trigonometry Refresher
- Properties and operations of vectors
- Components of the 2D Vector
- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion

Today's homework is homework #3, due 11pm, Friday, June 13!!

Announcements

- Reading Assignment
  - CH3.7
- Quiz #2
  - Beginning of the class this Thursday, June, 12
  - Covers CH3.1 to what we finish tomorrow, Wednesday, June 11
  - Bring your calculator but DO NOT input formula into it!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam → no solutions, derivations or definitions!
    - No additional formulae or values of constants will be provided!

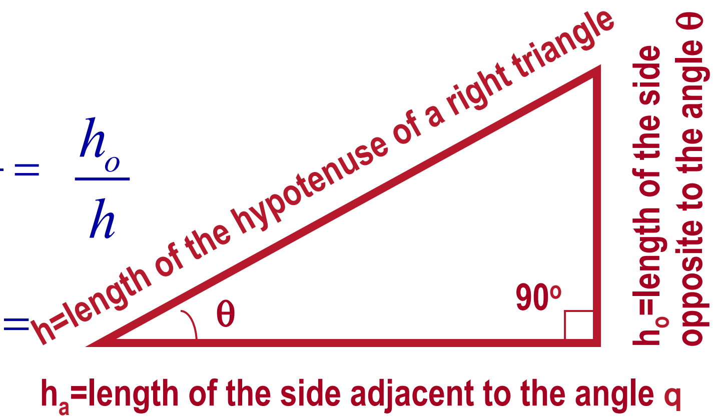
Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Monday, June 16
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in this lecture note and in the book!

Trigonometry Refresher

- Definitions of sinθ, cosθ and tanθ

$$\sin\theta = \frac{\text{Length of the opposite side to } \theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_o}{h}$$



$$\cos\theta = \frac{\text{Length of the adjacent side to } \theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_a}{h}$$

Solve for θ

$$\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$$

$$\tan\theta = \frac{\text{Length of the opposite side to } \theta}{\text{Length of the adjacent side to } \theta} = \frac{h_o}{h_a}$$

Solve for θ

$$\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{h_o}{\cancel{h}}}{\frac{h_a}{\cancel{h}}} = \frac{h_o}{h_a}$$

Pythagorian theorem: For right angle triangles

$$h^2 = h_o^2 + h_a^2 \Rightarrow h = \sqrt{h_o^2 + h_a^2}$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Velocity, acceleration, force, momentum, etc

Normally denoted in **BOLD** letters, $\boldsymbol{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|\boldsymbol{F}\right|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

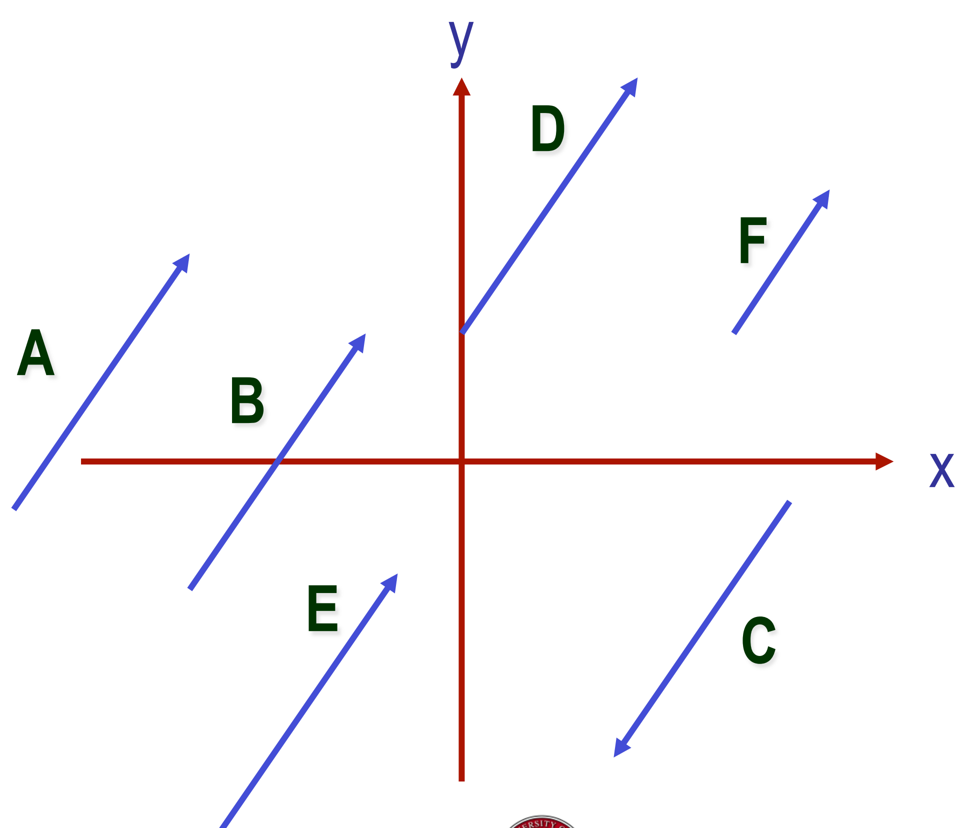
Normally denoted in normal letters, E

Speed, energy, heat, mass, time, etc

Both have units!!!

Properties of Vectors

- Two vectors are the same if their **sizes** and the **directions** are the same, no matter where they are on a coordinate system!! ➔ You can move them around as you wish as long as their directions and sizes are kept the same.



Which ones are the same vectors?

A=B=E=D

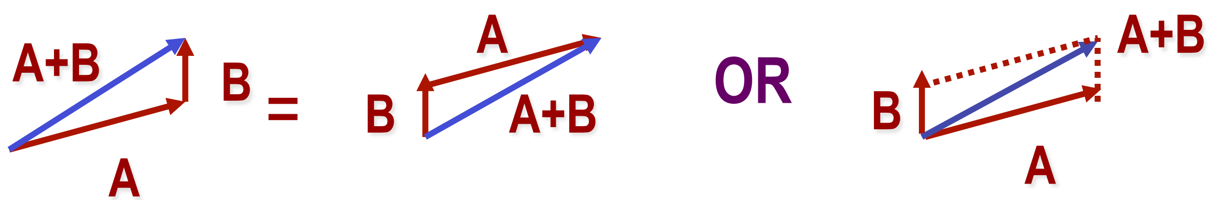
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:** A negative vector

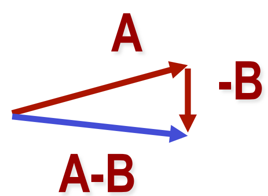
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**



- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



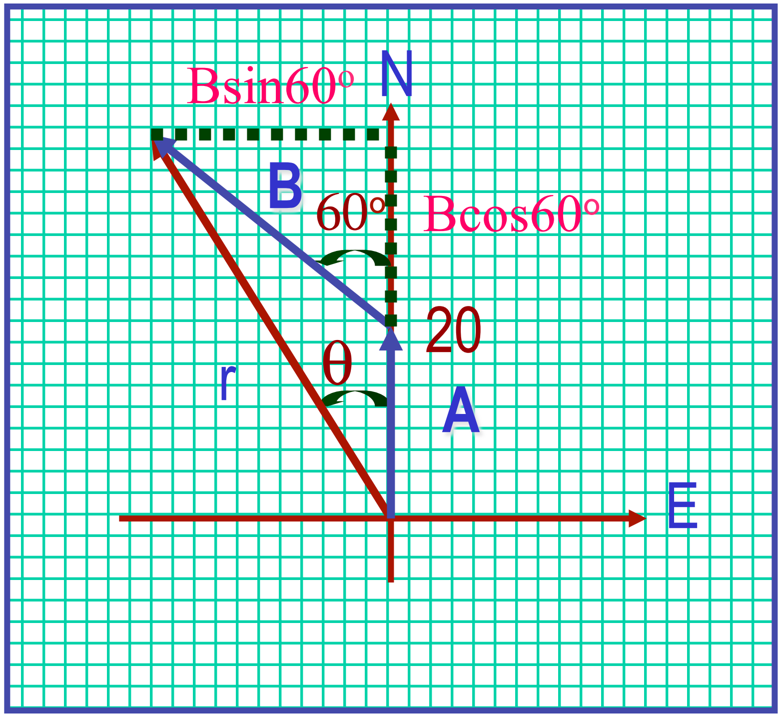
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**

$$\boxed{|\mathcal{B}| = 2|\mathcal{A}|}$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º West of North. Find the magnitude and direction of resultant displacement.



$$r = \sqrt{\left(A + B\cos\theta\right)^2 + \left(B\sin\theta\right)^2}$$

$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$

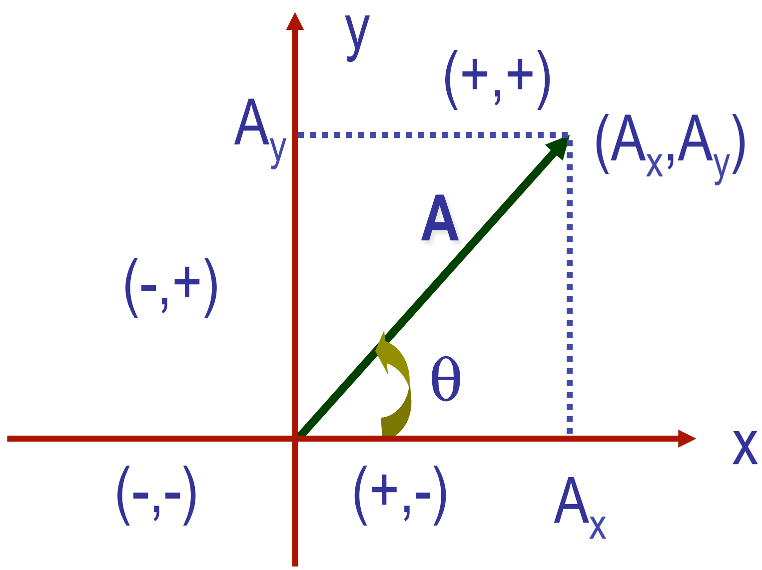
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Do this using components!!

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right|\cos\theta$$

$$A_y = \left|\vec{A}\right|\sin\theta$$

} Components

$$\left|\vec{A}\right| = \sqrt{A_x^{\ 2} + A_y^{\ 2}}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
  - Very powerful and makes vector notation and operations much easier!
- **Dimensionless**
- **Magnitudes these vectors are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or
  $\vec{i},\ \vec{j},\ \vec{k}$

So a vector **A** can be expressed as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C}=\vec{A}+\vec{B}=\left(2.0\vec{i}+2.0\vec{j}\right)+\left(2.0\vec{i}-4.0\vec{j}\right)$$
$$=\left(2.0+2.0\right)\vec{i}\ +\left(2.0-4.0\right)\vec{j}\ =4.0\vec{i}\ \ -2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right|=\sqrt{\left(4.0\right)^2+\left(-2.0\right)^2}$$
$$=\sqrt{16+4.0}=\sqrt{20}=4.5(m)$$

$$\theta=\ \tan^{-1}\frac{C_y}{C_x}=\tan^{-1}\frac{-2.0}{4.0}=-27^\circ$$

Find the resultant displacement of three consecutive displacements: $\mathbf{d_1}$=(15**i**+30**j** +12**k**)cm, $\mathbf{d_2}$=(23**i**+14**j** -5.0**k**)cm, and $\mathbf{d_3}$=(-13**i**+15**j**)cm

$$\overrightarrow{D}=\vec{d}_1+\vec{d}_2+\vec{d}_3=\left(15\vec{i}+30\vec{j}+12\vec{k}\right)+\left(23\vec{i}+14\vec{j}-5.0\vec{k}\right)+\left(-13\vec{i}+15\vec{j}\right)$$
$$=\left(15+23-13\right)\vec{i}+\left(30+14+15\right)\vec{j}\ +\left(12-5.0\right)\vec{k}=25\vec{i}\ +59\vec{j}+7.0\vec{k}(cm)$$

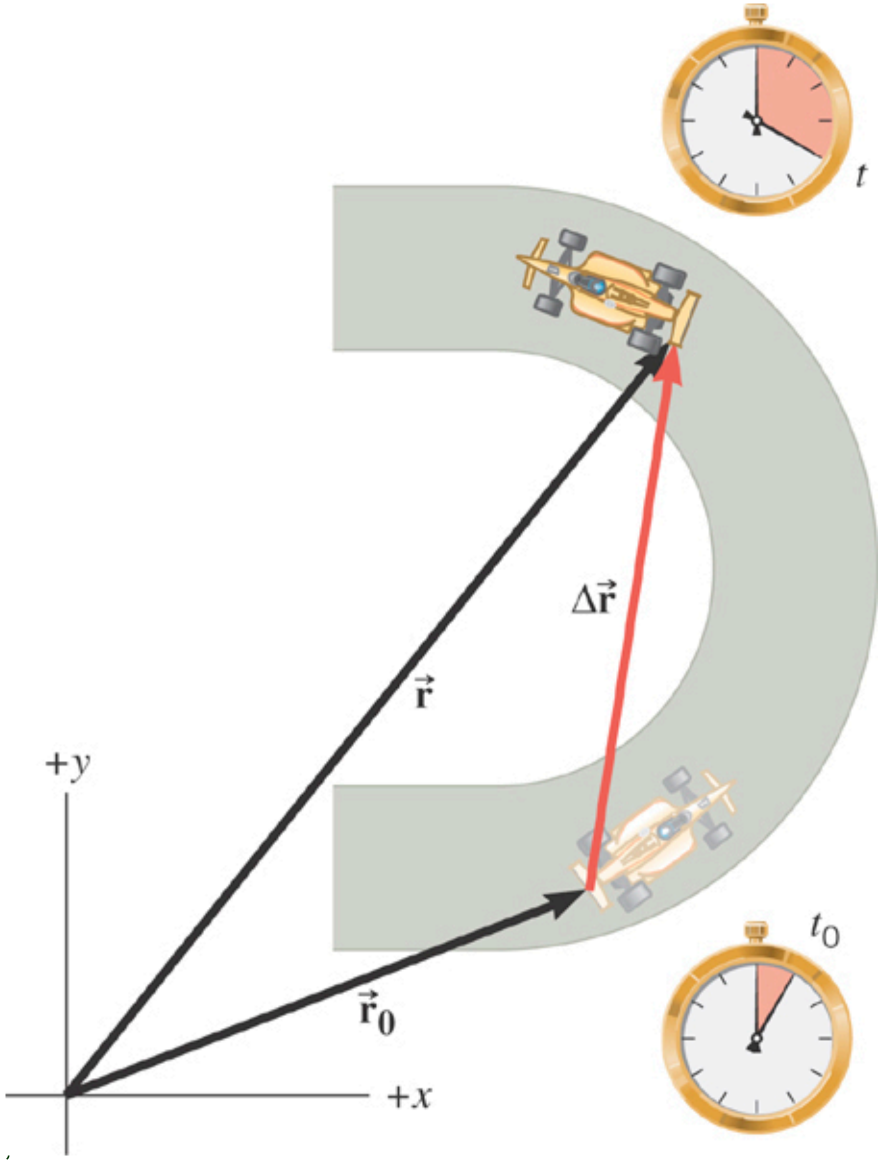
Magnitude $$\left|\vec{D}\right|=\sqrt{\left(25\right)^2+\left(59\right)^2+\left(7.0\right)^2}=65(cm)$$

2D Displacement

$\vec{\mathbf{r}}_o =$ initial position

$\vec{\mathbf{r}} =$ final position

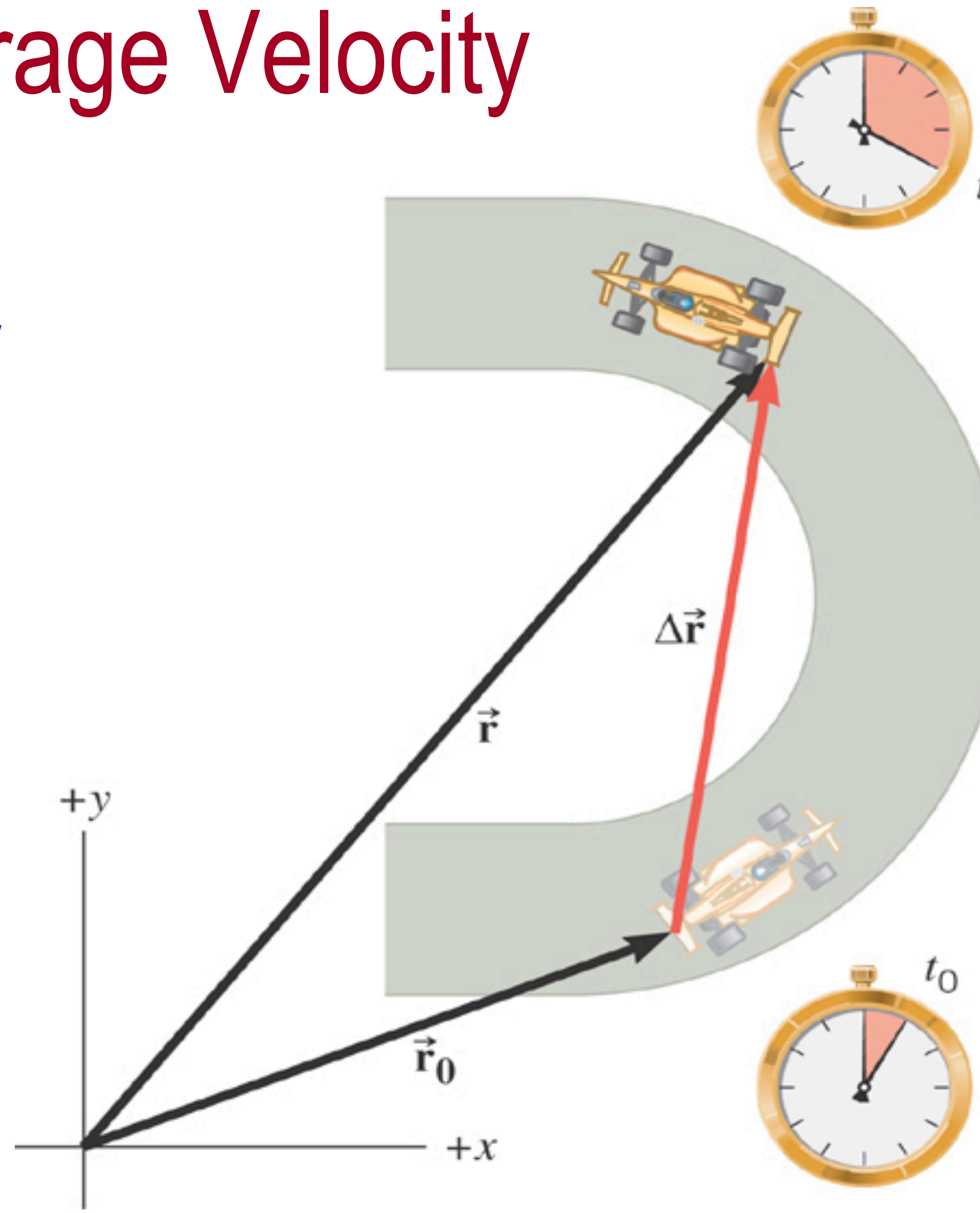
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

Average velocity is the displacement divided by the elapsed time.

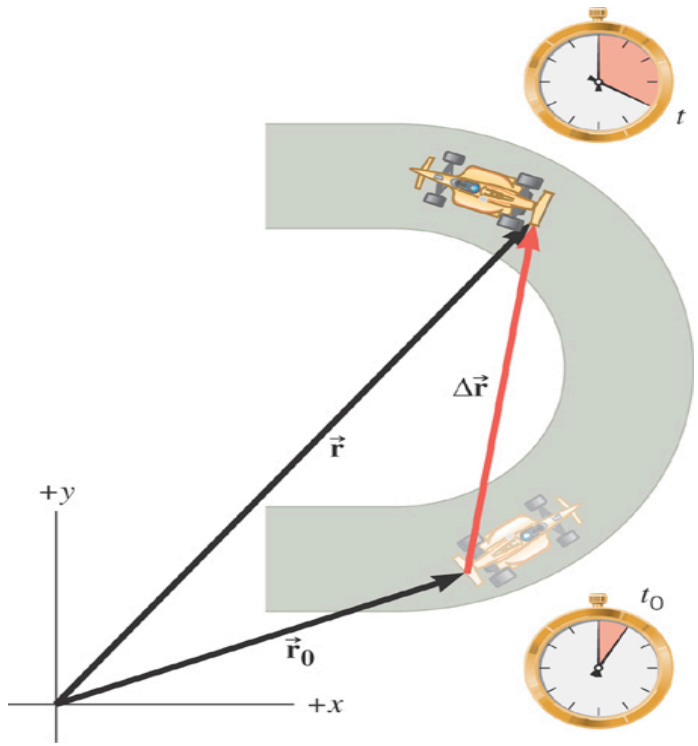
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

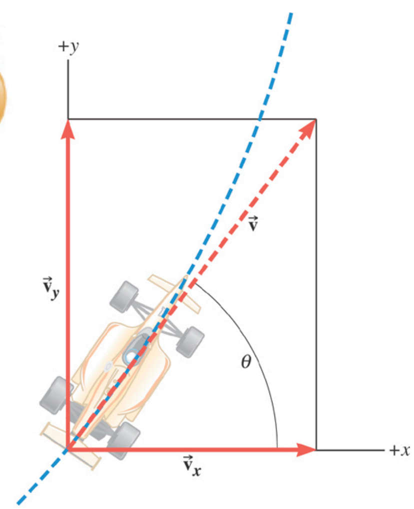


2D Instantaneous Velocity

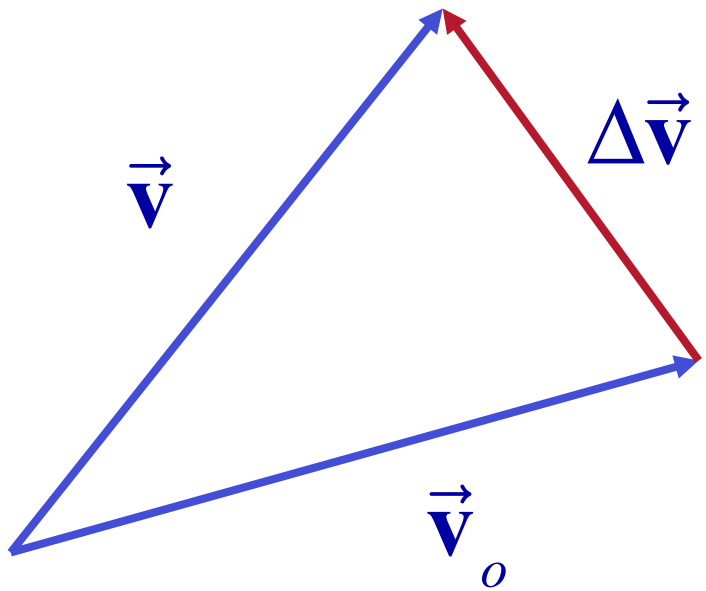
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration: $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$

What is the difference between 1D and 2D quantities?