PHYS 1441 – Section 001
Lecture #6

Wednesday, June 11, 2014
Dr. Jaehoon Yu

- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion

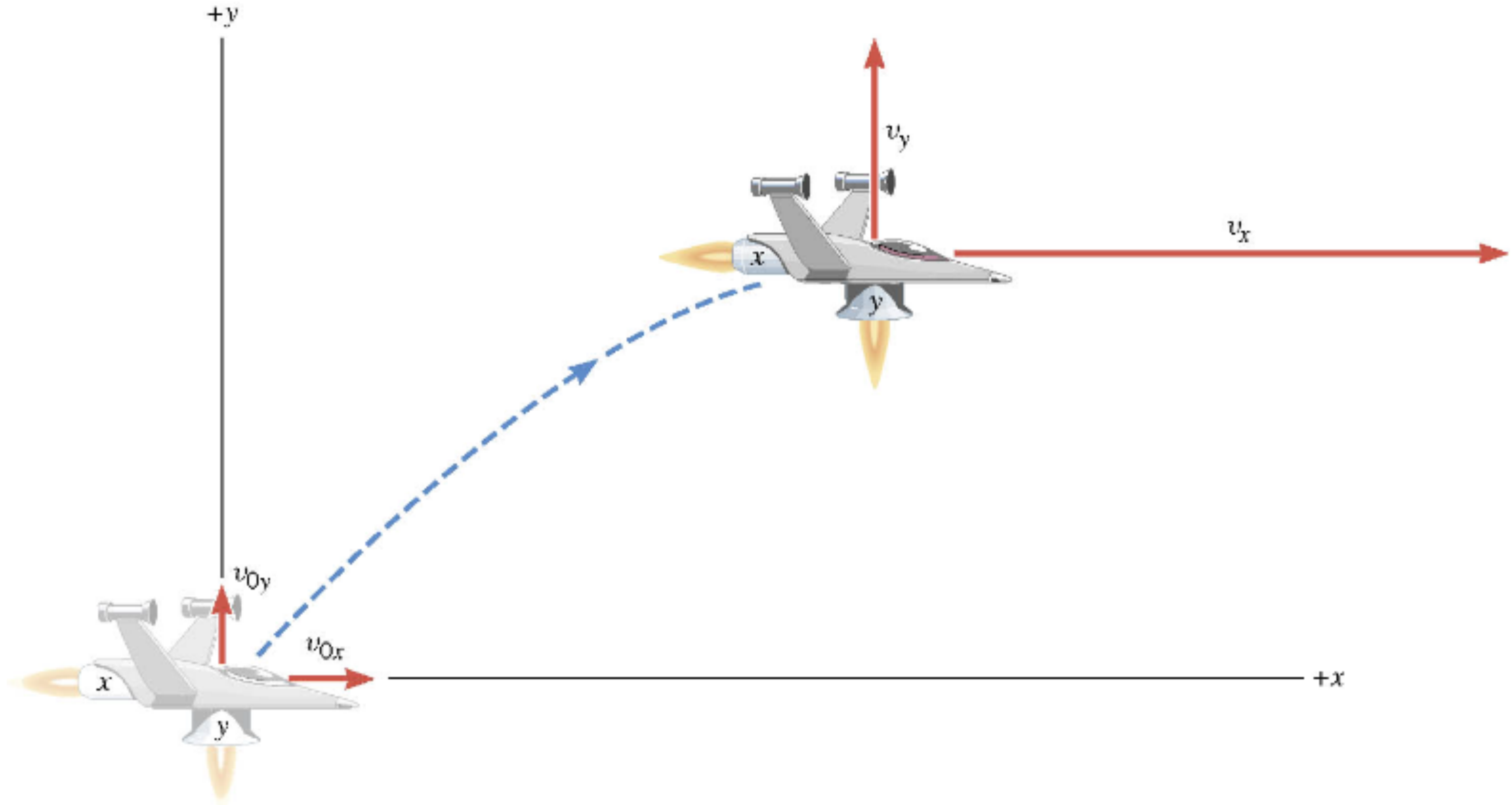
Announcements

- Reading Assignment
  – CH3.7
- Quiz #2
  – Beginning of the class tomorrow, Thursday, June, 12
  – Covers CH3.1 to what we finish today, Wednesday, June 11
  – Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  – You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam ➔ no solutions, derivations or definitions!
    - No additional formulae or values of constants will be provided!
- 1st Term exam results
  – Class average: 61.2/99
    - Equivalent to 61.8/100
  – Top score: 99/99

Reminder: Special Project #2

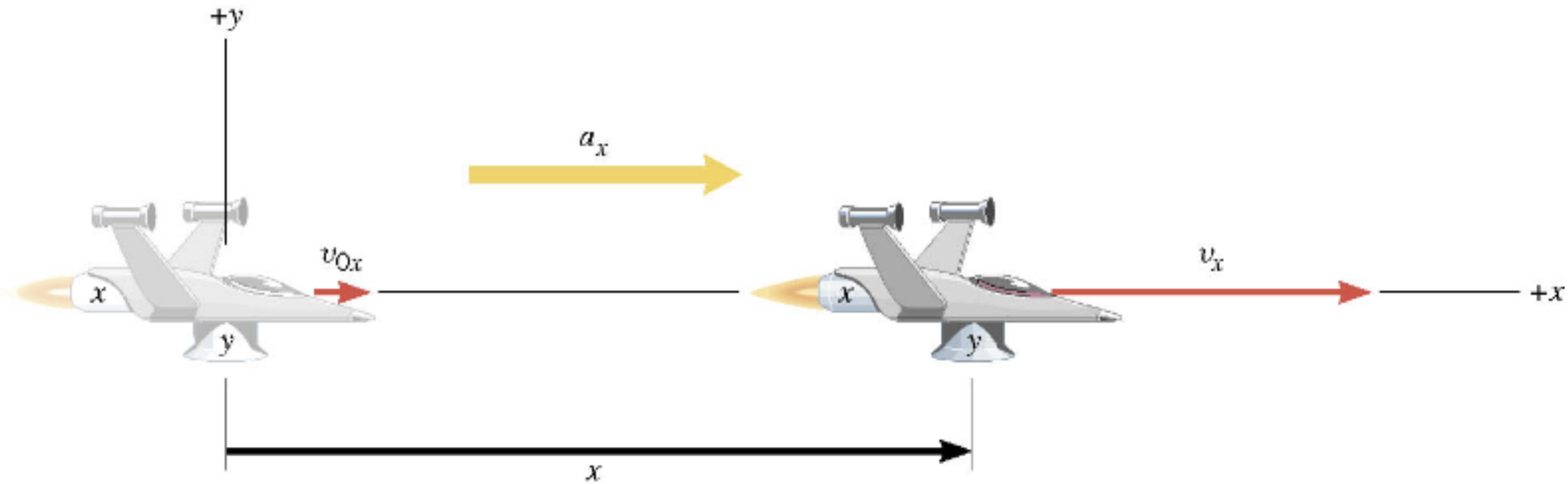
- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Monday, June 16
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in this lecture note and in the book!

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one. (superposition…)

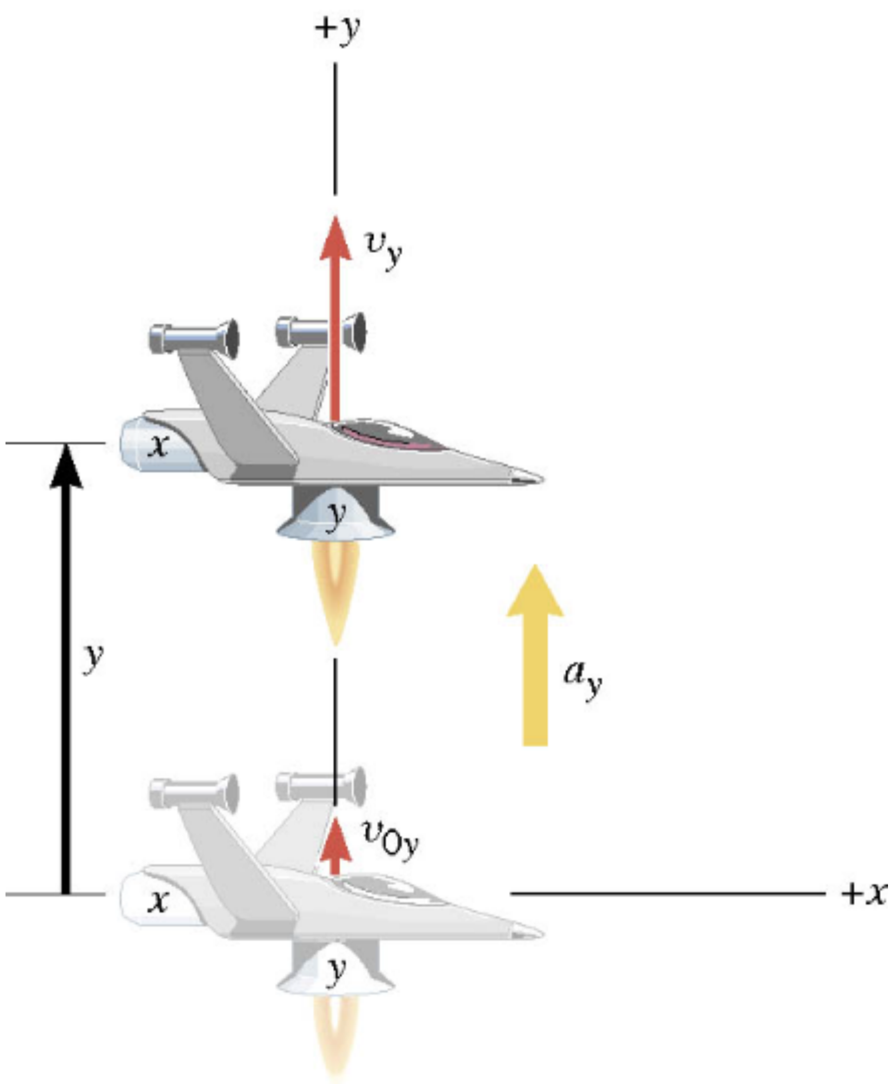
Motion in horizontal direction (x)



$$v_x = v_{xo} + a_x t \qquad x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

Motion in vertical direction (y)



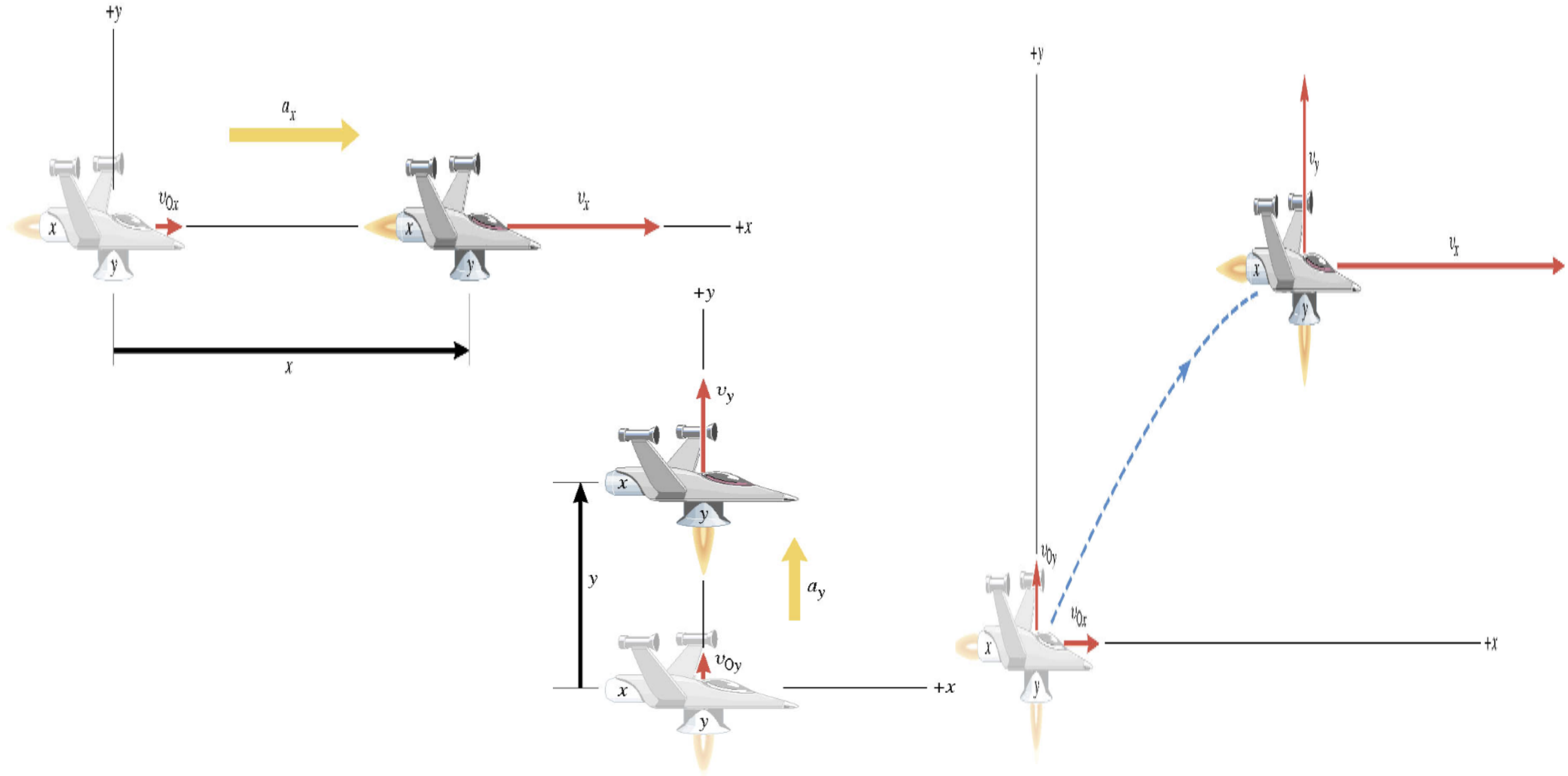
$$v_y = v_{yo} + a_y t$$

$$y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \tfrac{1}{2}a_y t^2$$

A Motion in 2 Dimension



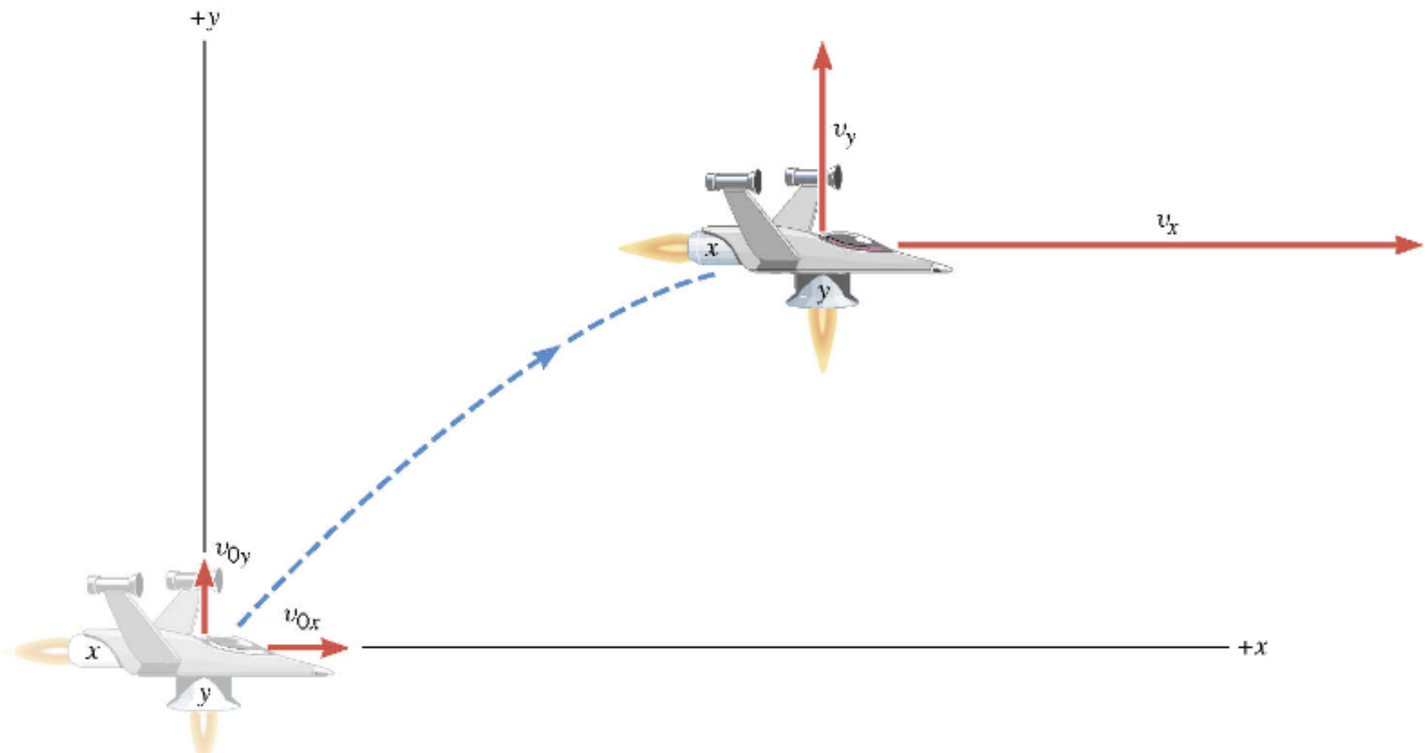
Imagine you add the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component	y-component
$v_x = v_{xo} + a_x t$	$v_y = v_{yo} + a_y t$
$x = \frac{1}{2}\left(v_{xo} + v_x\right) t$	$y = \frac{1}{2}\left(v_{yo} + v_y\right) t$
$v_x^2 = v_{xo}^2 + 2a_x x$	$v_y^2 = v_{yo}^2 + 2a_y y$
$\Delta x = v_{xo} t + \frac{1}{2} a_x t^2$	$\Delta y = v_{yo} t + \frac{1}{2} a_y t^2$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-). Normal convention is the right-hand rule.
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y* *separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the *x* direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

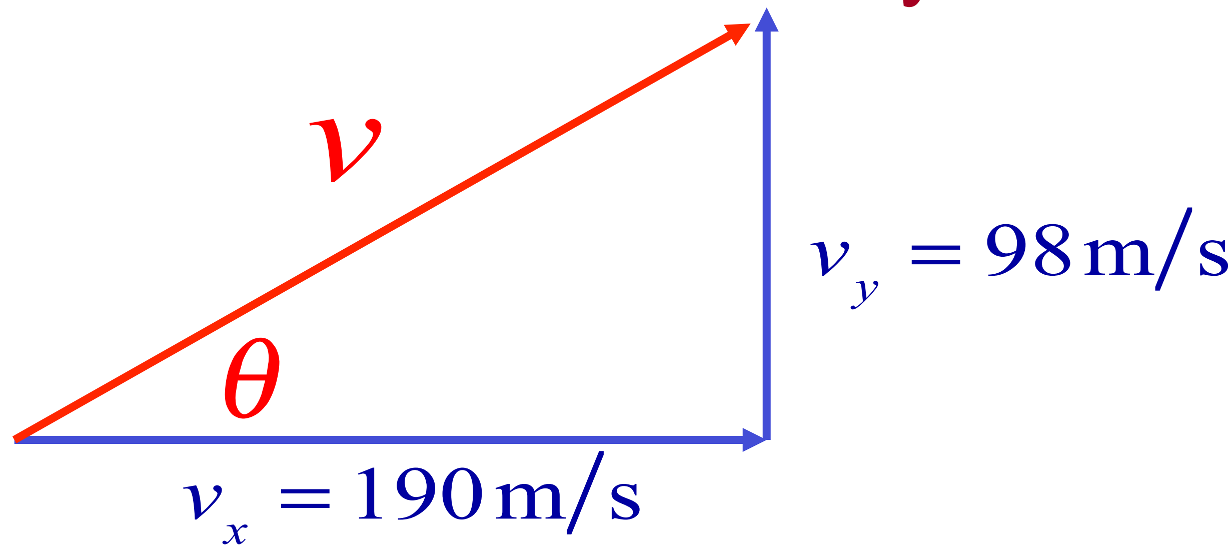
Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{oy} t + \tfrac{1}{2} a_y t^2$$
$$= (14\,\text{m/s})(7.0\ \text{s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\ \text{s})^2 = +390\ \text{m}$$

$$v_y = v_{oy} + a_y t$$
$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\ \text{s}) = +98\,\text{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

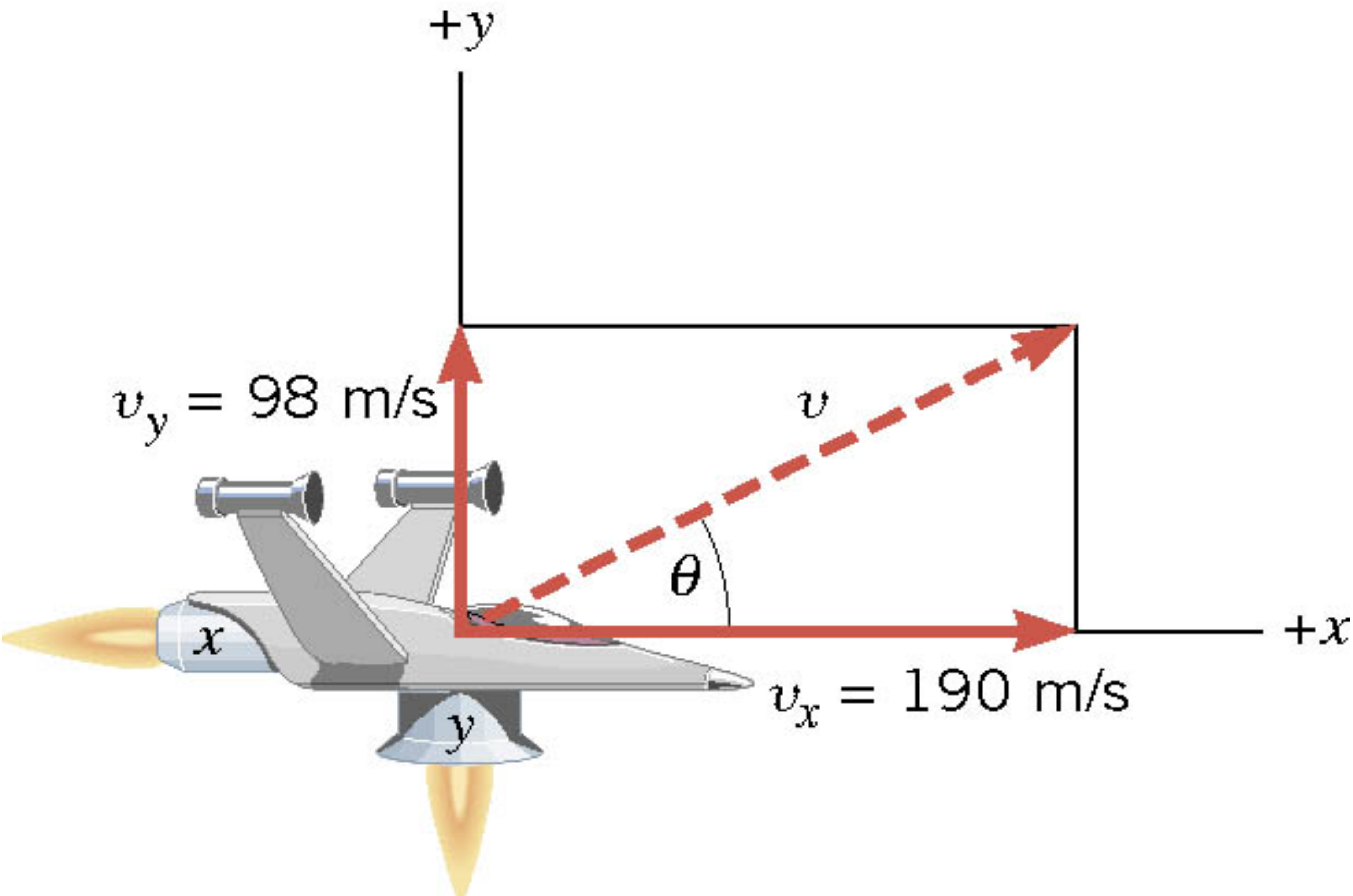
$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x \vec{i} + v_y \vec{j} = (190\vec{i} + 98\vec{j})\,m/s$$

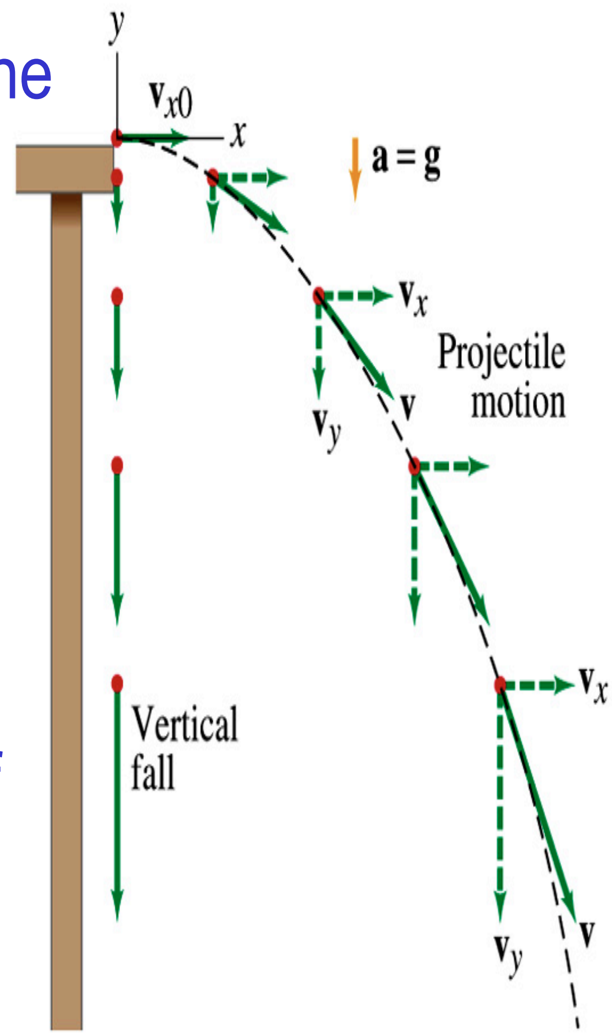
If we visualize the motion…



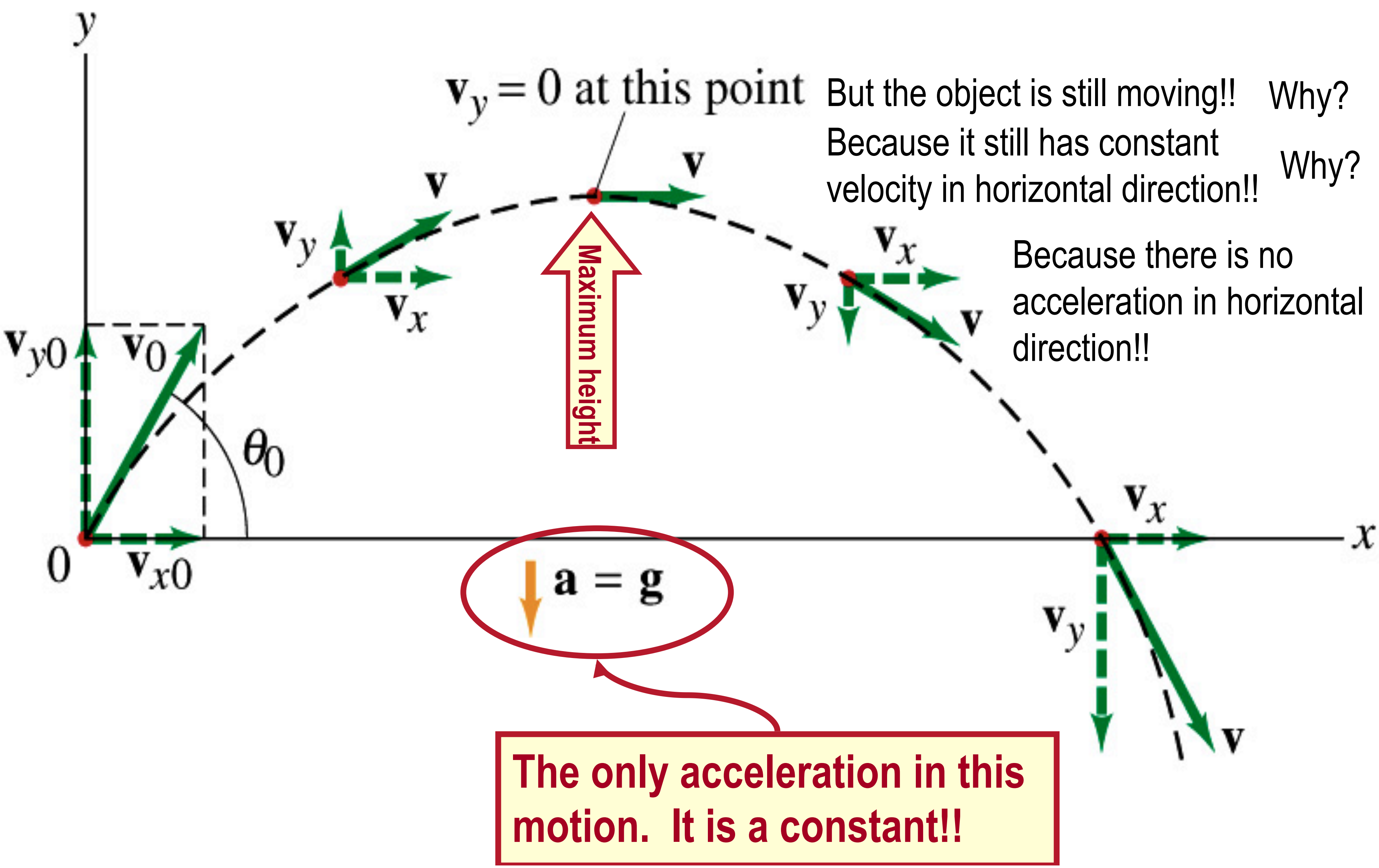
What is the Projectile Motion?

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is **constant** over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (**no acceleration**) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration (g)

$$v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$$



Projectile Motion



Kinematic Equations for a projectile motion

x-component

$$a_x = 0$$

$$v_x = v_{xo}$$

$$\Delta x = v_{xo} t$$

$$v_{x0}^2 = v_{xo}^2$$

$$\Delta x = v_{xo} t$$

y-component

$$a_y = -\left|\vec{g}\right| = -9.8\, m/s^2$$

$$v_y = v_{yo} - gt$$

$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right) t$$

$$v_y^2 = v_{yo}^2 - 2gy$$

$$\Delta y = v_{yo} t - \tfrac{1}{2} g t^2$$

Show that a projectile motion is a parabola!!!

x-component $$v_{xi} = v_i \cos\theta_i$$

y-component $$v_{yi} = v_i \sin\theta_i$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i \cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2}$$

What kind of parabola is this?

Example for a Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance from the original position when the ball lands.

Which component determines the flight time and the distance?

Flight time is determined by the y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore \ t \approx 8\sec$$

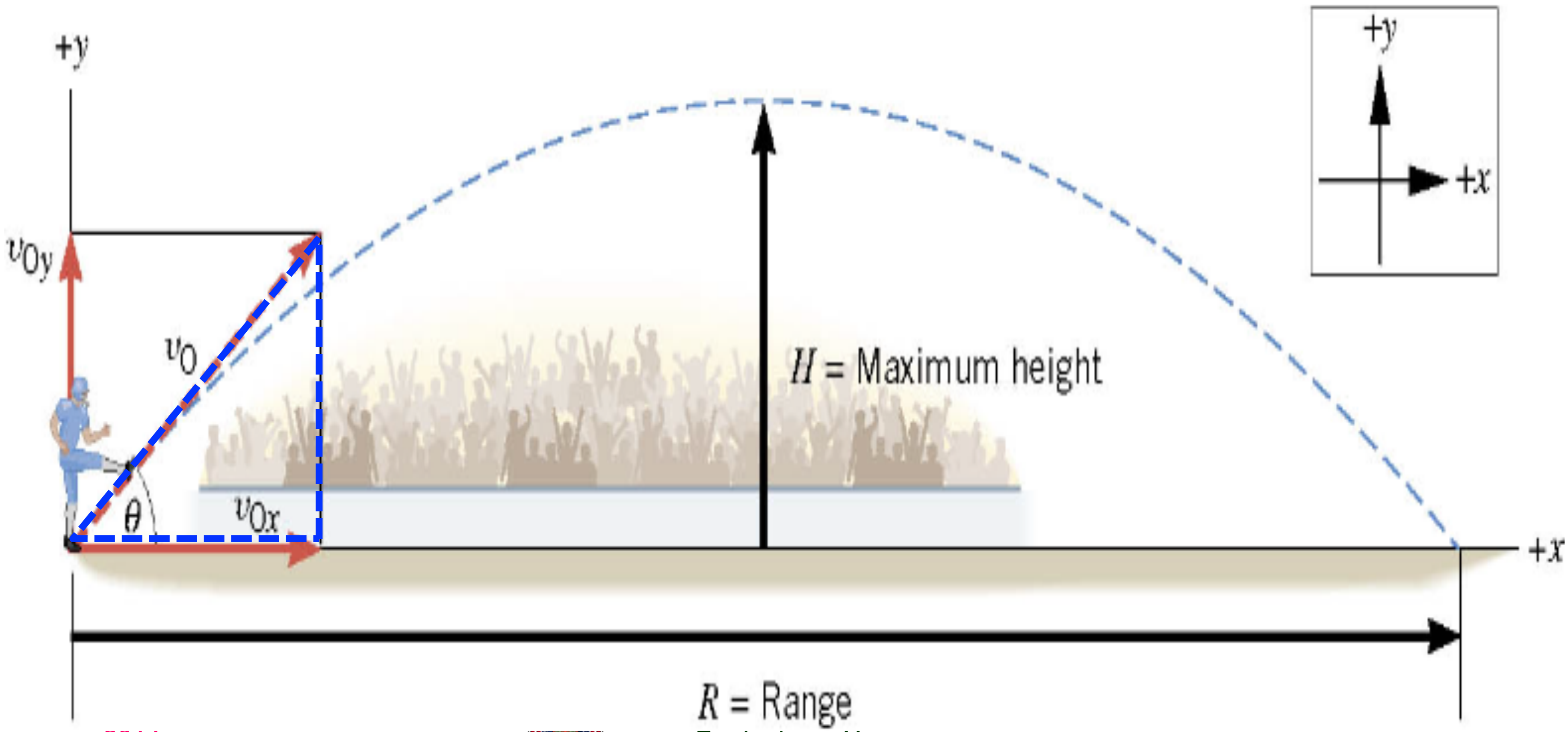
Why isn't t 0 the solution?

Distance is determined by the x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

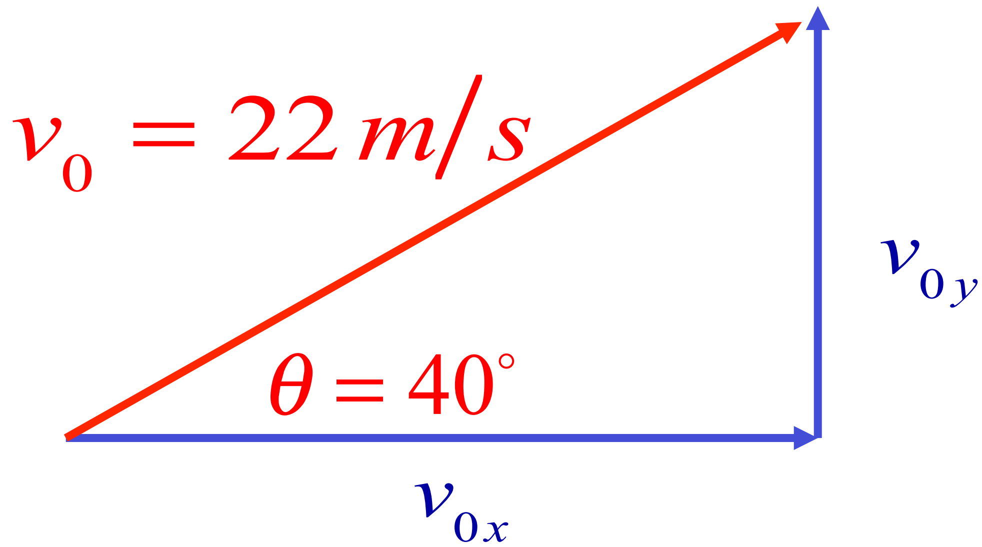
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Ex.3.5 The Height of a Kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



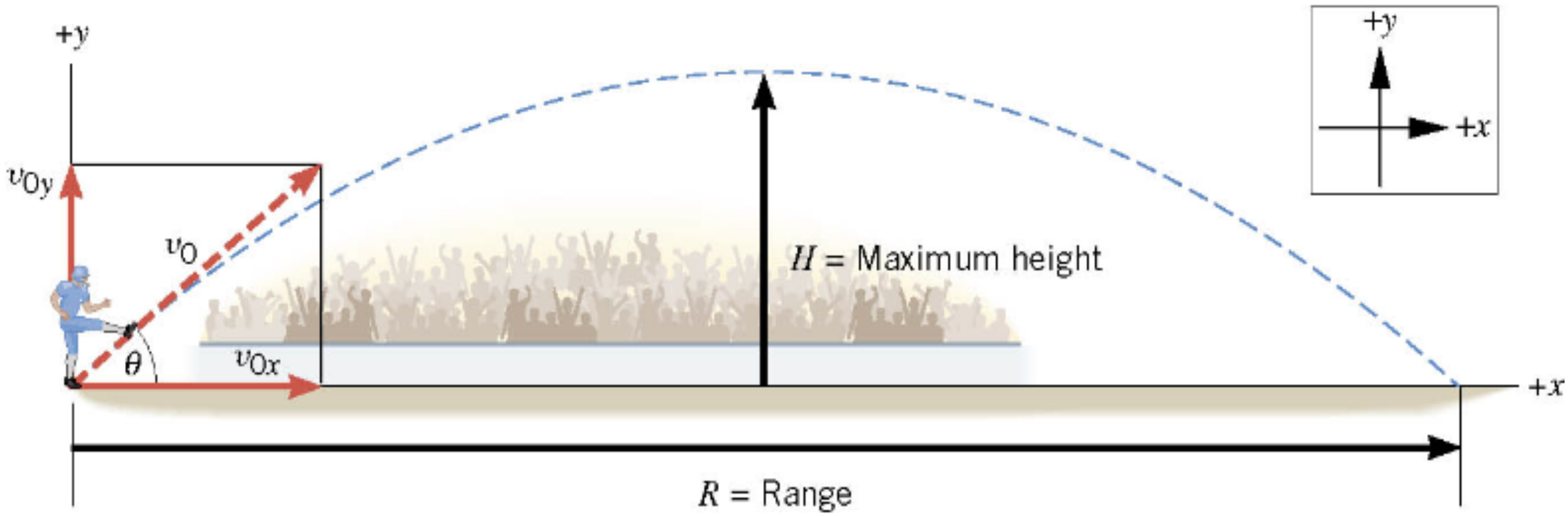
First, the initial velocity components



$$v_{0x} = v_o \cos\theta = \left(22\,\mathrm{m/s}\right)\cos 40^\circ = 17\,\mathrm{m/s}$$

$$v_{0y} = v_o \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^\circ = 14\,\mathrm{m/s}$$

Motion in y-direction is of the interest..



y	a_y	v_y	v_{0y}	t
?	$-9.8\ m/s^2$	0 m/s	+14 m/s	

Now the nitty, gritty calculations…

y	a_y	v_y	v_{0y}	t
?	-9.80 m/s^2	0	14 m/s	

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

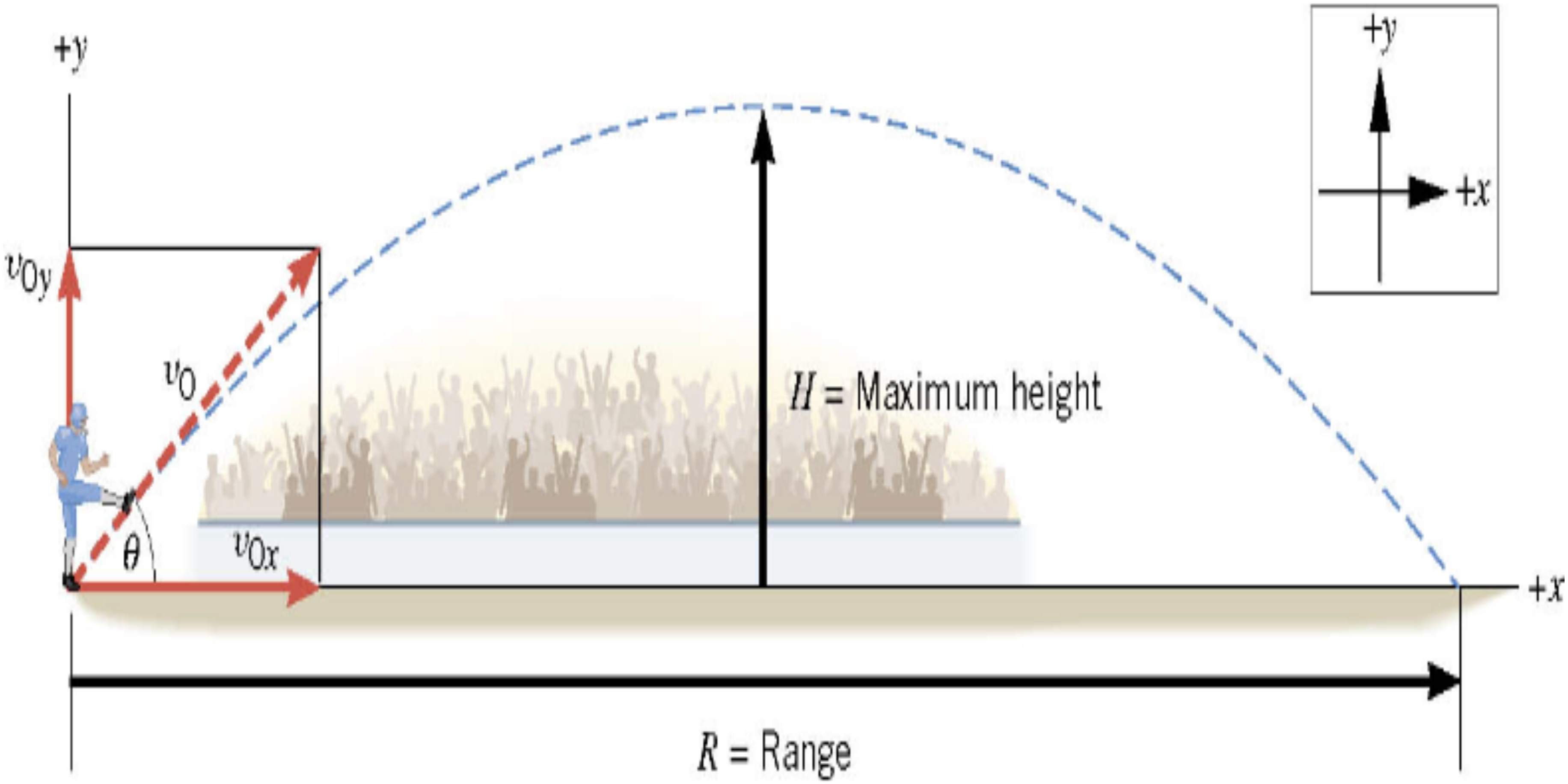
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

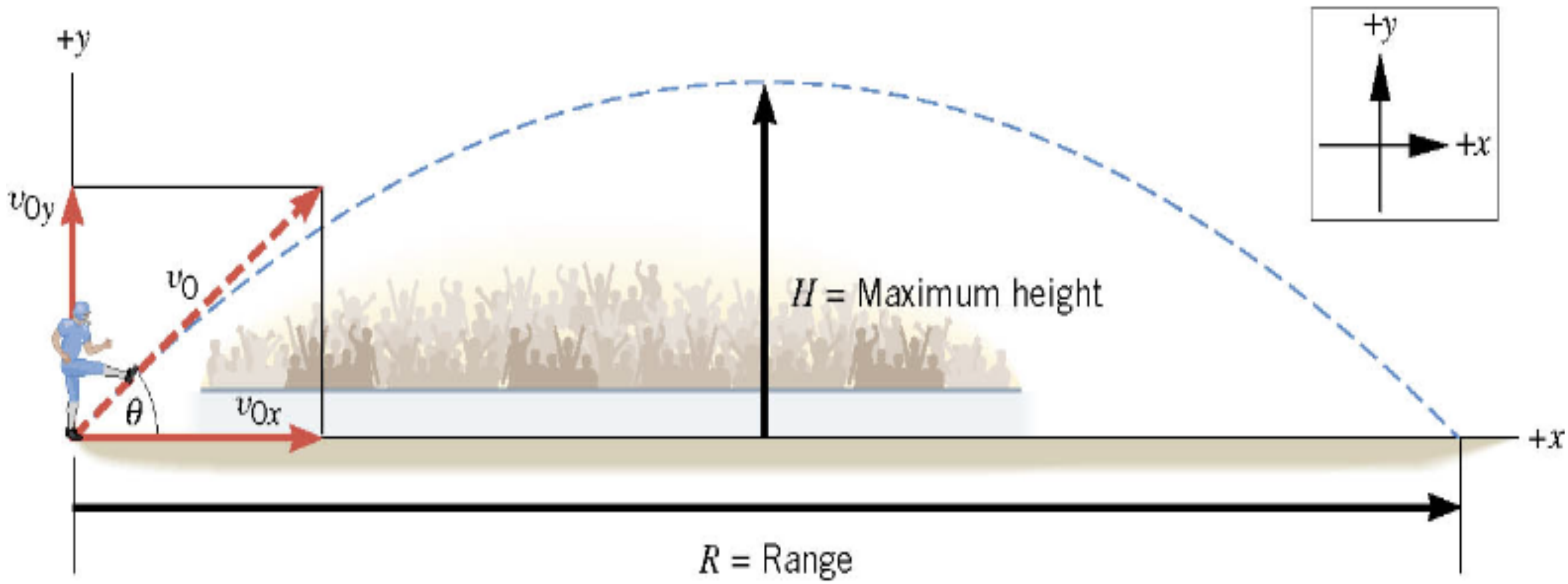
$$y = \frac{0 - \left(14\,\mathrm{m/s}\right)^2}{2\left(-9.8\,\mathrm{m/s^2}\right)} = +10\ \mathrm{m}$$

Ex.3.9 extended: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



What is y when it reached the max range?



y	a_y	v_y	v_{oy}	t
0 m	-9.80 m/s^2		14 m/s	?

Now solve the kinematic equations in y direction!!

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

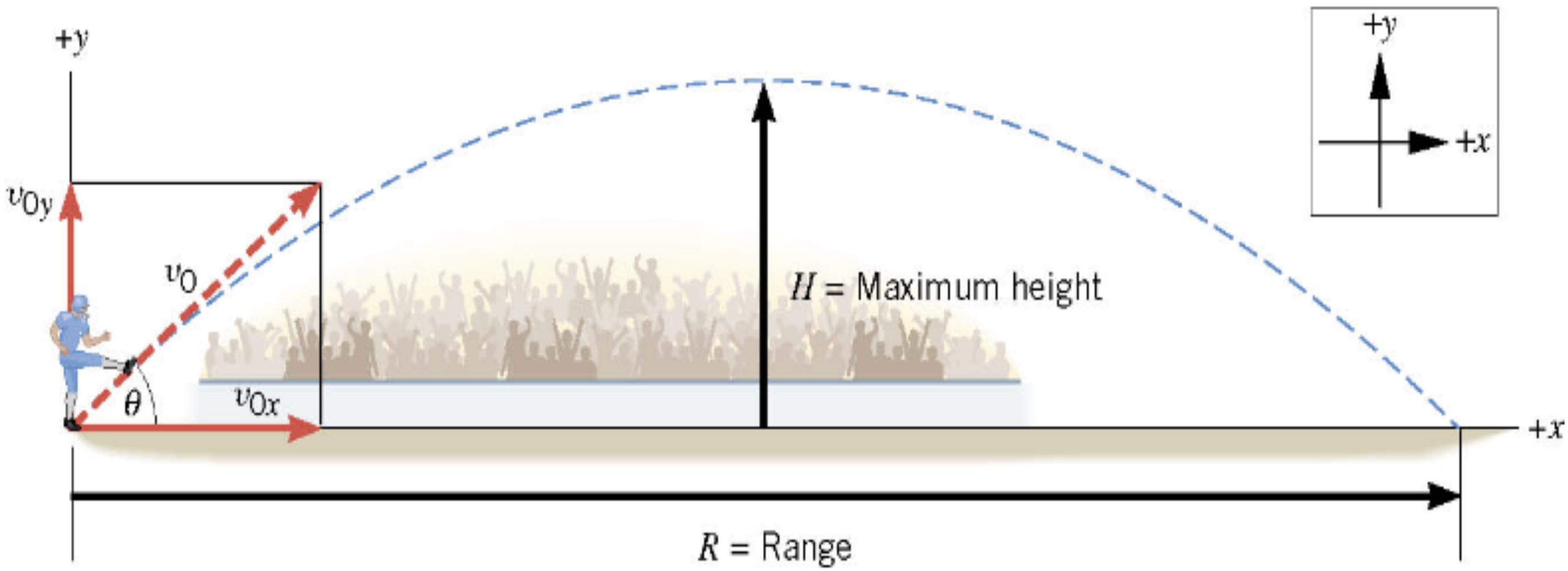
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \xrightarrow{\text{Since } y=0} 0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

$$v_{oy} + \tfrac{1}{2}a_y t = 0 \xrightarrow{\text{Solve for t}} t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2\cdot 14}{-9.8} = 2.9s$$

Ex.3.9 The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = \left(17\ \mathrm{m/s}\right)\left(2.9\ \mathrm{s}\right) = +49\ \mathrm{m}$$