PHYS 1441 – Section 001
Lecture #8

Monday, June 16, 2014

Dr. Jaehoon Yu

- What is the Force?
- Newton's Second Law
- Free Body Diagram
- Newton's Third Law
- Categories of forces
- Application of Newton's Laws
- Uniform Circular Motion

Today's homework is homework #5, due 11pm, Friday, June 20!!

Announcements

- Mid-term exam
  - In the class tomorrow, Tuesday, June 17
  - Comprehensive exam
    - Covers CH1.1 to what we finish today + Appendix A
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam → no solutions, derivations or definitions!
    - No additional formulae or values of constants will be provided!
- Quiz #2 results
  - Class average: 26.8/45
    - Equivalent to: 59.6/100
    - Previous quiz: 69.3/100
  - Top score: 45

Force

We've been learning kinematics; describing motion without understanding what the cause of the motion is. Now we are going to learn dynamics!!

Can someone tell me what FORCE is?

FORCE is what causes an object to move.

The above statement is not entirely correct. Why?

Because when an object is moving with a constant velocity no force is exerted on the object!!!

FORCEs are what cause changes to the velocity of an object!!

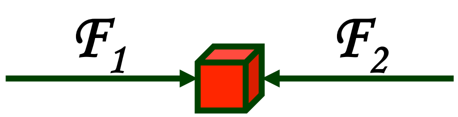
What does this statement mean?

When there is force, there is change of velocity!!

What does the force cause? An acceleration.!!

What happens if there are several forces being exerted on an object?

Forces are vector quantities, so vector sum of all forces, the NET FORCE, determines the direction of the acceleration of the object.



NET FORCE, $\mathbf{F} = F_1 + F_2$

When the net force on an object is **0***, it has constant velocity and is at its equilibrium!!*

More Forces

There are various types of forces

Contact Forces: Forces exerted by physical contact of objects

Examples of Contact Forces: Baseball hit by a bat, Car collisions

Field Forces: Forces exerted without physical contact of objects

Examples of Field Forces: Gravitational Force, Electro-magnetic force

What are possible ways to measure strength of a force?

A calibrated spring whose length changes linearly with the force exerted .

Forces are vector quantities, so the addition of multiple forces must be done following the rules of vector additions.

Newton's First Law and Inertial Frames

Aristotle (384-322BC): *A natural state of a body is rest. Thus force is required to move an object. To move faster, ones needs larger forces.*

Galileo's statement on natural states of matter: *Any velocity once imparted to a moving body will be rigidly maintained as long as the external causes of retardation are removed!!*

Galileo's statement is formulated by Newton into the **1st law of motion (Law of Inertia)**: *In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.*

What does this statement tell us?

- When no net force is exerted on an object, the acceleration of the object is 0.
- Any isolated object, the object that do not interact with its surroundings, is either at rest or moving at a constant velocity.
- Objects would like to keep its current state of motion, as long as there are no net force that interferes with the motion. This tendency is called the Inertia.

A frame of reference that is moving at a constant velocity is called the ***Inertial Frame***

Is a frame of reference with an acceleration an *Inertial Frame?*

NO!

Mass

Mass: *A measure of the inertia of an object* or *the quantity of matter*

- Independent of the object's surroundings: The same no matter where you go.
- Independent of the method of measurement: The same no matter how you measure it.

The heavier the object, the bigger the inertia !!

It is harder to make changes of motion of a heavier object than a lighter one.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}$$

Note that the mass and the weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon but the mass won't!!

Unit of mass? kg

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\vec{a} = \frac{\sum_i \vec{F}_i}{m}$$

From this we obtain

$$\sum_i \vec{F}_i = m\vec{a}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component must also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

Unit of the Force

From the vector expression in the previous page, what do you conclude the dimension and the unit of the force are?

$$\sum_i \vec{F_i} = m\vec{a}$$

The dimension of force is $[m][a] = [M][LT^{-2}]$

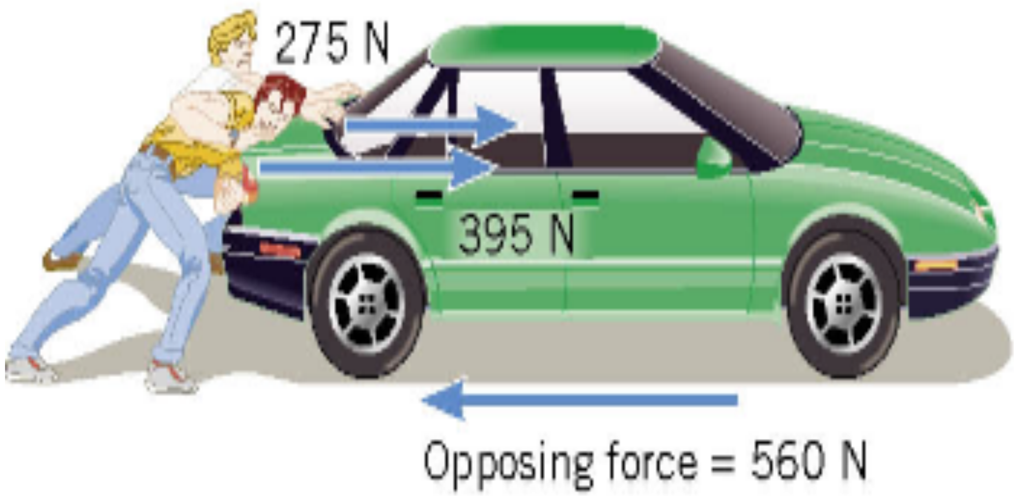
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = (\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right) = kg \cdot m / s^2$

For ease of use, we define a new derived unit called, Newton (N)

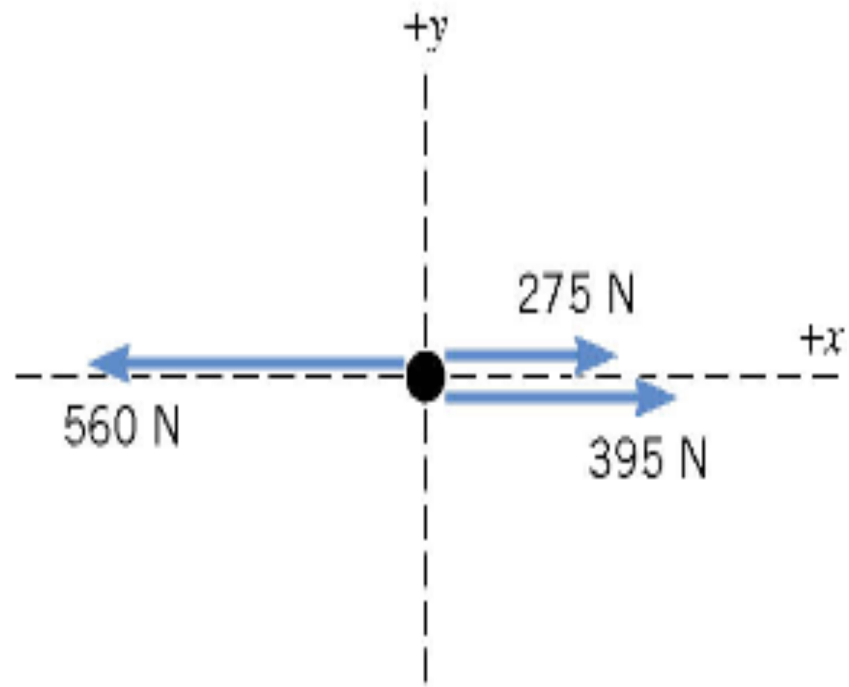
$$1N \equiv 1kg \cdot m / s^2 \approx \frac{1}{4} lbs$$

Free Body Diagram

A ***free-body-diagram*** is a diagram that represents the object and the forces that act on it.

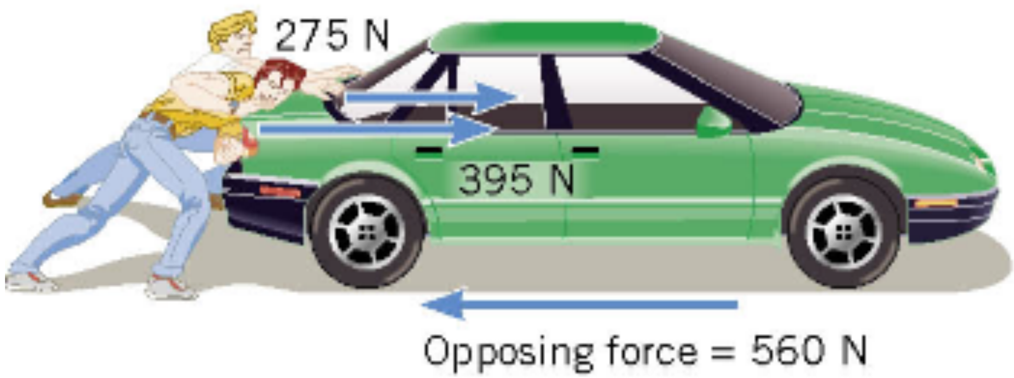


(a)

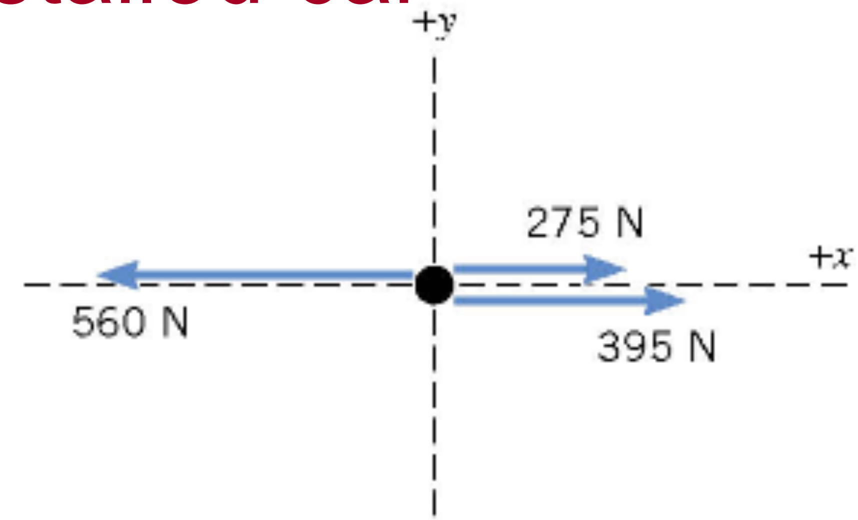


(b) Free-body diagram of the car

Ex. Pushing a stalled car



(a)



(b) Free-body diagram of the car

What is the net force in this example?

F= 275 N + 395 N – 560 N = +110 N

Which direction? The + x axis of the coordinate system.

What is the acceleration the car receives?

If the mass of the car is 1850 kg, then by Newton's second law, the acceleration is

$$\sum \vec{F} = m\vec{a}$$

Since the motion is in 1 dimension

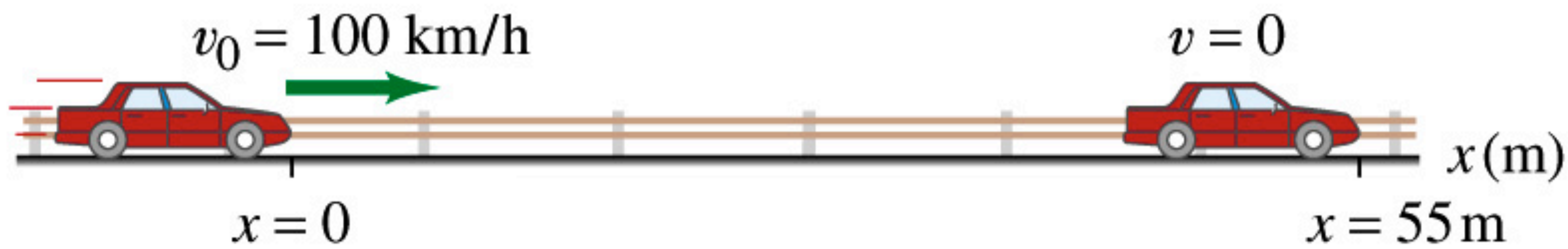
$$\sum F = ma$$

Now we solve this equation for a

$$a = \frac{\sum F}{m} = \frac{+110\,\mathrm{N}}{1850\,\mathrm{kg}} = +0.059\,\mathrm{m/s^2}$$

Example 4.3

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force? **Acceleration!!**

What are given? Initial speed: $v_{xi} = 100km/h = 28m/s$ Final speed: $v_{xf} = 0m/s$

Displacement: $\Delta x = x_f - x_i = 55m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Acceleration →

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{-(28m/s)^2}{2(55m)} = -7.1m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times (-7.1m/s^2) = -1.1 \times 10^4 N$$

Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2ma_x} = \frac{m(v_{xf}^2 - v_{xi}^2)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Vector Nature of the Force

The direction of the force and the acceleration vectors can be taken into account by using x and y components.

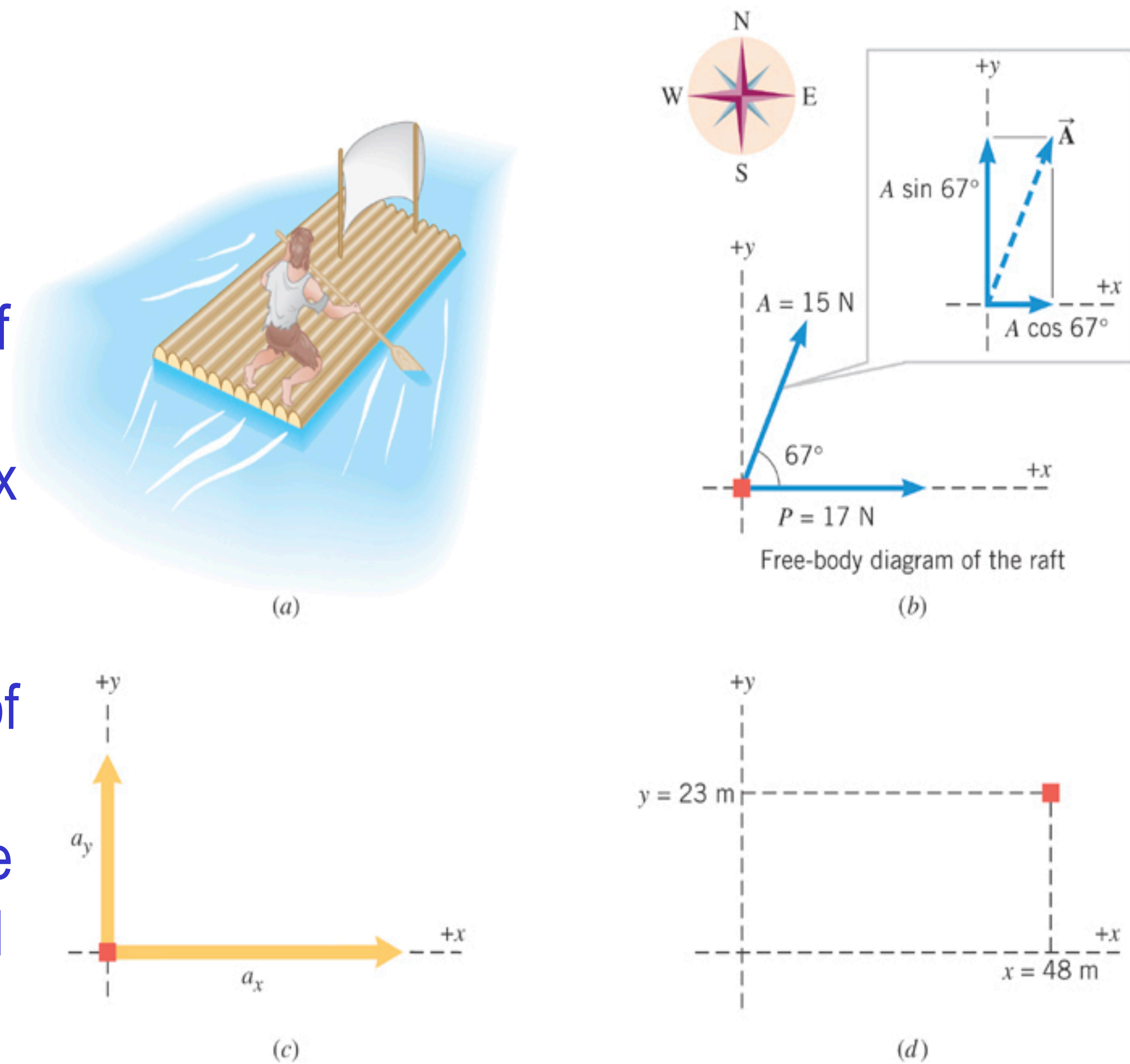
$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$$

is equivalent to

$$\sum F_y = ma_y \qquad \sum F_x = ma_x$$

Ex. Stranded man on a raft

A man is stranded on a raft (mass of man and raft = 1300kg) as shown in the figure. By paddling, he causes an average force P of 17N to be applied to the raft in a direction due east (the +x direction). The wind also exerts a force A on the raft. This force has a magnitude of 15N and points 67° north of east. Ignoring any resistance from the water, find the x and y components of the rafts acceleration.



First, let's compute the net force on the raft as follows:

Force	*x* component	*y* component
$\vec{\mathbf{P}}$	+17 N	0 N
$\vec{\mathbf{A}}$	$+(15\text{N})\cos 67^{\circ}$	$+(15\text{N})\sin 67^{\circ}$
$\vec{F} = \vec{P} + \vec{A}$	$+17+15\cos 67^{\circ} = +23(\text{N})$	$+15\sin 67^{\circ} = +14(\text{N})$

Now compute the acceleration components in x and y directions!!

$$a_x = \frac{\sum F_x}{m} = \frac{+23\ \text{N}}{1300\ \text{kg}} = +0.018\ \text{m/s}^2$$

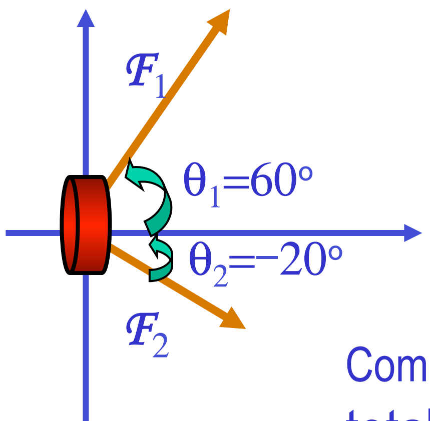
$$a_y = \frac{\sum F_y}{m} = \frac{+14\ \text{N}}{1300\ \text{kg}} = +0.011\ \text{m/s}^2$$

And put them all together for the overall acceleration:

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = \left(0.018\vec{i} + 0.011\vec{j}\right) m/s^2$$

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of the acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, **F1** and **F2,** as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right|\cos\theta_1 = 8.0\times\cos\left(60^{o}\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right|\sin\theta_1 = 8.0\times\sin\left(60^{o}\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right|\cos\theta_2 = 5.0\times\cos\left(-20^{o}\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right|\sin\theta_2 = 5.0\times\sin\left(-20^{o}\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2} = 34m/s^2$$

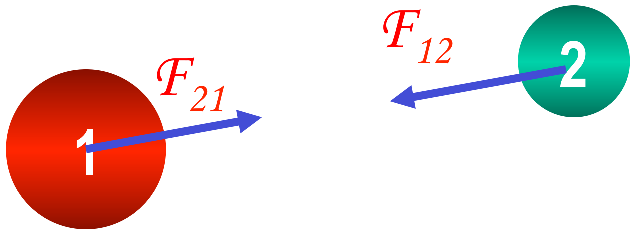
$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^{o}$$

Acceleration Vector *a*

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force F_{21} that object 2 exerts on object 1 is equal in magnitude and opposite in direction to the force F_{12} object 1 exerts on object 2.



$$\vec{F}_{12} = -\vec{F}_{21}$$

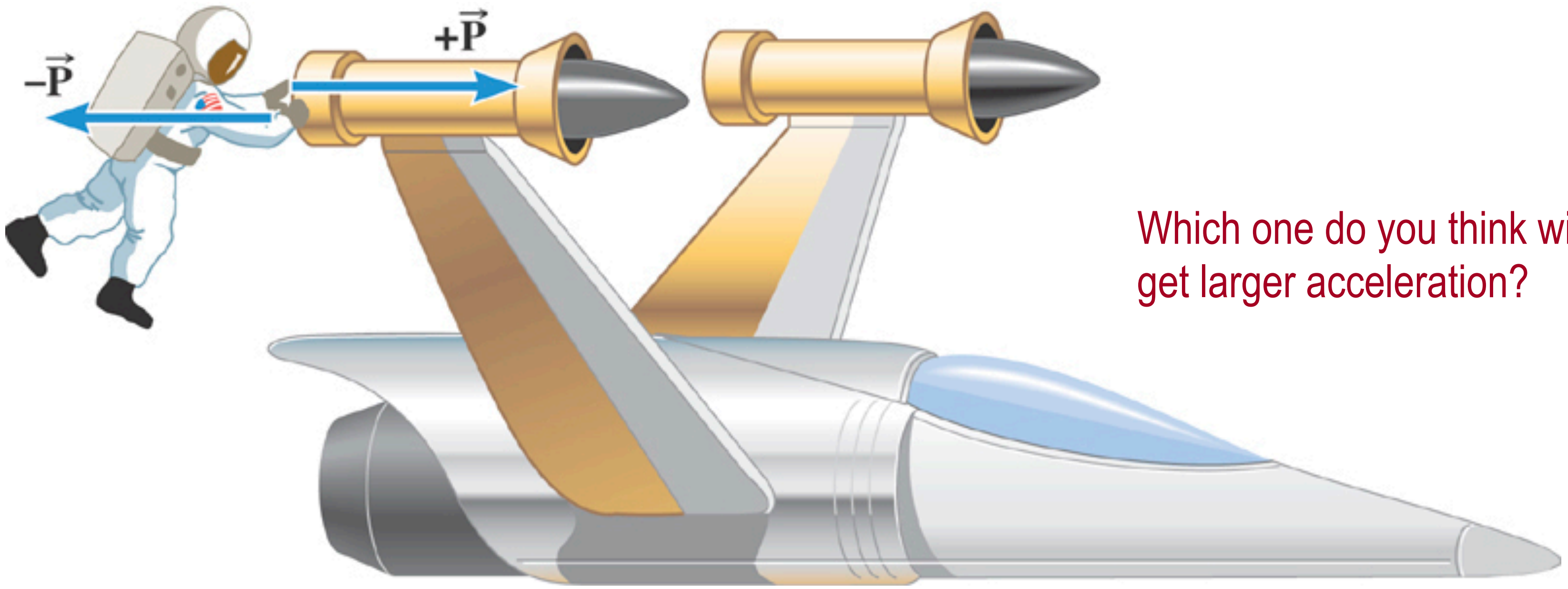
The reaction force is equal in magnitude to the action force but in opposite direction. These two forces always act ***on different objects.***

What is the reaction force to the force of a free falling object?

The gravitational force the object exerts on the Earth!

Stationary objects on top of a table has a reaction force (called the normal force) from table to balance the action force, the gravitational force.

Ex. The Accelerations Produced by Action and Reaction Forces



Which one do you think will get larger acceleration?

Suppose that the magnitude of the force P is 36 N. If the mass of the spacecraft is 11,000 kg and the mass of the astronaut is 92 kg, what are the accelerations?

Ex. continued

Force exerted on the space craft by the astronaut $\sum \vec{\mathbf{F}} = \vec{\mathbf{P}}$

Force exerted on the astronaut by the space craft $\sum \vec{\mathbf{F}} = -\vec{\mathbf{P}}$

space craft's acceleration

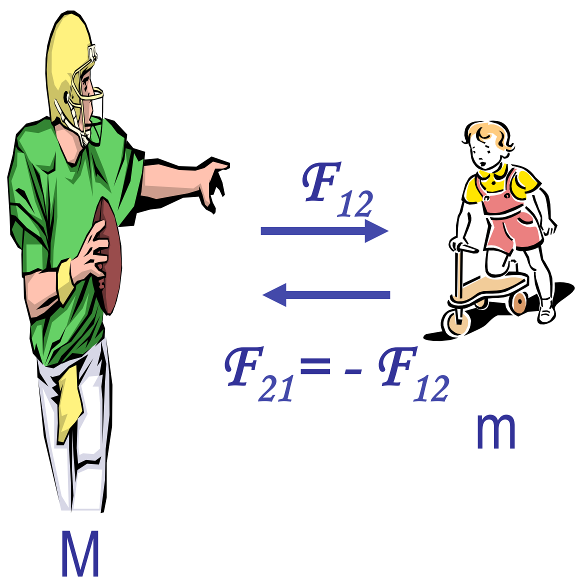
$$\vec{\mathbf{a}}_s = \frac{\vec{\mathbf{P}}}{m_s} = \frac{+36\,\vec{i}\ \mathrm{N}}{11{,}000\ \mathrm{kg}} = +0.0033\,\vec{i}\ \mathrm{m/s^2}$$

Astronaut's acceleration

$$\vec{\mathbf{a}}_A = \frac{-\vec{\mathbf{P}}}{m_A} = \frac{-36\,\vec{i}\ \mathrm{N}}{92\ \mathrm{kg}} = -0.39\,\vec{i}\mathrm{m/s^2}$$

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on **frictionless ice**. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \Rightarrow F_{12x} = ma_{bx}, \quad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \Rightarrow F_{21x} = Ma_{Mx}, \quad F_{21y} = Ma_{My} = 0$$

Since $\vec{F}_{12} = -\vec{F}_{21}$ and $\left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F$

Establish the equation $\Rightarrow$ $ma_{bx} = F = Ma_{Mx}$ **Divide by m** $\Rightarrow$ $a_{bx} = \frac{F}{m} = \frac{M}{m} a_{Mx}$

Example of Newton's 3rd Law, cnt'd

Man's velocity $$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

Boy's velocity $$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m} a_{Mx}t = \frac{M}{m} v_{Mxf}$$

So boy's velocity is higher than man's, if M>m, by the ratio of the masses.

b) Who moves farther while their hands are in contact?

Boy's displacement $$x_b = v_{bxi}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{2m} a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(\frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m} x_M$$

Man's displacement

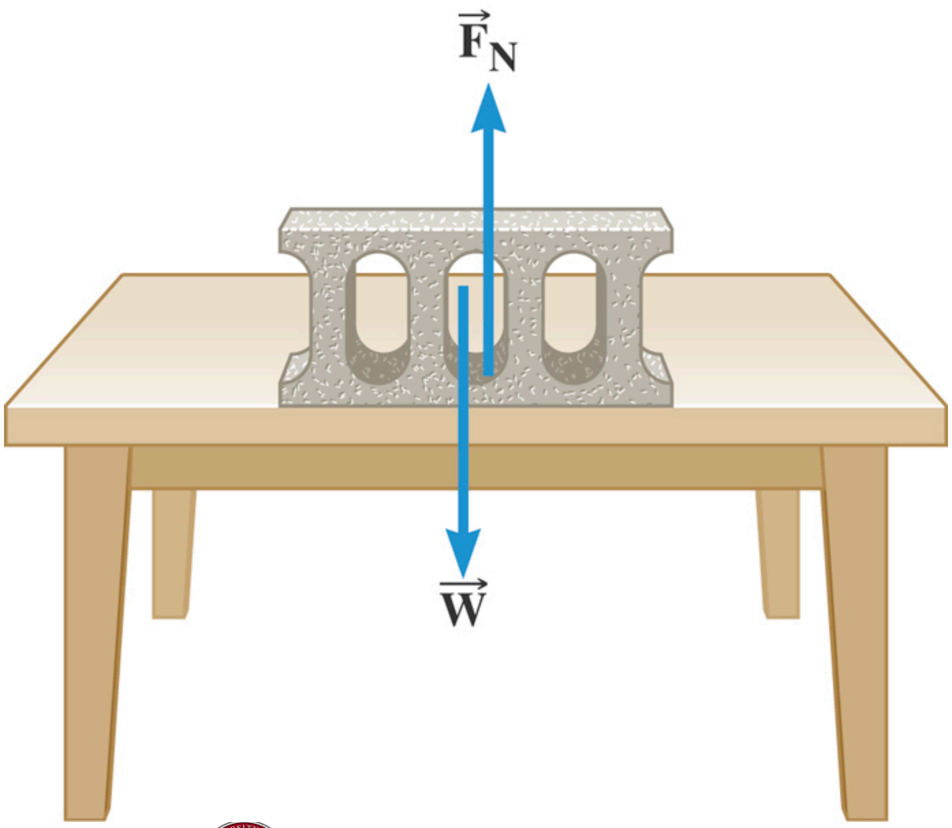
Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.

Categories of Forces

- Fundamental Forces: Truly unique forces that cannot be derived from any other forces
  - Total of three fundamental forces
    - Gravitational Force
    - Electro-Weak Force
    - Strong Nuclear Force
- Non-fundamental forces: Forces that can be derived from fundamental forces
  - Friction
  - Tension in a rope
  - Normal or support forces

The Normal Force

The normal force is one component of the force that a surface exerts on an object with which it is in contact – namely, the component that is **perpendicular to the surface**.



Some normal force exercises

Case 1: Hand pushing down on the book

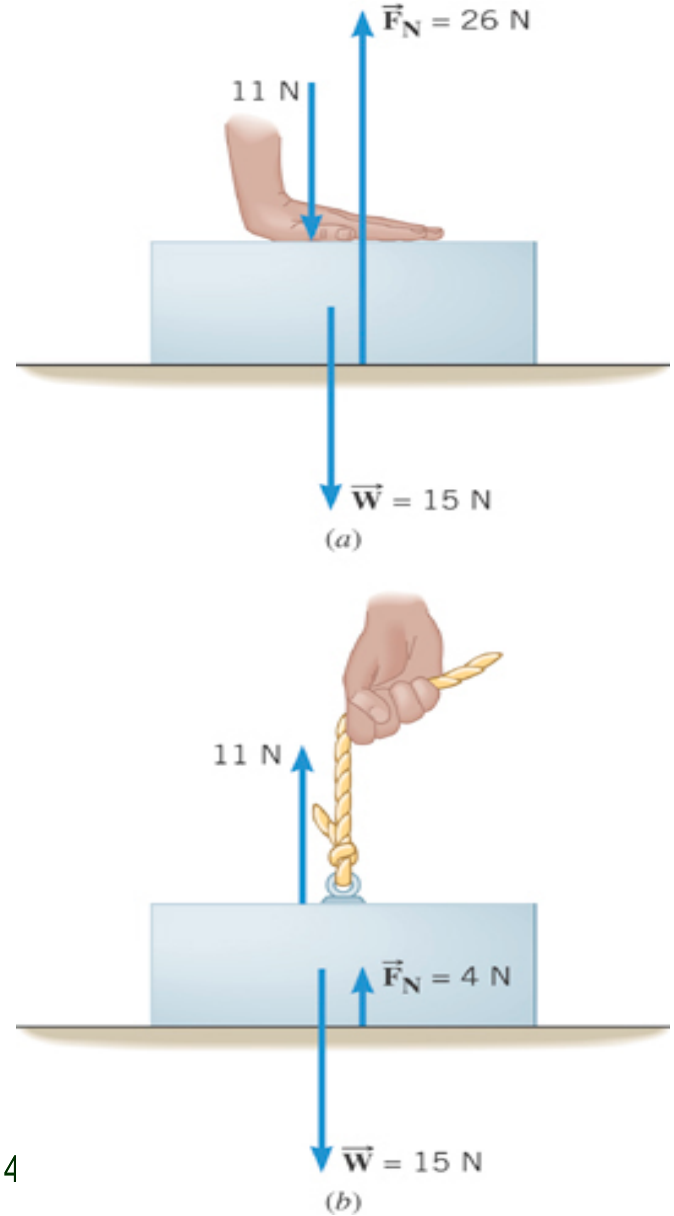
$$F_N - 11\text{ N} - 15\text{ N} = 0$$

$$F_N = 26\text{ N}$$

Case 2: Hand pulling up the book

$$F_N + 11\text{ N} - 15\text{ N} = 0$$

$$F_N = 4\text{ N}$$



Some Basic Information

When Newton's laws are applied, *external forces* are only of interest!!

Why?

Because, as described in Newton's first law, an object will keep its current motion unless non-zero net external force is applied.

Normal Force, ***n*:**

The force that reacts to action forces due to the surface structure of an object. Its direction is perpendicular to the surface.

Tension, ***T*:**

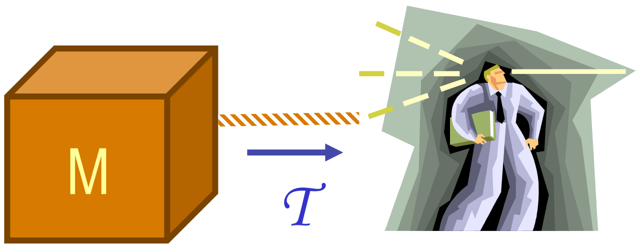
The reactionary force by a stringy object against an external force exerted on it.

Free-body diagram

A graphical tool which is a ***diagram of external forces on an object*** *and is extremely useful analyzing forces and motion!! Drawn only on an object.*

Applications of Newton's Laws

Suppose you are pulling a box on frictionless ice, using a rope.



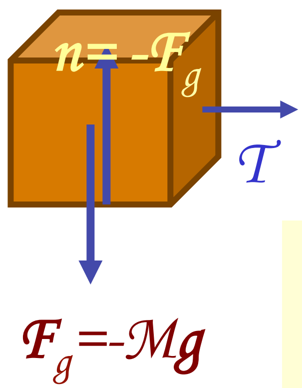
What are the forces being exerted on the box?

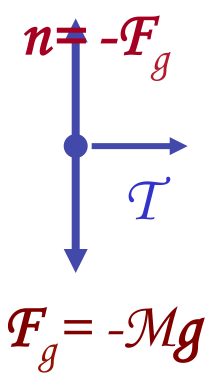
Gravitational force: $\boldsymbol{F_g}$

Normal force: $\boldsymbol{n}$

Tension force: $\boldsymbol{T}$

Free-body diagram





Total vector force:
$\boldsymbol{F}=\boldsymbol{F_g}+\boldsymbol{n}+\boldsymbol{T}=\boldsymbol{T}$

$$\sum F_x = T = Ma_x \qquad a_x = \frac{T}{M}$$

$$\sum F_y = -F_g + n = Ma_y = 0 \qquad a_y = 0$$

If T is a constant force, a_x, is constant

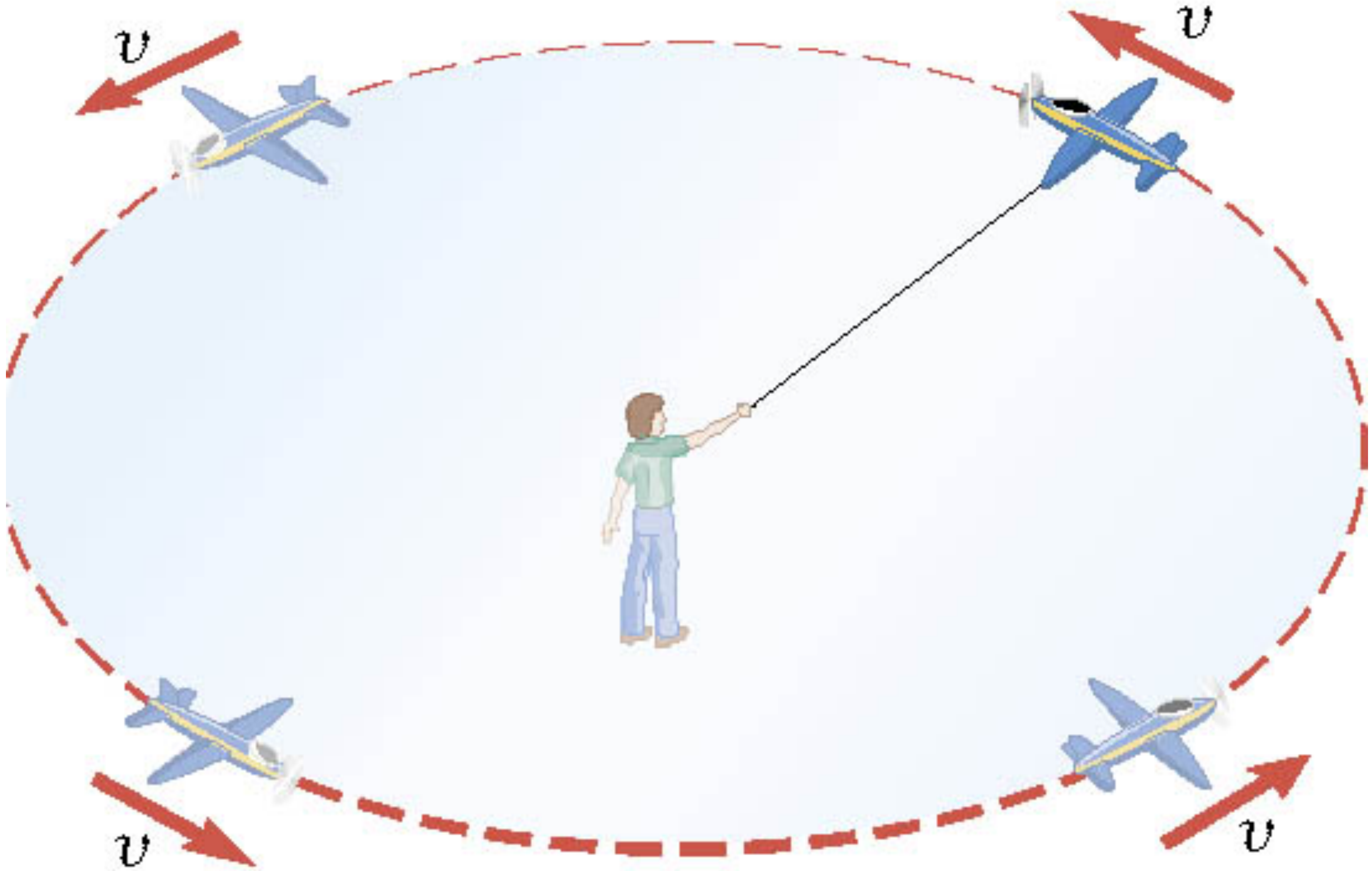
$$v_{xf} = v_{xi} + a_x t = v_{xi} + \left(\frac{T}{M}\right)t$$

$$\Delta x = x_f - x_i = v_{xi}t + \frac{1}{2}\left(\frac{T}{M}\right)t^2$$

What happened to the motion in y-direction?

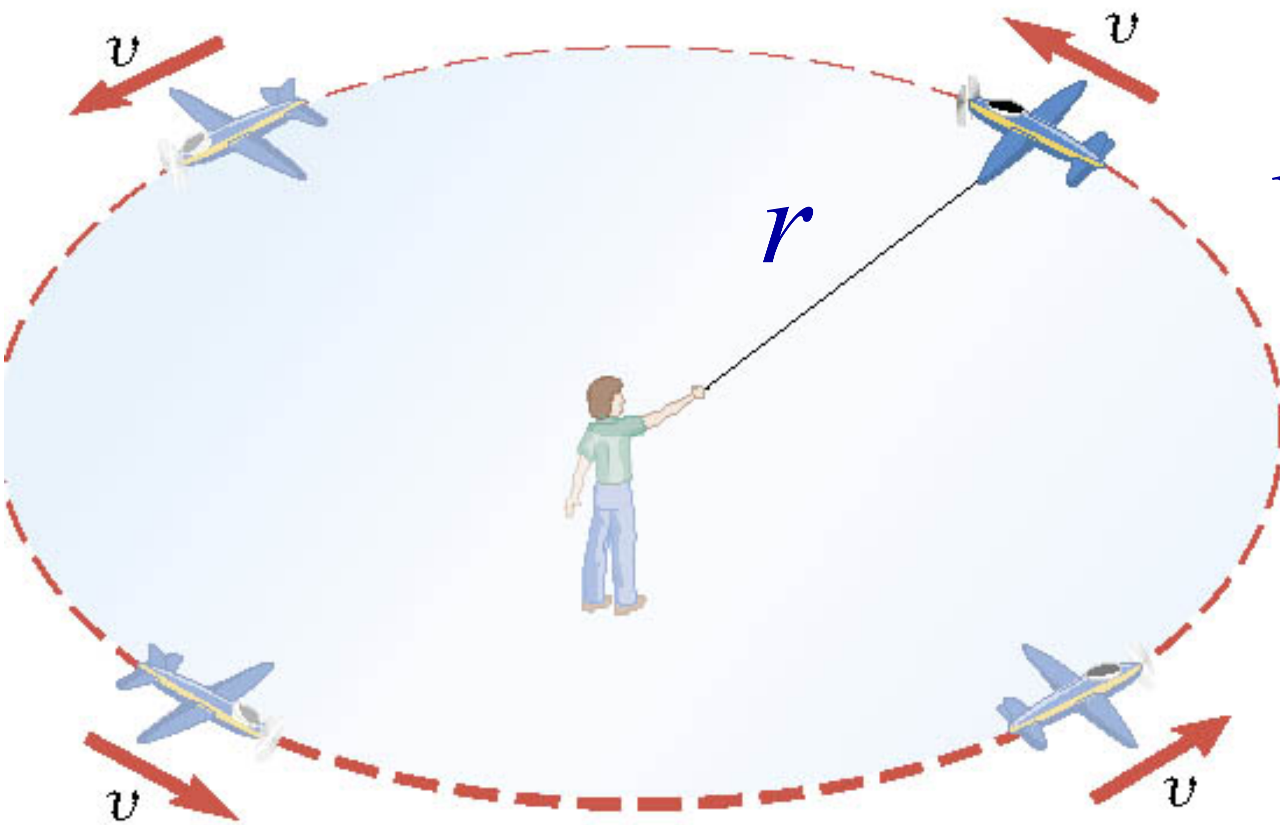
Definition of the Uniform Circular Motion

Uniform circular motion is the motion of an object traveling at a constant **speed** on a circular path.



Speed of a uniform circular motion?

Let *T* be the period of this motion, the time it takes for the object to travel once around the complete circle whose radius is r.



$$v = \frac{\text{distance}}{\text{time}}$$

$$= \frac{2\pi r}{T}$$

Ex. : A Tire-Balancing Machine

The wheel of a car has a radius of 0.29m and is being rotated at 830 revolutions per minute on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

$$\frac{1}{830\,\text{revolutions/min}} = 1.2\times 10^{-3}\ \text{min/revolution}$$

$$T = 1.2\times 10^{-3}\ \text{min} = 0.072\ \text{s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.29\ \text{m})}{0.072\ \text{s}} = 25\,\text{m/s}$$