PHYS 1441 – Section 001
Lecture #11

Monday, June 23, 2014
Dr. Jaehoon Yu

- Newton's Law of Universal Gravitation
- Weightlessness
- Work done by a constant force
- Multiplication of Vectors
- Work-Kinetic Energy Theorem

Announcements

- Term exam #2
  - In class this Wednesday, June 25
  - Non-comprehensive exam
  - Covers CH 4.7 to what we finish tomorrow, Tuesday, June 24
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam → no solutions, derivations or definitions!
    - No additional formulae or values of constants will be provided!

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long~ time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every object in the Universe attracts every other object with a force that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance** between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

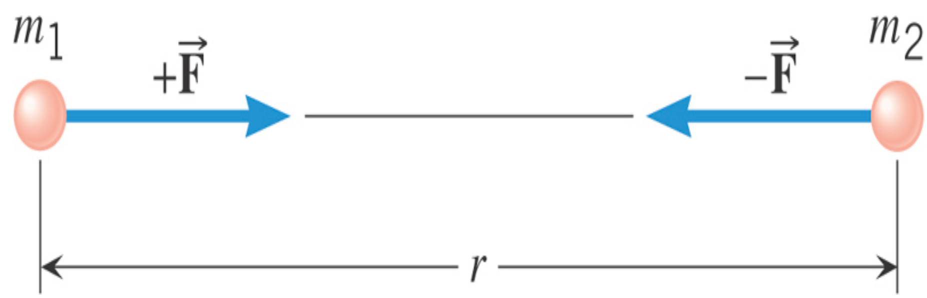
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as **the inverse-square law**, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

Ex. Gravitational Attraction

What is the magnitude of the gravitational force that acts on each particle in the figure, assuming m_1=12kg, m_2=25kg, and r=1.2m?

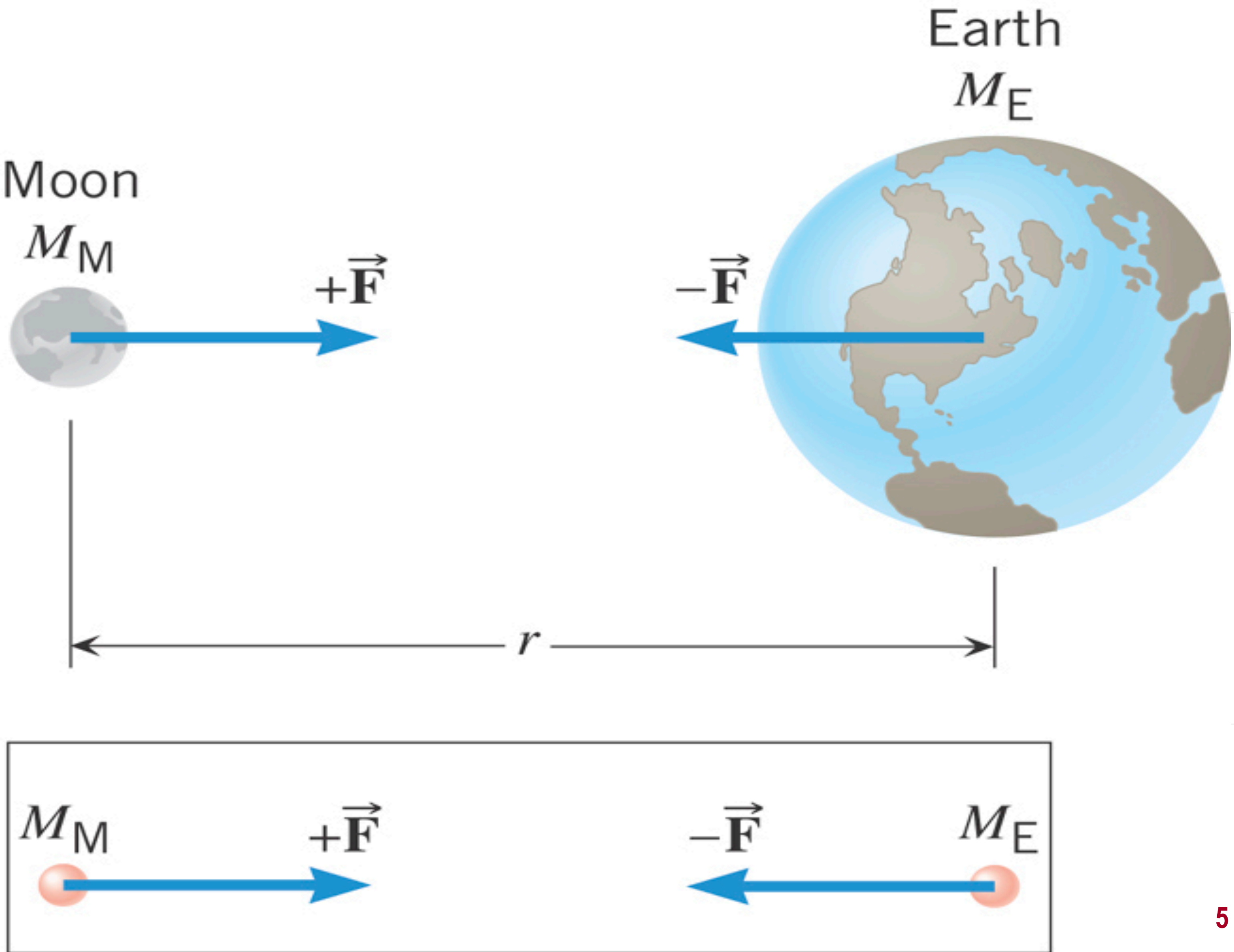


$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{(12\ \mathrm{kg})(25\ \mathrm{kg})}{(1.2\ \mathrm{m})^2}$$

$$= 1.4\times10^{-8}\ \mathrm{N}$$

Why does the Moon orbit the Earth?



Gravitational Force and Weight

Gravitational Force, F_g

The attractive force exerted on an object by another object

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

What is the SI unit of weight?

N

*Since weight depends on the magnitude of gravitational acceleration, **g**, it varies depending on geographical location.*

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Gravitational Acceleration

$$W = G\frac{M_E m}{r^2}$$

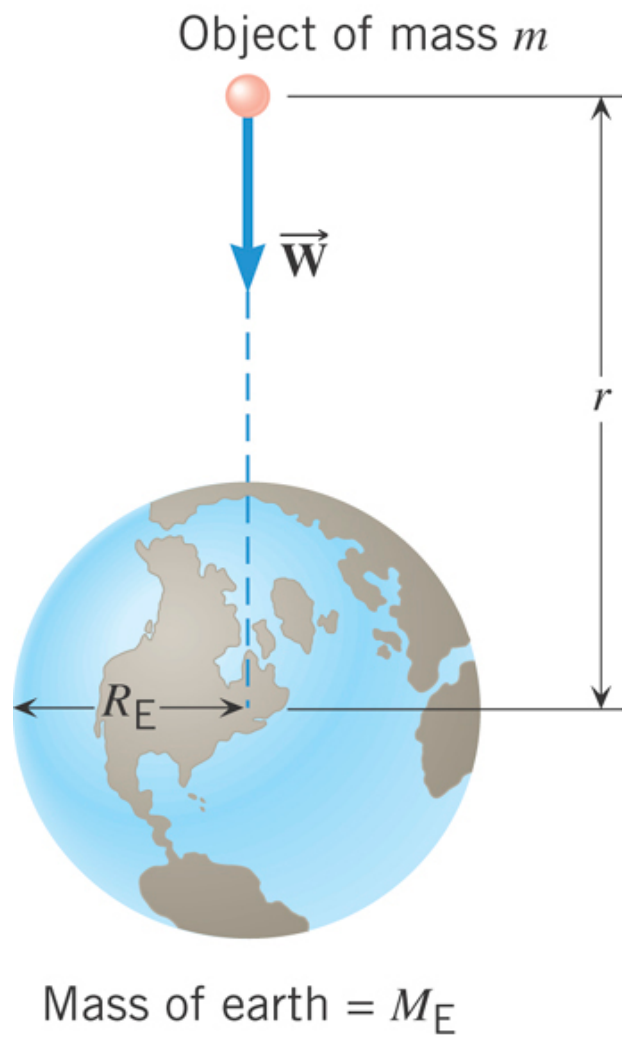
$$W = mg$$

$$\cancel{m}g = G\frac{M_E \cancel{m}}{r^2}$$

$$g = G\frac{M_E}{r^2}$$

Gravitational acceleration at distance r from the center of the earth!

What is the SI unit of g? $\boxed{\mathrm{m/s^2}}$



Magnitude of the gravitational acceleration on the surface of the Earth

Gravitational force on the surface of the earth:

$$F_G = G\frac{M_E m}{r^2} = G\frac{M_E \cancel{m}}{R_E^2} = \cancel{m}g$$

$$g = G\frac{M_E}{R_E^2}$$

$G = 6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}$

$M_E = 5.98\times10^{24}\ \mathrm{kg};\quad R_E = 6.38\times10^{6}\ m$

$$= \left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.38\times10^{6}\ \mathrm{m}\right)^2}$$

$$= 9.80\ \mathrm{m/s^2}$$

Example for Universal Gravitation

Using the fact that g=9.80m/s^2 on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^{\ 2}} = 6.67\times10^{-11}\frac{M_E}{R_E^{\ 2}}$$

Solving for M_E

$$M_E = \frac{R_E^{\ 2} g}{G}$$

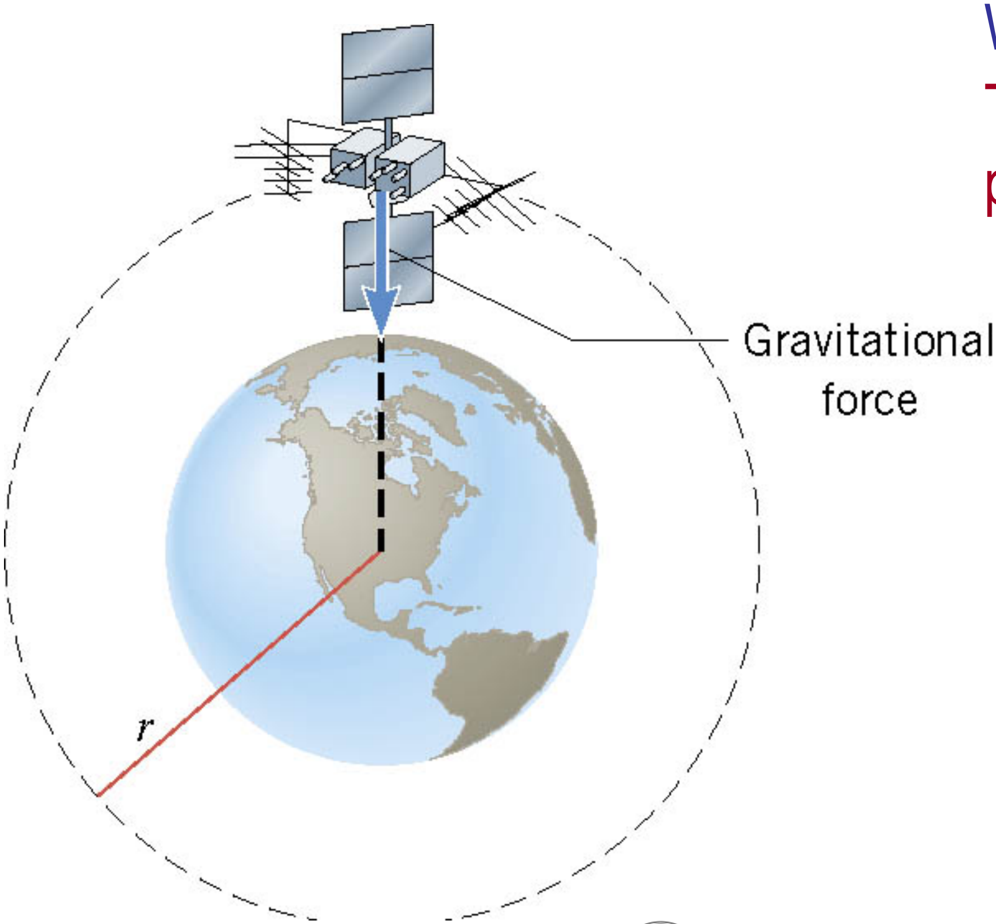
Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^{\ 2} g}{G}}{\frac{4\pi}{3} R_E^{\ 3}} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times9.80}{4\pi\times6.67\times10^{-11}\times6.37\times10^{6}} = 5.50\times10^{3}\, kg/m^{3}$$

Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



What acts as the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

Ex. Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{6.38\times10^{6}\,\mathrm{m}+598\times10^{3}\,\mathrm{m}}}$$

$$= 7.56\times10^{3}\,\mathrm{m/s} \qquad \left(16900\,\mathrm{mi/h}\right)$$

Period of a Satellite in an Orbit

Speed of a satellite $$v=\sqrt{\frac{GM_E}{r}}=\frac{2\pi r}{T}$$

$$\frac{GM_E}{r}=\left(\frac{2\pi r}{T}\right)^2$$

Square either side and solve for T2

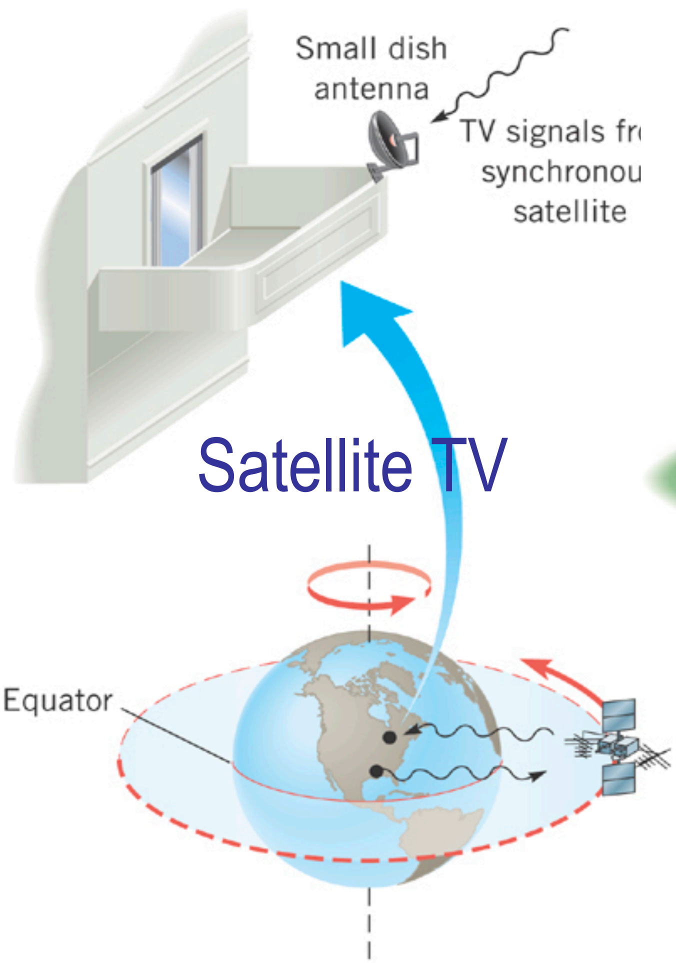
$$T^2=\frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $$T=\frac{2\pi r^{3/2}}{\sqrt{GM_E}}$$ Kepler's 3rd Law

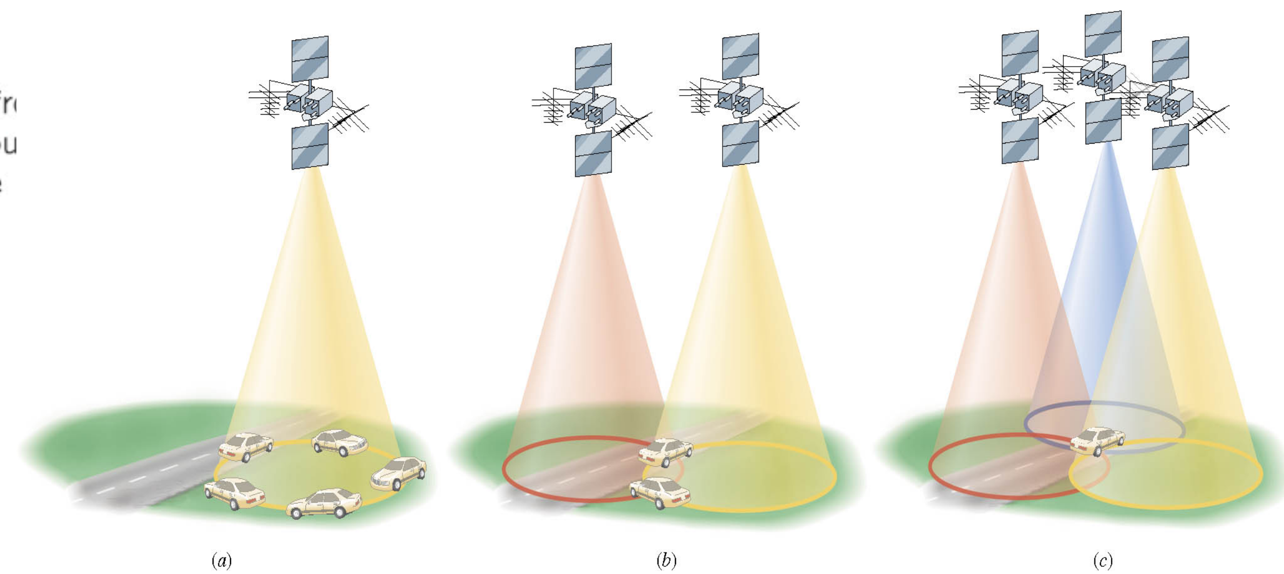
This is applicable to any satellite or even for planets and moons.

Geo-synchronous Satellites

Global Positioning System (GPS)



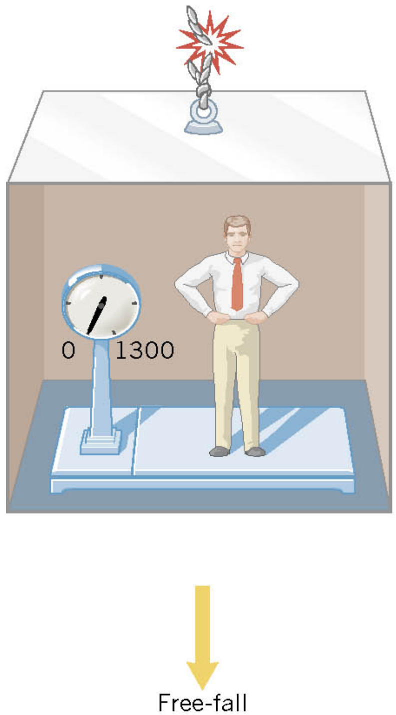
Satellite TV



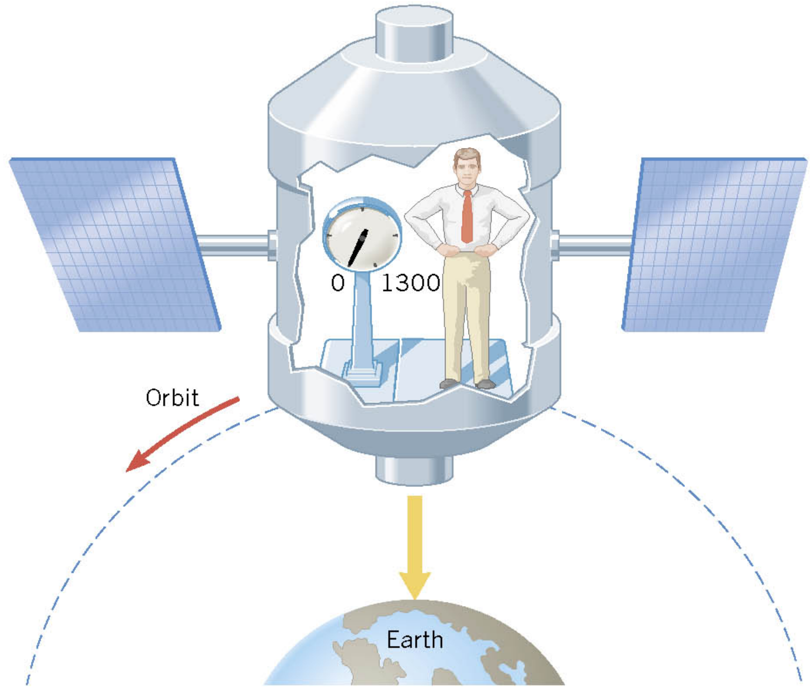
What period should these satellites have?

The same as the earth!! 24 hours

Ex. Apparent Weightlessness and Free Fall



(a)



(b)

0 0

In each case, what is the weight recorded by the scale?

Ex. The Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

$$= \sqrt{(1700\text{ m})(9.80\,\text{m/s}^2)}$$

$$= 130\,m/s$$