PHYS 1441 – Section 001
Lecture #13

Thursday, June 26, 2014

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- Potential Energy
- Gravitational Potential Energy
- Elastic Potential Energy
- Mechanical Energy Conservation
- Power
- Linear Momentum
- Linear Momentum and Impulse

Today's homework is homework #8, due 11pm, Tuesday, July 1!!

Special Project #4

1. A ball of mass M at rest is dropped from the height h above the ground onto a spring on the ground, whose spring constant is k. Neglecting air resistance and assuming that the spring is in its equilibrium, express, in terms of the quantities given in this problem and the gravitational acceleration g, the distance x of which the spring is pressed down when the ball completely loses its energy. (10 points)
2. Find the x above if the ball's initial speed is v_i. (10 points)
3. Due for the project is this Monday, June 30
4. You must show the detail of your OWN work in order to obtain any credit.

Potential Energy

Energy associated with a system of objects ➔ *Stored energy which has the potential or the possibility to work or to convert to kinetic energy*

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the conservative force which results in the principle of ***conservation of mechanical energy.***

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are the forms of energies in the universe?

Mechanical Energy | *Chemical Energy* | *Biological Energy*

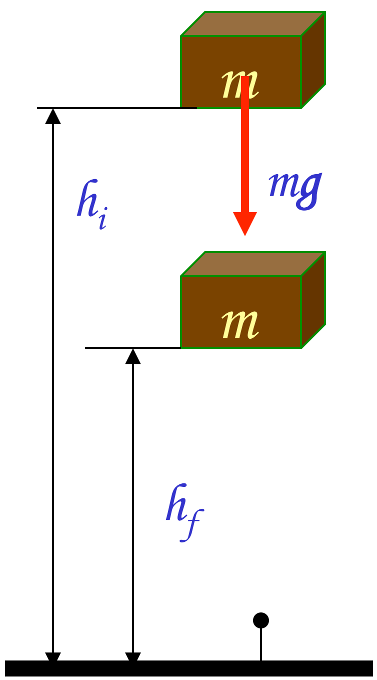
Electromagnetic Energy | *Nuclear Energy* | *Thermal Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

This potential energy is given to an object by the gravitational field in the system of Earth by virtue of the object's height from an arbitrary zero level



When an object is falling, the gravitational force, mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at a height y, __the potential to do work,__ is expressed as

$$PE = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgh \qquad PE \equiv mgh$$

The work done on the object by the gravitational force as the brick drops from y_i to y_f is:

$$W_g = PE_i - PE_f$$
$$= mgh_i - mgh_f = -\Delta PE$$

(since $\Delta PE = PE_f - PE_i$*)*

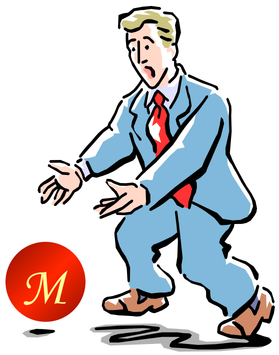
What does this mean?

Work by the gravitational force as the brick drops from y_i to y_f is the negative change of the system's potential energy

➔ Potential energy was spent in order for the gravitational force to increase the brick's kinetic energy.

Example for Potential Energy

A bowler drops a bowling ball of mass 7kg on his toe. Choosing the floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls on the toe.



Let's assume that the top of the toe is 3cm from the floor and the hand was at 50cm above the floor.

$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3J \quad U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

The kinetic energy of a 60g bullet flying at 33m/s (100fts) or 119km/hr.

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of the ball in his hand and on his toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7\times 9.8\times\left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7\times 9.8\times\left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Elastic Potential Energy

Potential energy given to an object by a spring or an object with elasticity in the system that consists of an object and the spring.

The force spring exerts on an object when it is distorted from its equilibrium by a distance x is

$$F_s = -kx$$

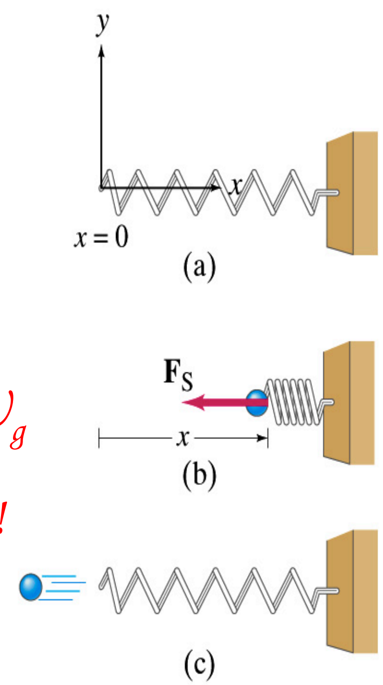
Hooke's Law

The work performed on the object by the spring is

$$W_s = \int_{x_i}^{x_f} (-kx)dx \quad = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} \quad = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 \quad = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

The potential energy of this system is

$$U_s \equiv \frac{1}{2}kx^2$$



What do you see from the above equations?

The work done on the object by the spring depends only on the initial and final position of the distorted spring.

Where else did you see this trend?

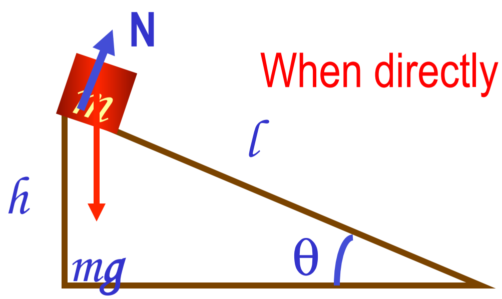
The gravitational potential energy, U_g

So what does this tell you about the elastic force?

A conservative force!!!

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path in the absence of a retardation force.



When directly falls, the work done on the object by the gravitation force is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg\sin\theta \times l$$
$$= mg(l\sin\theta) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$W_g = mgh$$

So the work done by the gravitational force on an object is independent of the path of the object's motion. It only depends on the difference of the object's initial and final position in the direction of the force.

Forces like gravitational and elastic forces are called the conservative force

1. If the work performed by the force does not depend on the path.
2. If the work performed on a closed path is 0.

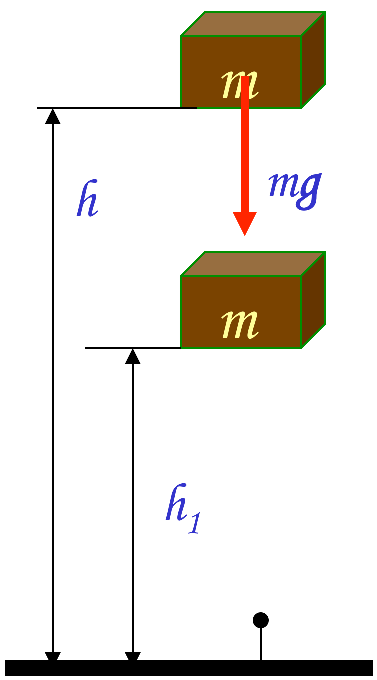
Total mechanical energy is conserved!!

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv KE + PE$$



Let's consider a brick of mass *m* at the height *h* from the ground

What is the brick's potential energy?

$$PE = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta PE = PE_f - PE_i = -Fs$$

The brick gains speed *By how much?* $v = gt$

So what? *The brick's kinetic energy increased* $K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$

And? *The lost potential energy is converted to kinetic energy!!*

What does this mean?

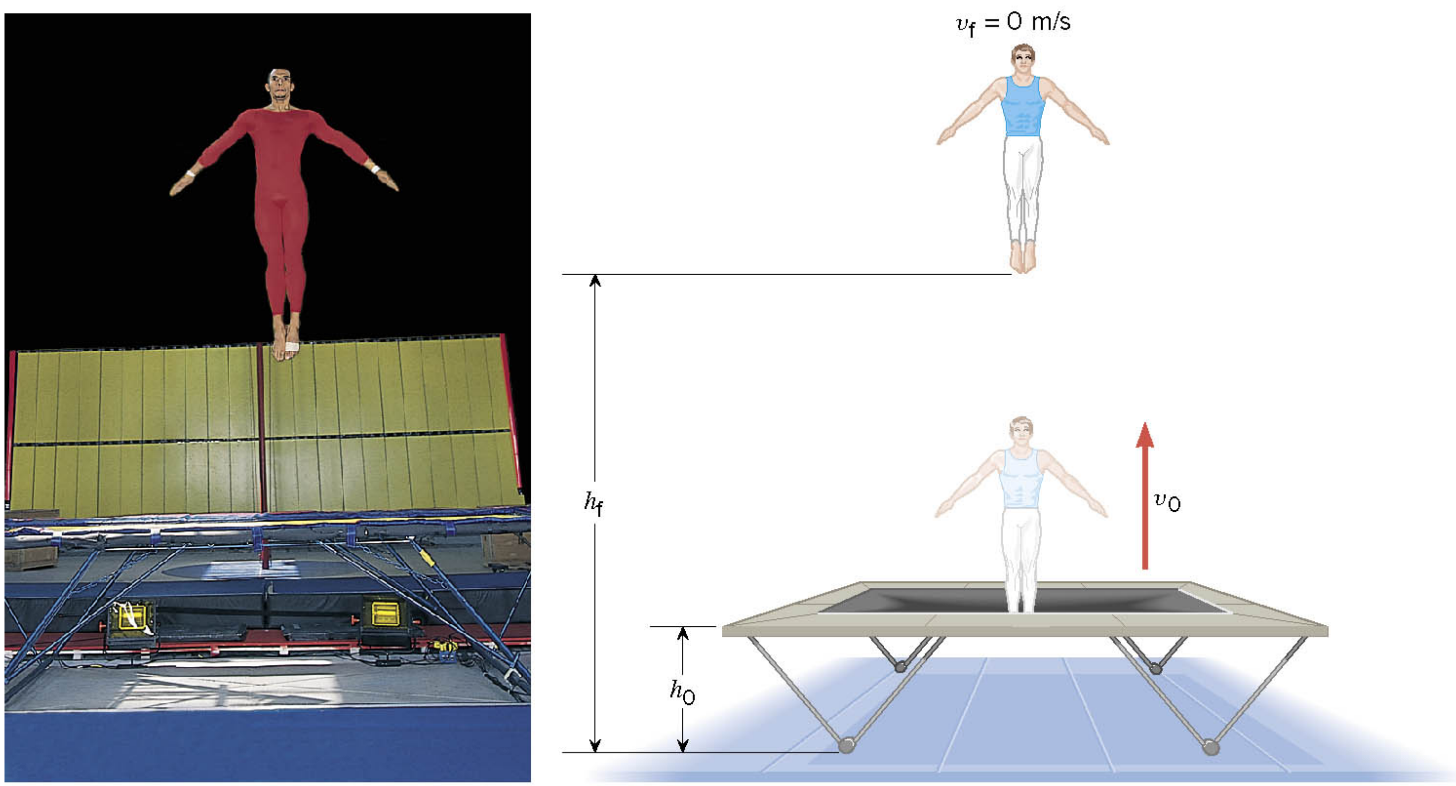
The total mechanical energy of a system remains constant in any isolated systems of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

$$E_i = E_f$$

$$KE_i + \sum PE_i = KE_f + \sum PE_f$$

Ex. A Gymnast on a Trampoline

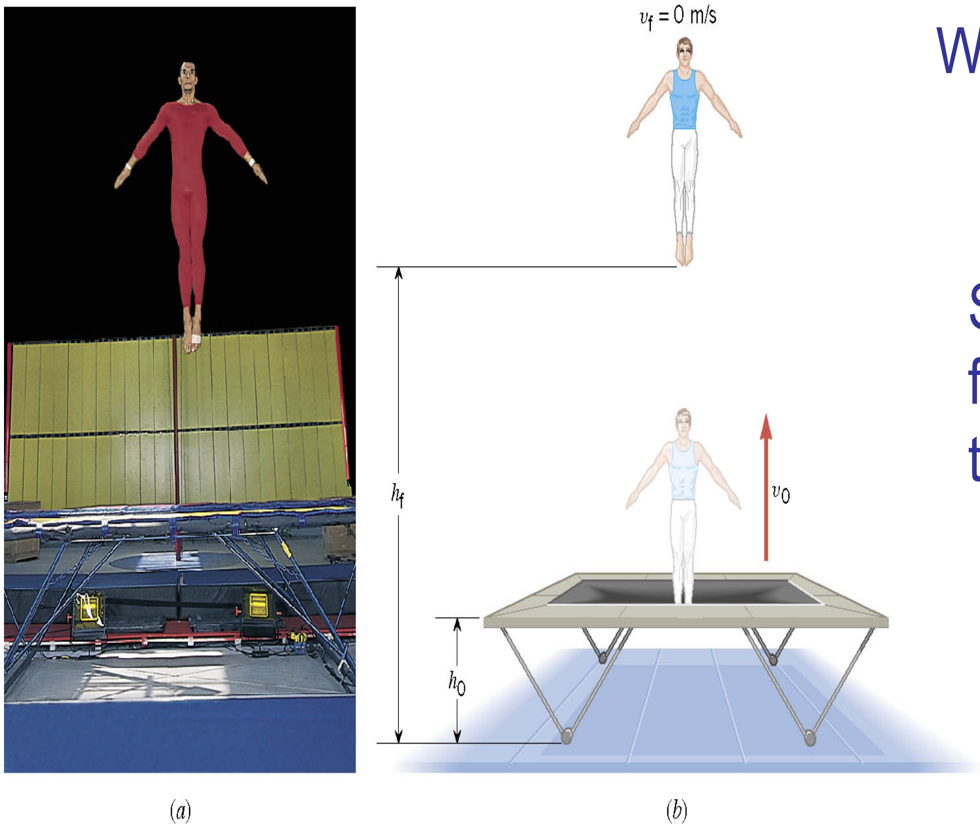
A gymnast leaves the trampoline at an initial height of 1.20 m and reaches a maximum height of 4.80 m before falling back down. What was the initial speed of the gymnast?



(a) (b)

Ex. Continued

From the work-kinetic energy theorem $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_o^2$



Work done by the gravitational force

$$W_{\text{gravity}} = mg\left(h_o - h_f\right)$$

Since at the maximum height, the final speed is 0. Using work-KE theorem, we obtain

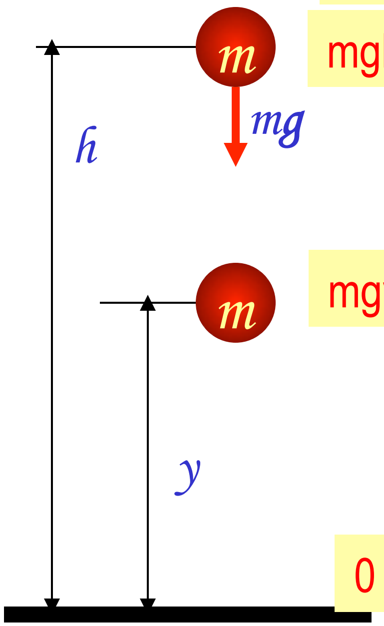
$$\cancel{m}g\left(h_o - h_f\right) = -\frac{1}{2}\cancel{m}v_o^2$$

$$v_o = \sqrt{-2g\left(h_o - h_f\right)}$$

$$\therefore v_o = \sqrt{-2\left(9.80\,\text{m/s}^2\right)\left(1.20\text{ m} - 4.80\text{ m}\right)} = 8.40\,\text{m/s}$$

Example

A ball of mass m at rest is dropped from the height h above the ground. a) Neglecting the air resistance, determine the speed of the ball when it is at the height y above the ground.



PE	KE	
mgh	0	$mv_i^2/2$
mgy	$mv^2/2$	$mv_f^2/2$
0		

Using the principle of mechanical energy conservation

$$K_i + PE_i = K_f + PE_f \quad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of the release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + PE_i = K_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

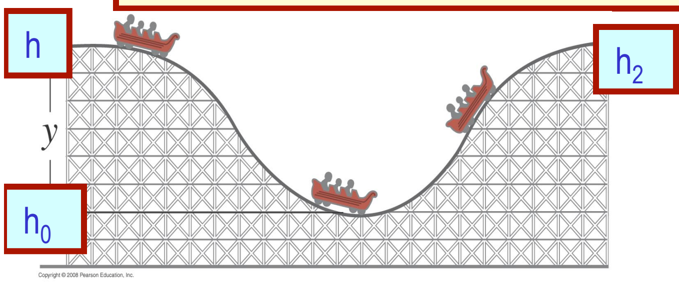
$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Ex. 6 – 8

Assuming the height of the hill in the figure is 40m, and the roller-coaster car starts from rest at the top, calculate (a) the speed if the roller coaster car at the bottom of the hill and

Using mechanical energy conservation



$$K_i + PE_i = K_f + PE_f = \quad 0 + mgh \; = \frac{1}{2}mv^2 + mgh_0$$

$$\frac{1}{2}mv^2 = mg\left(h - h_0\right)$$

$$\therefore v = \sqrt{2g\left(h - h_0\right)} = \sqrt{2 \cdot 9.8 \cdot 40} = 28\left(m/s\right)$$

b) Determine at what height (h2) of the second hill it will have half the speed at the bottom?

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + PE_i = K_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgh_0 \quad = \frac{1}{2}mv_f^2 + mgh_2$$

Reorganize the terms

$$mg\left(h_2 - h_0\right) = \frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 - \frac{1}{2}m\left(\frac{1}{2}v_i\right)^2 = \frac{3}{8}mv_i^2 \Rightarrow$$

Solving for h_2-h_0

$$h_2 - h_0 = \frac{3}{8}\frac{v_i^2}{g} = \frac{3}{8}\frac{\left(28\right)^2}{9.8} = 30m$$

Power

- Rate at which the work is done or the energy is transferred
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
  - ➔ The time… 8 cylinder car climbs up the hill faster!

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinders than 4.

Average power

$$\overline{P} \equiv \frac{\Delta W}{\Delta t} = \frac{Fs}{\Delta t} = F\frac{s}{\Delta t} = F\,\overline{v}$$

Scalar quantity

Unit? $J/s = Watts$

$$1HP \equiv 746Watts$$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^{6} J$

Energy

Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:
- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving the vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227N$$

Air Drag

$$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Human Metabolic Rates

Activity	Rate (watts)
Running (15 km/h)	1340 W
Skiing	1050 W
Biking	530 W
Walking (5 km/h)	280 W
Sleeping	77 W

[a]For a young 70-kg male.

Ex. The Power to Accelerate a Car

A 1.10×10^3kg car, starting from rest, accelerates for 5.00s. The magnitude of the acceleration is a=4.60m/s^2. Determine the average power generated by the net force that accelerates the vehicle.

What is the force that accelerates the car?

$$F = ma = \left(1.10\times 10^3\right)\cdot\left(4.60\, m/s^2\right) = 5060\, N$$

Since the acceleration is constant, we obtain

$$\bar{v} = \frac{v_0 + v_f}{2} = \frac{0 + v_f}{2} = \frac{v_f}{2}$$

From the kinematic formula

$$v_f = v_0 + at = 0 + \left(4.60\, m/s^2\right)\cdot\left(5.00 s\right) = 23.0\, m/s$$

Thus, the average speed is

$$\frac{v_f}{2} = \frac{23.0}{2} = 11.5\, m/s$$

And, the average power is

$$\bar{P} = F\bar{v} = \left(5060 N\right)\cdot\left(11.5\, m/s\right) = 5.82\times 10^4 W$$

$$= 78.0 hp$$