PHYS 1441 – Section 001
Lecture #15

Tuesday, July 1, 2014

Dr. Jaehoon Yu

- Concept of the Center of Mass
- Fundamentals of the Rotational Motion
- Rotational Kinematics
- Equations of Rotational Kinematics
- Relationship Between Angular and Linear Quantities
- Rolling Motion of a Rigid Body

Today's homework is homework #9, due 11pm, Saturday, July 5!!

Announcements

- Planetarium Extra Credit
  - Tape all ticket stubs on a sheet of paper
  - Tape one side of each ticket stub with the title on the surface so that I can see the signature on the other side
  - Put your name on the extra credit sheet
  - Bring the sheet coming Monday, July 7, at the final exam
- Final exam
  - 10:30am – 12:30pm, Monday, July 7
  - Comprehensive exam, covers from CH1.1 – what we finish this Thursday, July 3, plus appendices A1 – A8
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam ➔ no solutions, no derived formulae, derivations or definitions!
    - No additional formulae or values of constants will be provided!

Extra-Credit Special Project #5

- Derive express the final velocities of the two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in a far greater detail than in the lecture note. (20 points)
- Show mathematically what happens to the final velocities if m_1=m_2 and describe in words the resulting motion. (5 points)
- Due: Monday, July 7, 2014

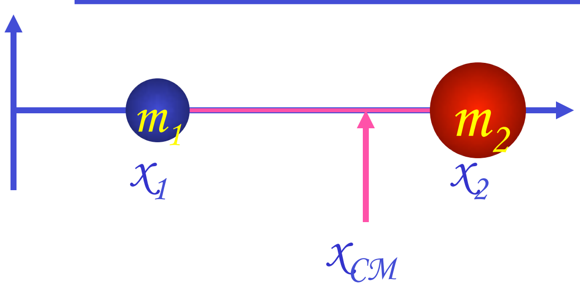
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if the entire mass of the system is on the center of mass.*



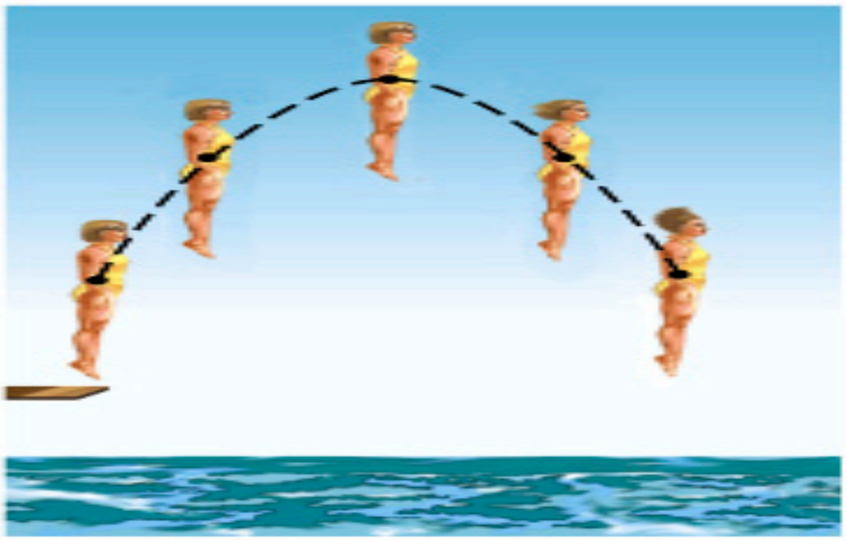
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

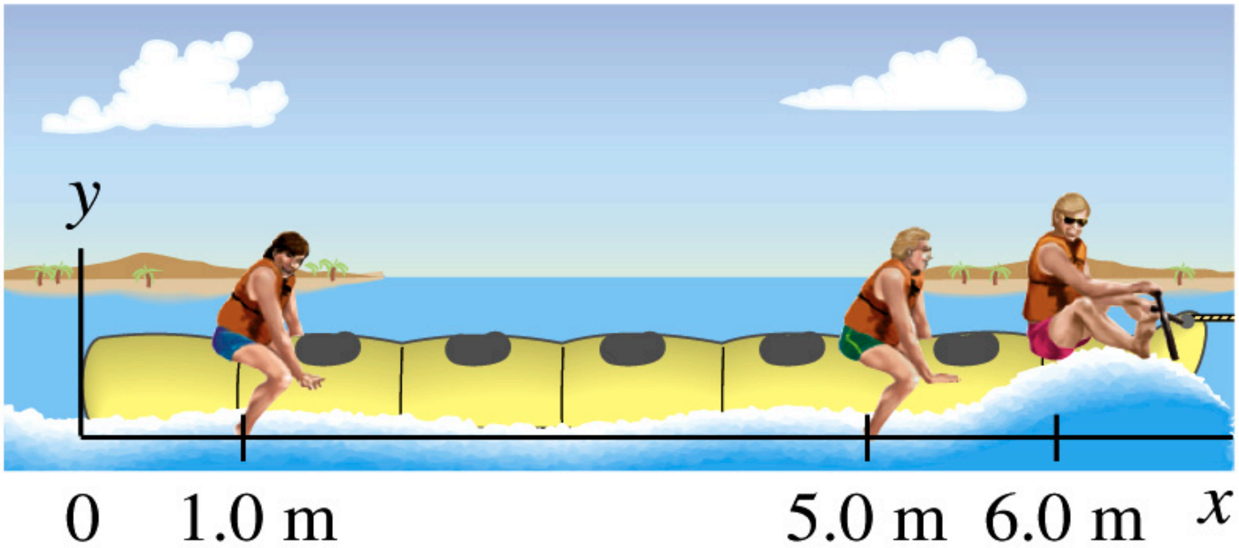
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Ex. 7 – 12 Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.

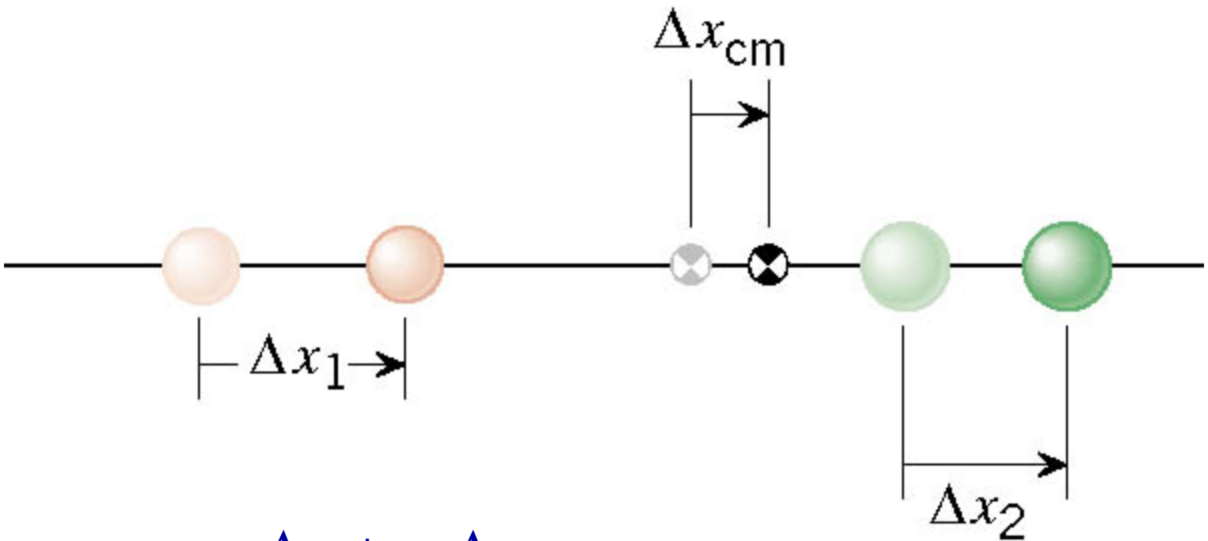


Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Velocity of the Center of Mass



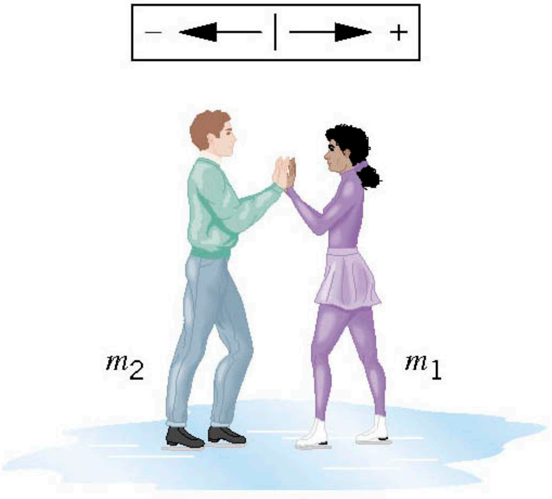
$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1 / \Delta t + m_2 \Delta x_2 / \Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

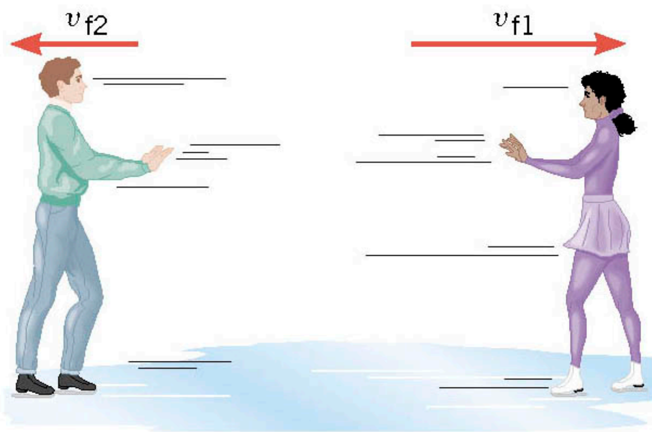
In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Another Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a velocity of +2.5 m/s.



(a) Before



(b) After

$$v_{10} = 0\,m/s \qquad v_{20} = 0\,m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

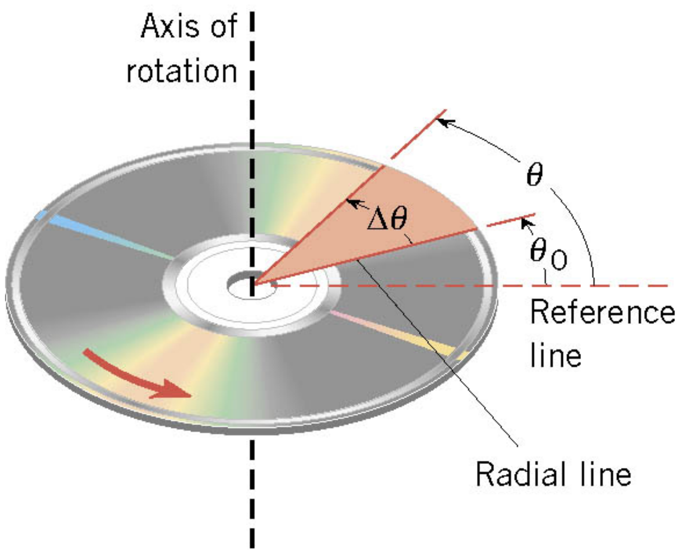
$$v_{1f} = +2.5\,m/s \qquad v_{2f} = -1.5\,m/s$$

$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54\cdot(+2.5) + 88\cdot(-1.5)}{54+88} = \frac{3}{142} = 0.02 \approx 0\,m/s$$

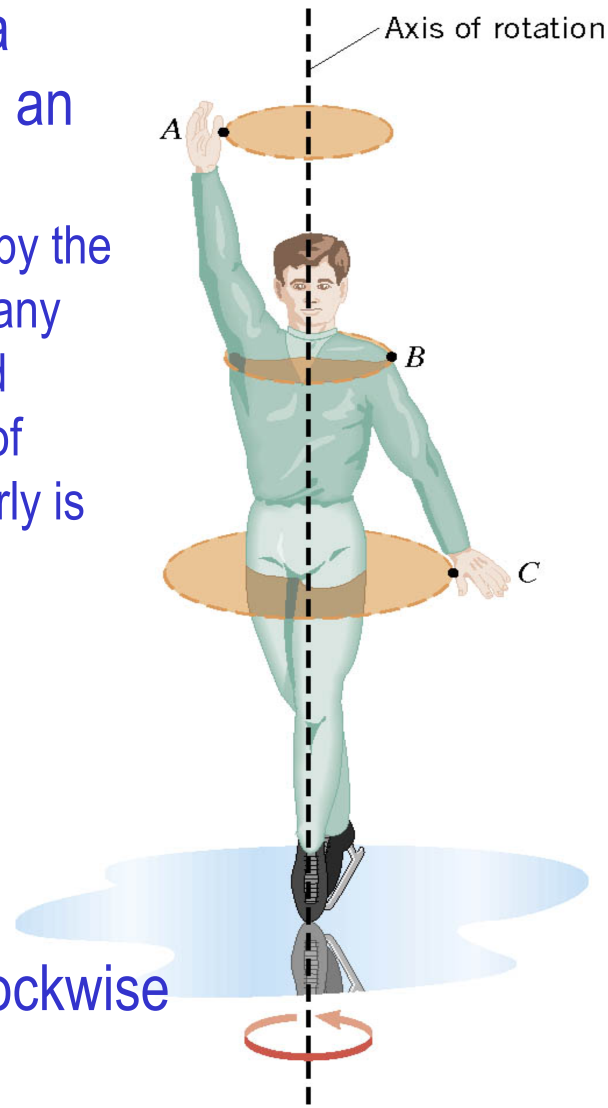
Rotational Motion and Angular Displacement

In the simplest kind of rotation, points on a rigid object move on circular paths around an ***axis of rotation.***



The angle swept out by the line passing through any point on the body and intersecting the axis of rotation perpendicularly is called the ***angular displacement.***

$$\Delta\theta = \theta - \theta_o$$



It's a vector!! So there must be a direction…

How do we define directions?

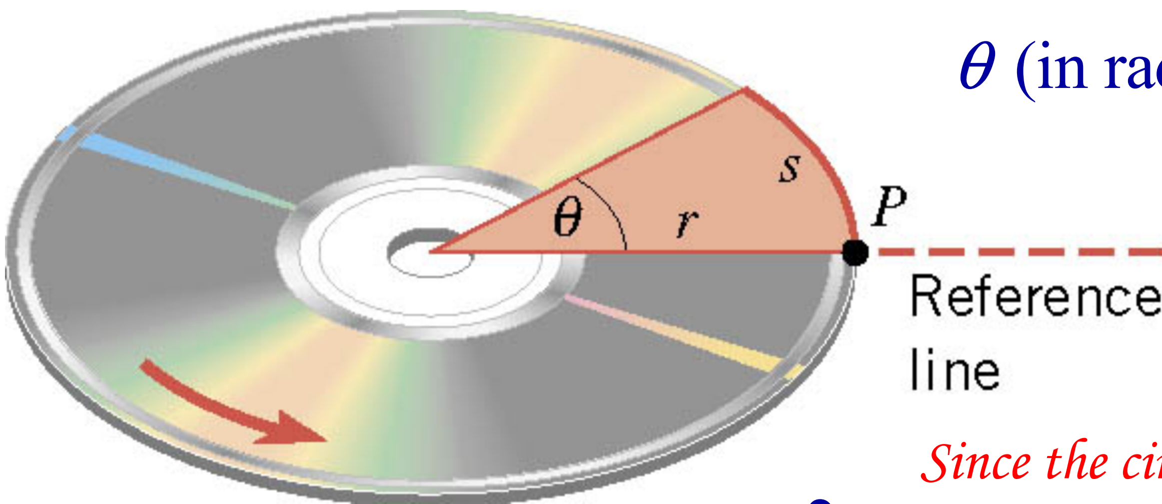
+:if counter-clockwise

-:if clockwise

The direction vector points gets determined based on the right-hand rule.

These are just conventions!!

SI Unit of the Angular Displacement



$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Dimension? None

For one full revolution:

Since the circumference of a circle is $2\pi r$

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \Rightarrow 2\pi \text{ rad} = 360^{o}$$

One radian is an angle subtended by an arc of the same length as the radius!

Unit of the Angular Displacement

How many degrees are in one radian?

1 radian is

$$1\text{ rad} = \frac{360^\circ}{2\pi rad}\cdot 1rad = \frac{180^\circ}{\pi} \cong \frac{180^\circ}{3.14} \cong 57.3^\circ$$

How radians is one degree?

And one degrees is

$$1^\circ = \frac{2\pi}{360^\circ}\cdot 1^\circ = \frac{\pi}{180^\circ}\cdot 1^\circ \cong \frac{3.14}{180^\circ}\cdot 1^\circ \cong 0.0175rad$$

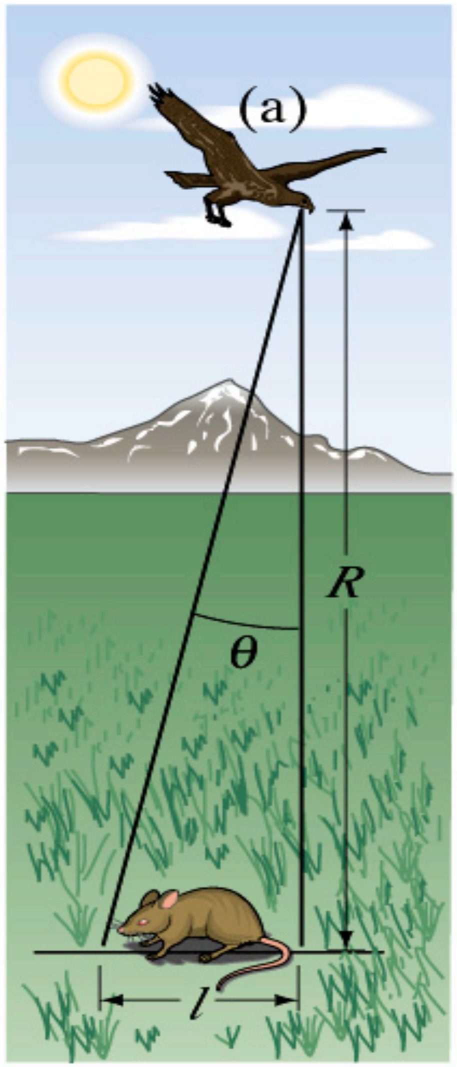
How many radians are in 10.5 revolutions?

$$10.5rev = 10.5rev\cdot 2\pi\frac{rad}{rev} = 21\pi(rad)$$

Very important: In solving angular problems, all units, degrees or revolutions, must be converted to radians.

Example 8-2

A particular bird's eyes can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (a) How many degrees is this? (b) How small an object can the bird just distinguish when flying at a height of 100m?



(a) One radian is 360°/2π. Thus

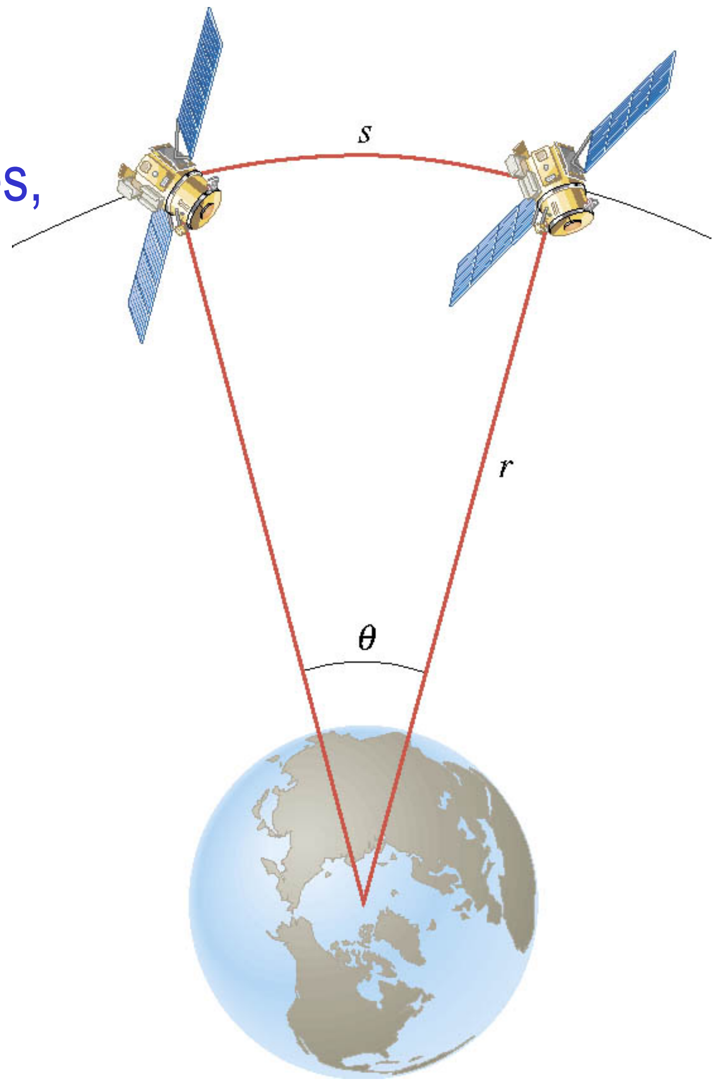
$$3\times10^{-4}\,rad = \left(3\times10^{-4}\,rad\right)\times\left(360^{o}/2\pi\,rad\right) = 0.017^{o}$$

(b) Since l=rθ and for small angle arc length is approximately the same as the chord length.

$$l = r\theta = 100m\times3\times10^{-4}\,rad = 3\times10^{-2}\,m = 3cm$$

Ex. Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^{7}m. If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.



What do we need to find out? The Arc length!!!

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Convert degrees to radians

$$2.00\,\text{deg}\left(\frac{2\pi \text{ rad}}{360\,\text{deg}}\right) = 0.0349 \text{ rad}$$

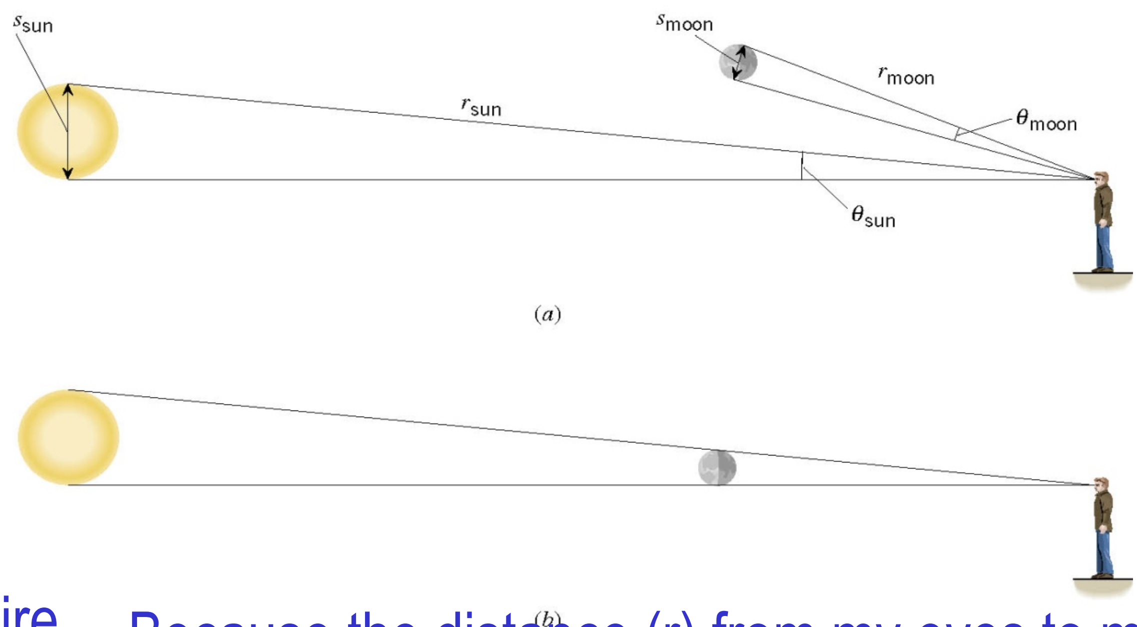
$$s = r\theta = \left(4.23\times10^{7}\,\text{m}\right)\left(0.0349 \text{ rad}\right)$$

$$= 1.48\times10^{6}\,\text{m} \;\; (920 \text{ miles})$$

Ex. A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon. For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



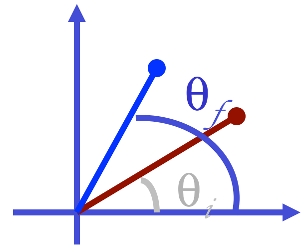
I can even cover the entire sun with my thumb!! Why?

Because the distance (r) from my eyes to my thumb is far shorter than that to the sun.

Angular Displacement, Velocity, and Acceleration

Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$



How about the average angular velocity, the rate of change of angular displacement?

$$\overline{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit? rad/s Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as…

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit? rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.