PHYS 1441 – Section 001
Lecture #4

Thursday, June 11, 2015

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- Chapter 2: One Dimensional Motion
  - Acceleration
  - Motion under constant acceleration
  - One dimensional Kinematic Equations
  - How do we solve kinematic problems?
  - Falling motions
- Chapter 3
  - Trigonometry Refresher
  - Properties and operations of vectors

Announcements

- Reading Assignment: CH2.8
- Homework: 100% enrolled but 69/71 completed HW1
- 1st non-comprehensive term exam
  - In class Monday, June 15
  - Covers: CH1.1 through CH2.8 plus appendix A
  - Bring your calculator but DO NOT input formula into it!
    - Cell phones or any types of computers cannot replace a calculator!
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the quiz
  - No derivations, word definitions or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Quiz Results
  - Class average: 54/72
    - Equivalent to 75/100
  - Top score: 72/72

Acceleration

Change of velocity in time (what kind of quantity is this?)

Vector!

- Definition of Average acceleration:

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Dimension? [LT^{-2}] Unit? m/s^2

- Definition of Instantaneous acceleration: Average acceleration over a very short amount of time.

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} \quad \text{analogs to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}}$$

The Direction (sign) of the Acceleration

- If the velocity **INCREASES**, the acceleration must be in the **SAME** direction as the velocity!!
  - If the positive velocity increases, what sign is the acceleration?
    - Positive!!
  - If the negative velocity increases, what sign is the acceleration?
    - Negative
- If the velocity **DECREASES**, the acceleration must be in the **OPPOSITE** direction to the velocity!!
  - If the positive velocity decreases, what sign is the acceleration?
    - Negative
  - If the negative velocity decreases, what sign is the acceleration?
    - Positive

Some questions on acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$).
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!

Displacement, Velocity, Speed & Acceleration

Displacement

$$\Delta x \equiv x_f - x_i$$

Dimension? [L]   Unit? m

Average velocity

$$\bar{v}_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Dimension? [LT^{-1}]   Unit? m/s

Average speed

$$\bar{v} \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Dimension? [LT^{-1}]   Unit? m/s

Average Acceleration

$$\bar{a}_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$

Dimension? [LT^{-2}]   Unit? m/s^2

Instantaneous velocity

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Dimension? [LT^{-1}]   Unit? m/s

Instantaneous speed

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

Dimension? [LT^{-1}]   Unit? m/s

Instantaneous Acceleration

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

Dimension? [LT^{-2}]   Unit? m/s^2

Derivation of Kinematic Eq.

- The simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, the velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad \text{(If } t_f=t \text{ and } t_i=0\text{)} \quad a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For a constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i} \quad \text{(If } t_f=t \text{ and } t_i=0\text{)} \quad \bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Derivation of Kinematic Eq. cont'd

Average velocity $\overline{v}_x = \frac{v_{xi} + v_{xf}}{2}$ $\Rightarrow$ $x_f = x_i + \overline{v}_x t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right)t$

Since $a_x = \frac{v_{xf} - v_{xi}}{t}$ $\xrightarrow{\text{Solving for t}}$ $t = \frac{v_{xf} - v_{xi}}{a_x}$

Substituting t in the above equation,

$$x_f = x_i + \left(\frac{v_{xf} + v_{xi}}{2}\right)\left(\frac{v_{xf} - v_{xi}}{a_x}\right) = x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x}$$

Resulting in

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right) t}$$

Displacement as a function of velocities and time

$$\boxed{x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinetic equations, depending on the information given to you for specific physical problems!!

How do we solve a problem using the kinematic formula for constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance?
  - Time?
- Identify what the problem wants you to figure out.
- Identify which kinematic formula is most appropriate and easiest to solve for what the problem wants.
  - Often multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that makes the problem easiest to solve.
- Solve the equation for the quantity wanted!

Example

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇒ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \dfrac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

The acceleration is $a_x = \dfrac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \dfrac{0 - \left(28m/s\right)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \dfrac{v_{xf} - v_{xi}}{a} = \dfrac{0 - 28m/s}{-390m/s^2} = 0.07s$

Falling Motion

- The falling motion is a motion under the influence of the gravitational pull (gravity) only; Which direction is a freely falling object moving? **Yes, down to the center of the earth!!**
  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the square of the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $g=9.80m/s^2$ on the surface of the earth.
- The direction of gravitational acceleration is **ALWAYS** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$ where +y points upward
- The difference is that the object initially moving upward will turn around and come down!

Example for Using 1D Kinematic Equations on a Falling object

A stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $a=g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What happens at the maximum height? The stone stops; V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$