# PHYS 1441 – Section 001
# Lecture #5

*Tuesday, June 16, 2015*

*Dr. Jaehoon Yu*

- Trigonometry Refresher
- Properties and operations of vectors
- Components of the 2D Vector
- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion

Today's homework is homework #3, due 11pm, Friday, June 19!!

# Announcements

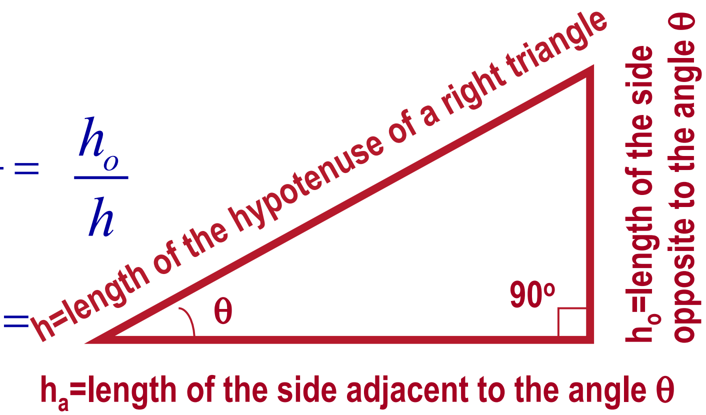
- Reading Assignment
  - CH3.7
- Quiz #2
  - Beginning of the class this Thursday, June, 18
  - Covers CH3.1 to what we finish tomorrow, Wednesday, June 17
  - Bring your calculator but DO NOT input formula into it!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam → no solutions, derivations or word definitions!
    - No additional formulae or values of constants will be provided!
- Term exam results
  - Class average: 63.9/99
    - Equivalent to 64.5/100
  - Top score: 101/99

# Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola and explain your answer.
  - 20 points
  - Due: Monday, June 22
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in this lecture note and in the book!

# Trigonometry Refresher

- Definitions of sinθ, cosθ and tanθ




$$\sin\theta = \frac{\text{Length of the opposite side to } \theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_o}{h}$$

$$\cos\theta = \frac{\text{Length of the adjacent side to } \theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_a}{h}$$

Solve for θ: $\theta = \cos^{-1}\left(\frac{h_a}{h}\right)$

$$\tan\theta = \frac{\text{Length of the opposite side to } \theta}{\text{Length of the adjacent side to } \theta} = \frac{h_o}{h_a}$$

Solve for θ: $\theta = \tan^{-1}\left(\frac{h_o}{h_a}\right)$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{h_o}{h}}{\frac{h_a}{h}} = \frac{h_o}{h_a}$$

Pythagorian theorem: For right angle triangles

$$h^2 = h_o^2 + h_a^2 \Rightarrow h = \sqrt{h_o^2 + h_a^2}$$

# Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

*Velocity, acceleration, force, momentum, etc*

Normally denoted in **BOLD** letters, ***F***, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, *F*, or absolute values: $\left|\vec{F}\right|$ or $|\mathbf{F}|$

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Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

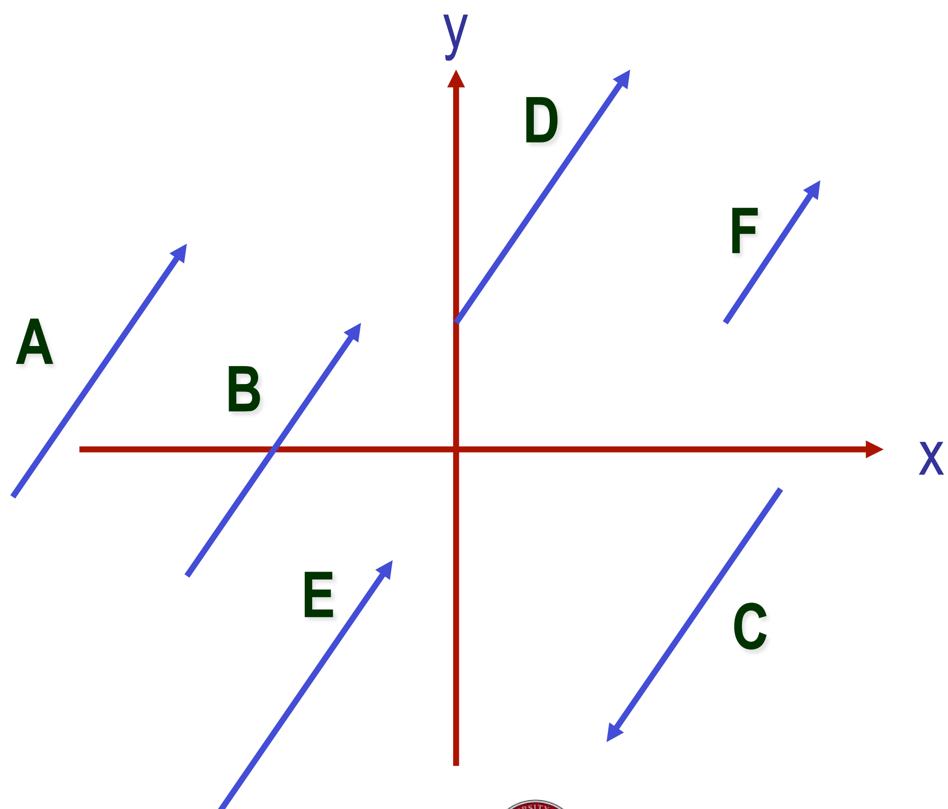
Normally denoted in normal letters, *E*

*Speed, energy, heat, mass, time, etc*

Both have units!!!

# Properties of Vectors

- Two vectors are the same if their **sizes** and the **directions** are the same, no matter where they are on a coordinate system!! ➔ You can move them around as you wish as long as their **directions** and **sizes** are kept the same.




Which ones are the same vectors?

**A=B=E=D**

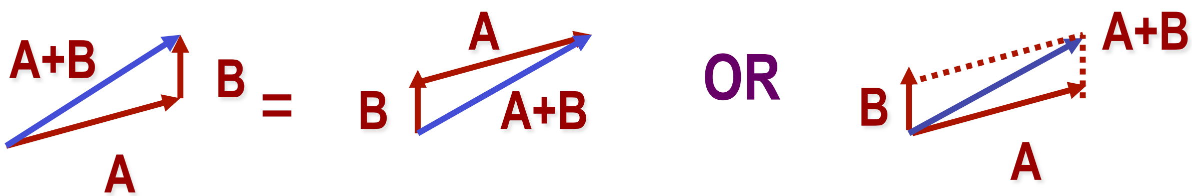
Why aren't the others?

**C:** The same magnitude but opposite direction: **C=-A:** A negative vector

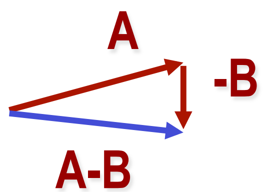
**F:** The same direction but different magnitude

# Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results **A+B=B+A**, **A+B+C+D+E=E+C+A+B+D**




- Subtraction:
  - The same as adding a negative vector: **A** - **B** = **A** + (-**B**)



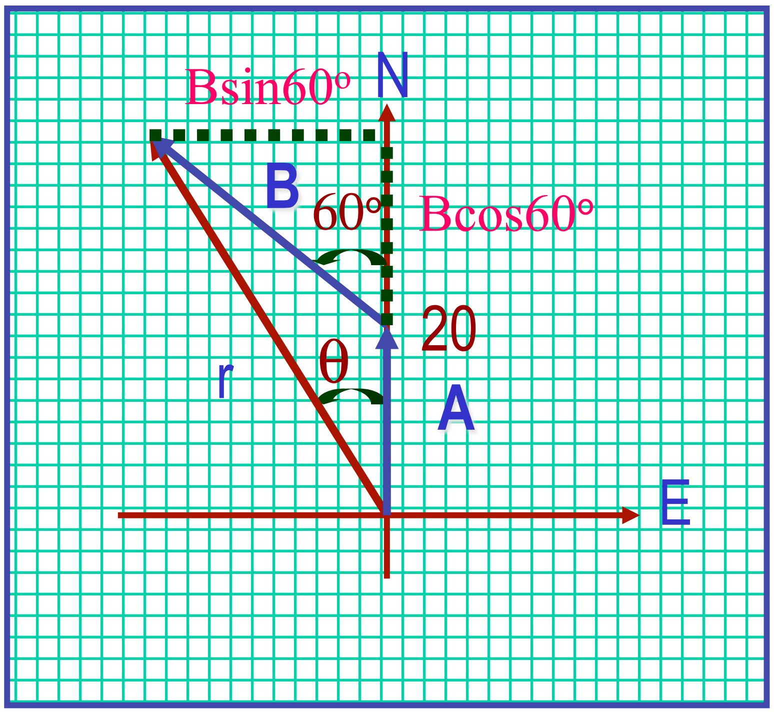

Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude **A**, **B**=2**A**


$$|\mathcal{B}| = 2|\mathcal{A}|$$

# Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0° West of North. Find the magnitude and direction of resultant displacement.




$$r = \sqrt{(A + B\cos 60)^2 + (B\sin 60)^2}$$

$$= \sqrt{A^2 + B^2(\cos^2 60 + \sin^2 60) + 2AB\cos 60}$$

$$= \sqrt{A^2 + B^2 + 2AB\cos 60}$$

$$= \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0\cos 60}$$

$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{|\vec{B}|\sin 60}{|\vec{A}| + |\vec{B}|\cos 60}$$

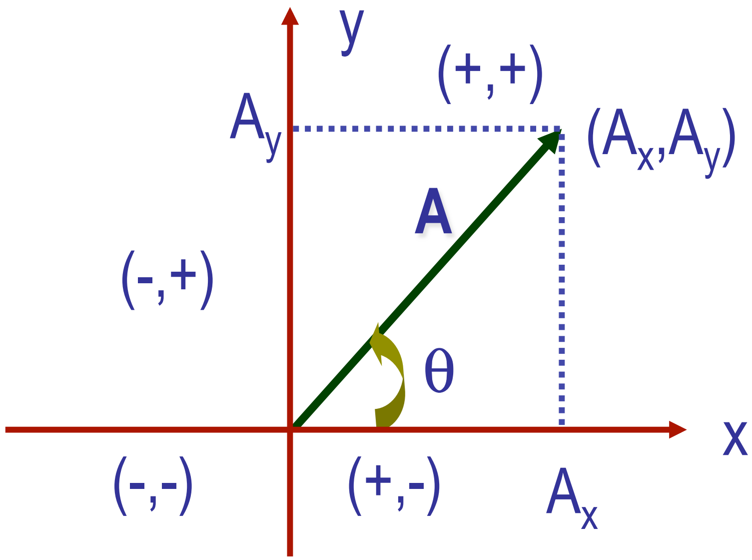
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$

$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^\circ \text{ to W wrt N}$$

Do this using components!!

# Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components




$$A_x = \left|\vec{A}\right|\cos\theta$$

$$A_y = \left|\vec{A}\right|\sin\theta$$

} Components

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2}$$

} Magnitude

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

# Unit Vectors

- Unit vectors are the ones that tells us the directions (and only directions) of the components
  - Very powerful and makes vector notation and operations much easier!
- **<u>Dimensionless</u>**
- **<u>Magnitudes these vectors are exactly 1</u>**
- Unit vectors are usually expressed in **i, j, k** or $\vec{i},\ \vec{j},\ \vec{k}$

So a vector **A** can be expressed as

$$\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}$$

# Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\left(m\right)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

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Find the resultant displacement of three consecutive displacements: **d₁**=(15**i**+30**j** +12**k**)cm, **d₂**=(23**i**+14**j** -5.0**k**)cm, and **d₃**=(-13**i**+15**j**)cm

$$\overrightarrow{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}(cm)$$

Magnitude

$$\left|\overrightarrow{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$