PHYS 1441 – Section 001
Lecture #6

Wednesday, June 17, 2015

Dr. Jaehoon Yu

- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion
- Maximum Range and Height

Announcements

- Quiz #2
  – Beginning of the class tomorrow, Thursday, June 18
  – Covers CH3.1 to what we finish today, Wednesday, June 17
  – Bring your calculator but DO NOT input formula into it!
  – You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam ➔ no solutions of any kind, derivations or word definitions!
    - No additional formulae or values of constants will be provided!

Reminder: Special Project #2 for Extra Credit

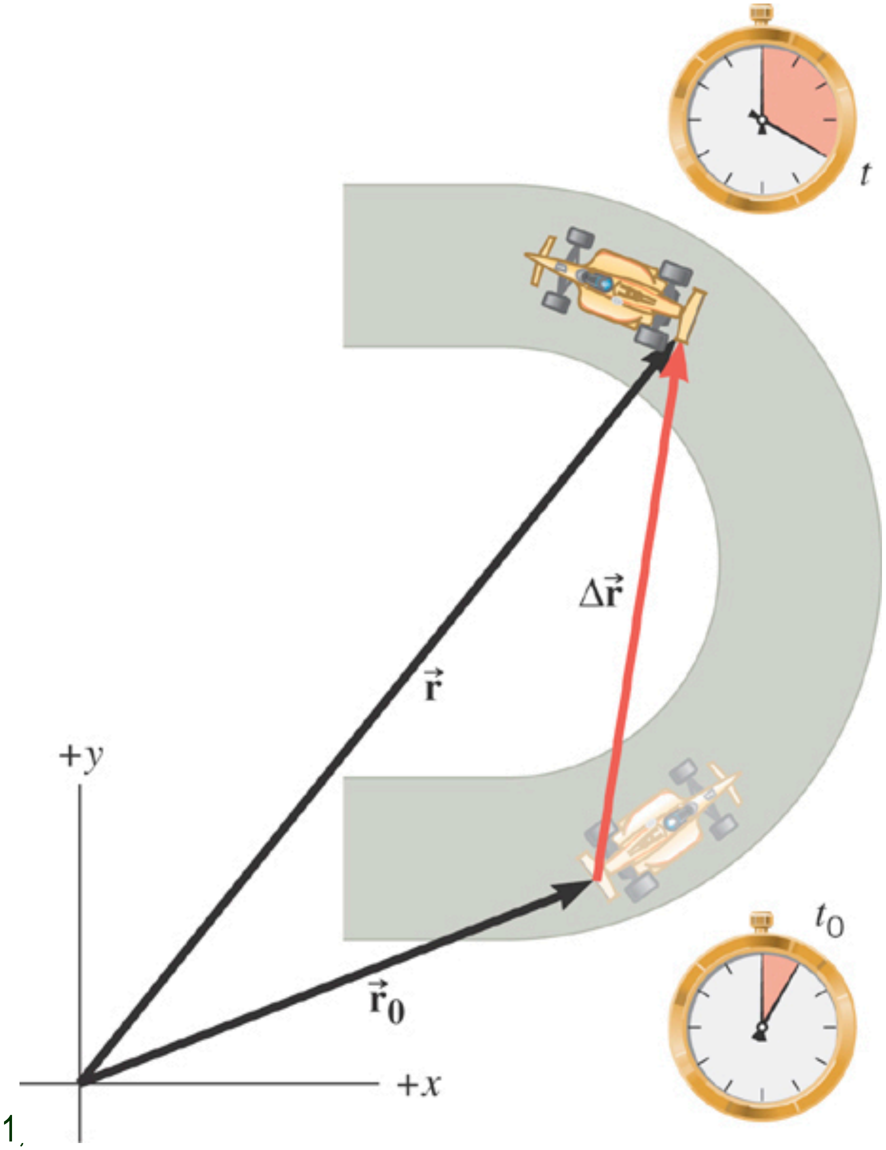
- Show that the trajectory of a projectile motion is a parabola and explain your answer.
  - 20 points
  - Due: Monday, June 22
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in this lecture note and in the book!

2D Displacement

$\vec{\mathbf{r}}_o =$ initial position

$\vec{\mathbf{r}} =$ final position

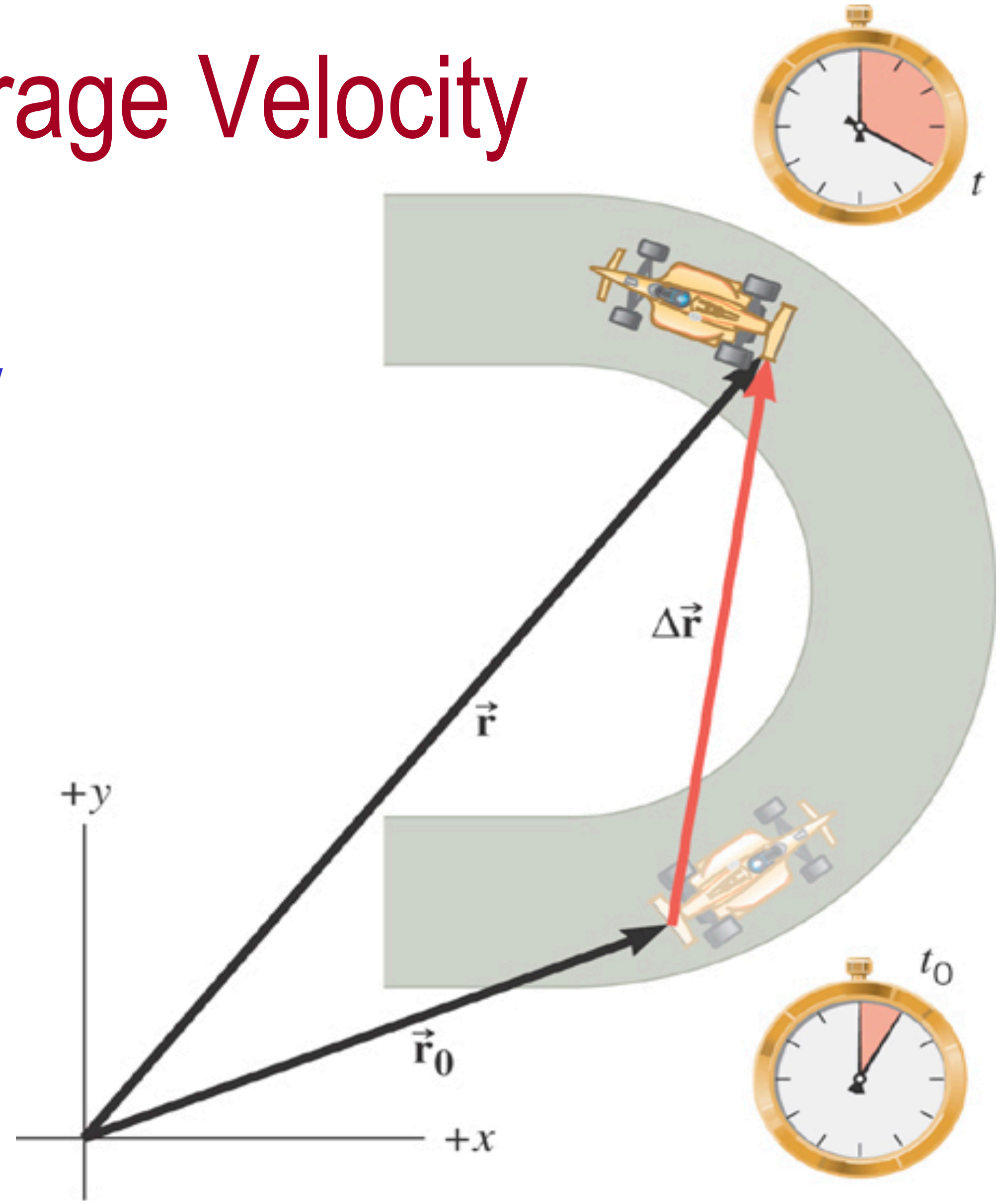
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

Average velocity is the displacement divided by the elapsed time.

$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



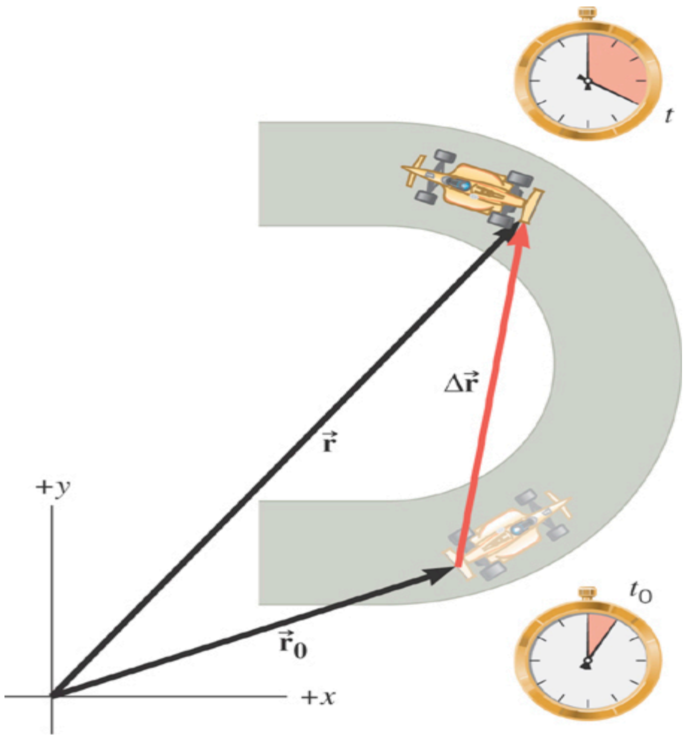
What is the direction of the Average velocity?

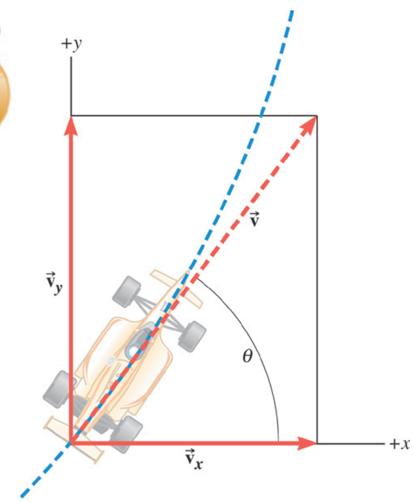
Right! The same as the displacement!

2D Instantaneous Velocity

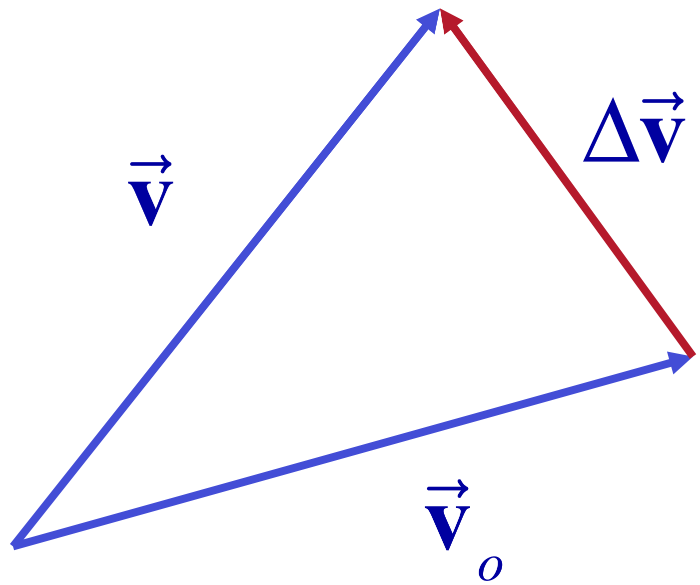
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration: $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

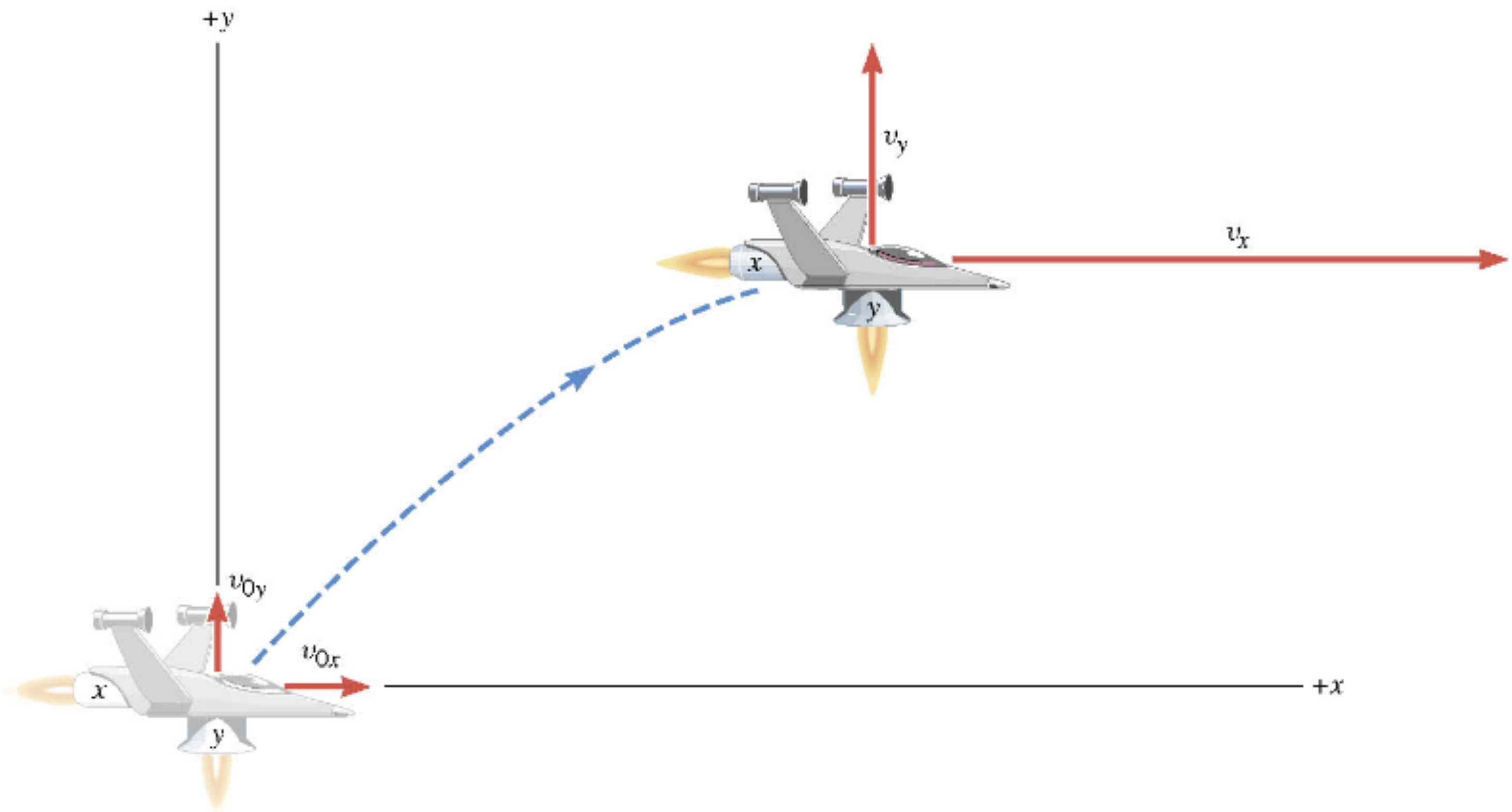
How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

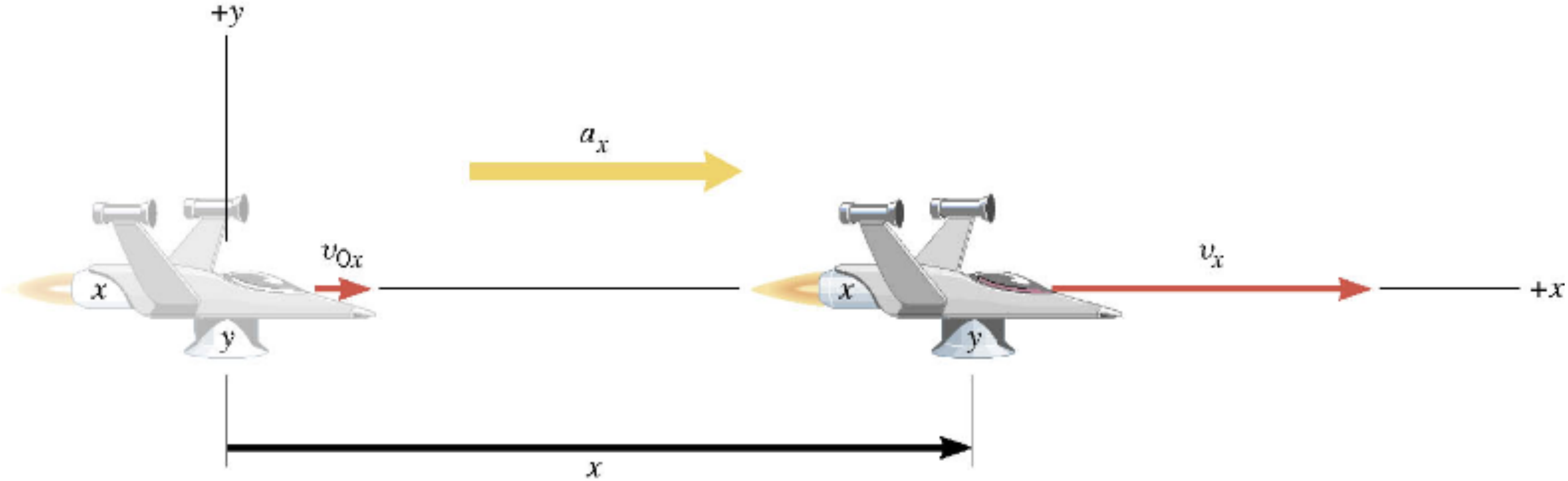
What is the difference between 1D and 2D quantities?

A Motion in 2 Dimension



This is a motion that could be viewed as two motions (x and y directions) combined into one. (superposition…)

Motion in horizontal direction (x)



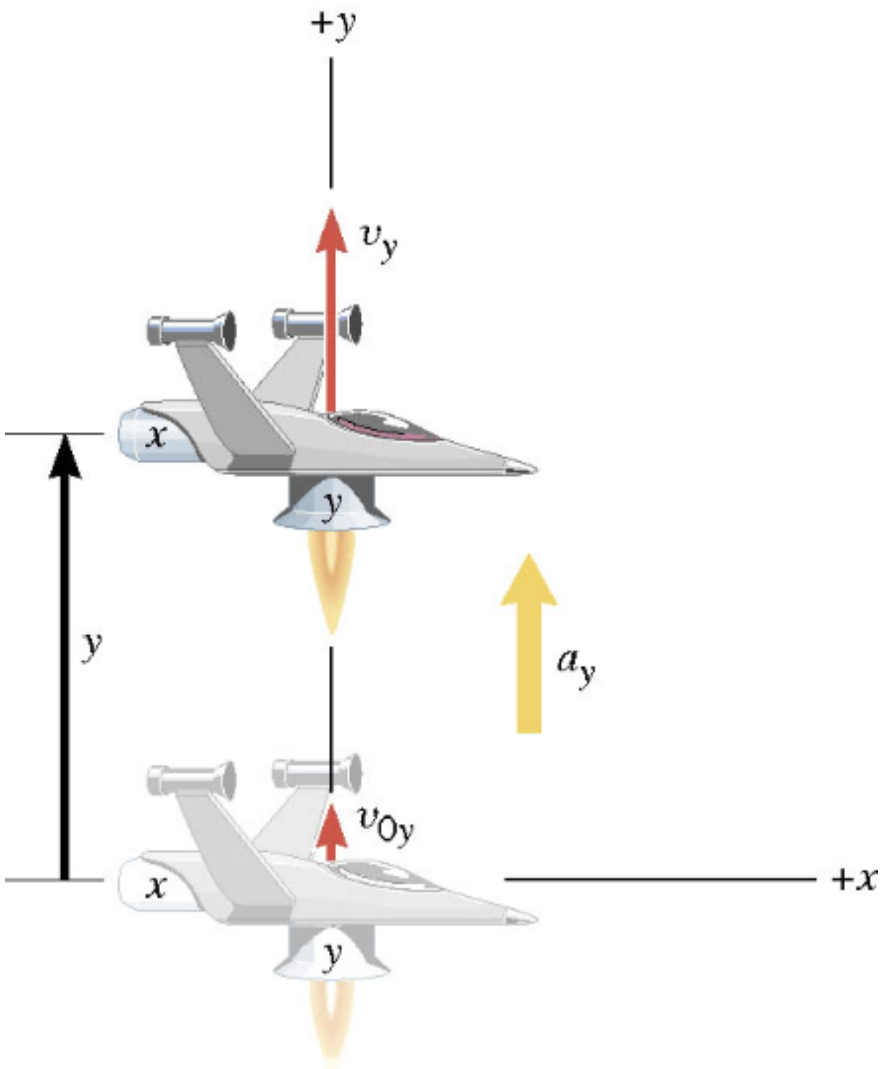
$$v_x = v_{xo} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{xo} + v_x\right) t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$\Delta x = v_{xo} t + \tfrac{1}{2} a_x t^2$$

Motion in vertical direction (y)



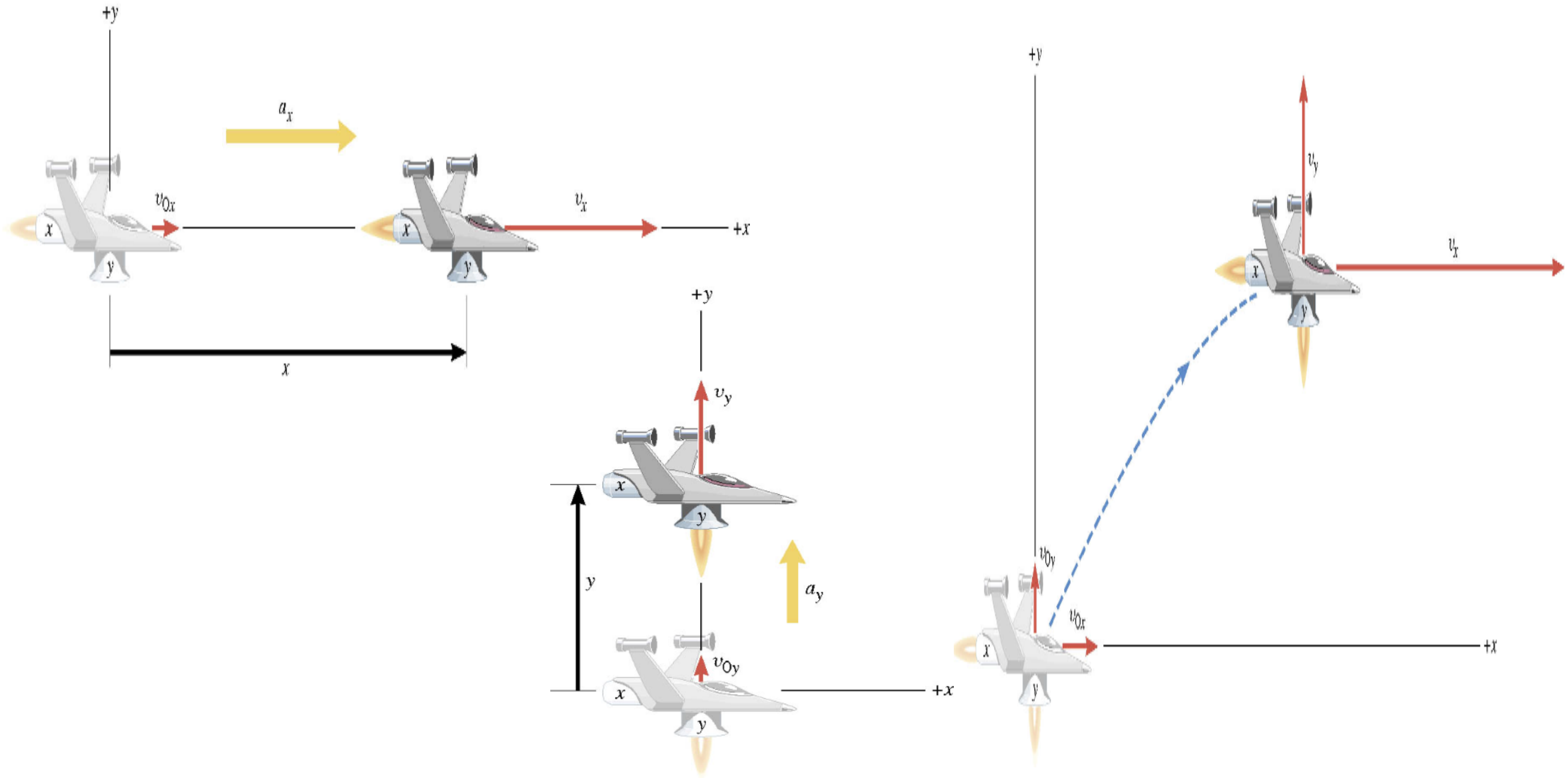
$$v_y = v_{yo} + a_y t$$

$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right) t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$\Delta y = v_{yo} t + \tfrac{1}{2} a_y t^2$$

A Motion in 2 Dimension



Imagine you add the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{xo} + a_x t$$

$$\Delta x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$\Delta x = v_{xo}t + \frac{1}{2}a_x t^2$$

y-component

$$v_y = v_{yo} + a_y t$$

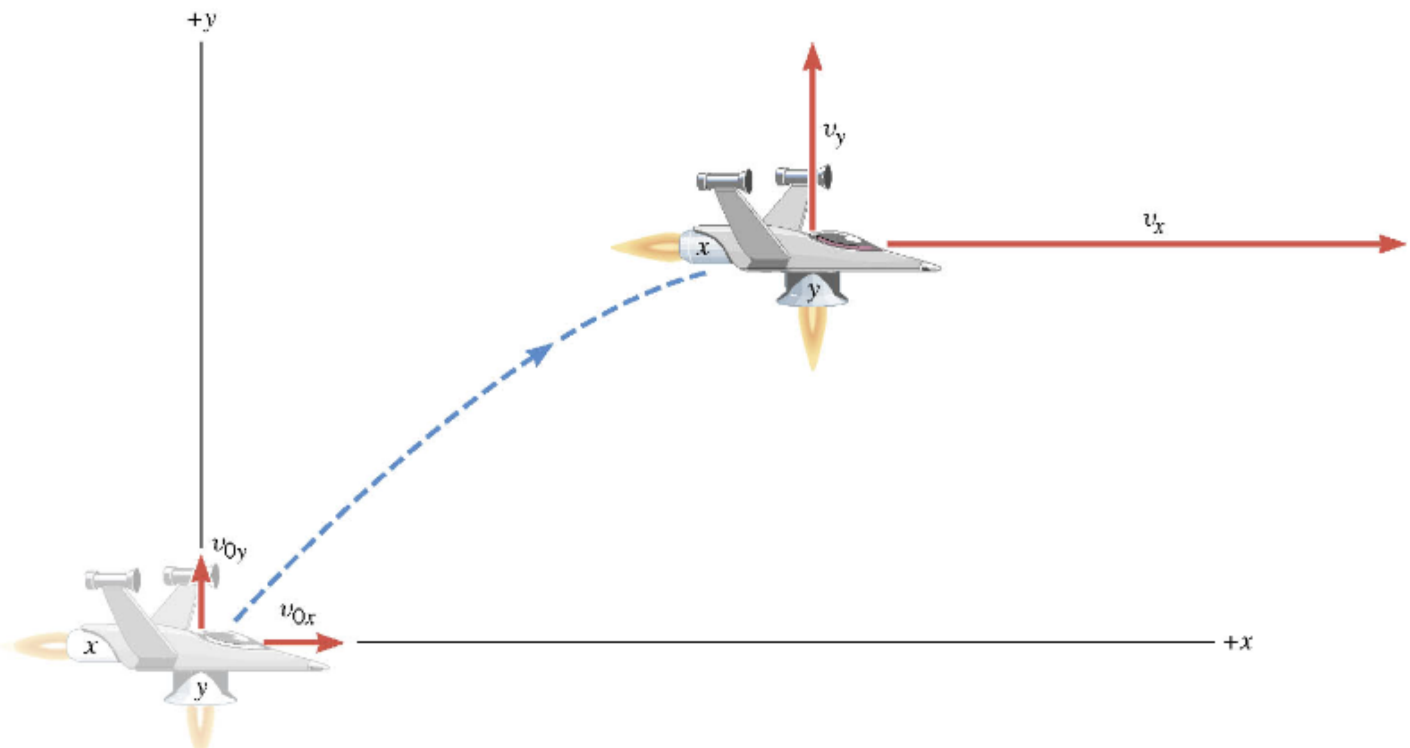
$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$\Delta y = v_{yo}t + \frac{1}{2}a_y t^2$$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-). Normal convention is the right-hand rule.
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y* *separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the *x* direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox} t + \tfrac{1}{2} a_x t^2$$

$$= (22\,\text{m/s})(7.0\ \text{s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\ \text{s})^2 = +740\ \text{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\ \text{s}) = +190\,\text{m/s}$$

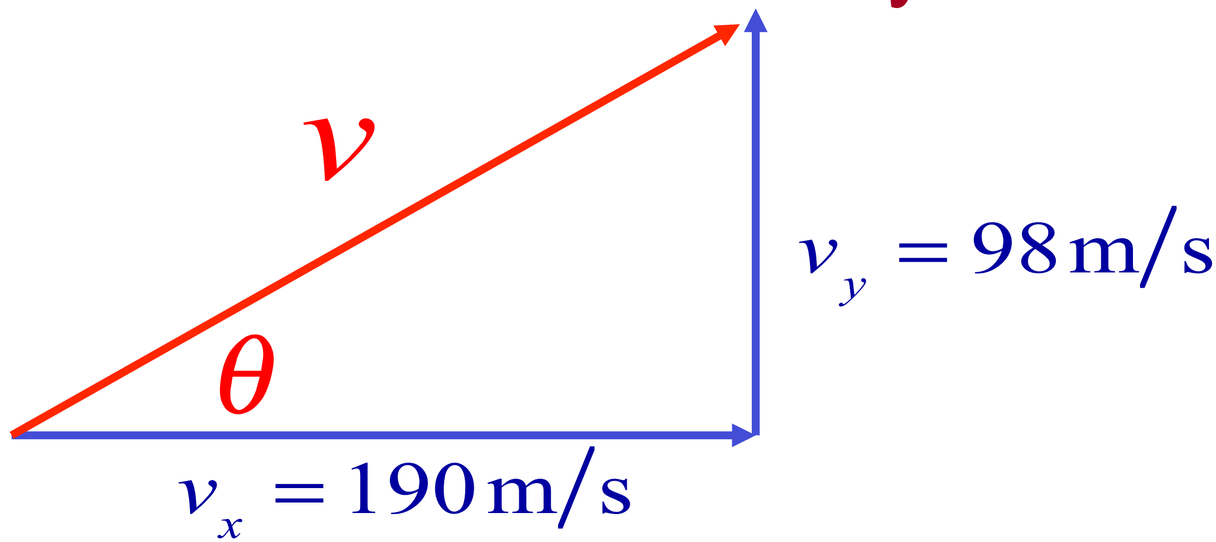
Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$
$$= (14\,\text{m/s})(7.0\text{ s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\text{ s})^2 = +390\text{ m}$$

$$v_y = v_{oy} + a_y t$$
$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\text{ s}) = +98\,\text{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

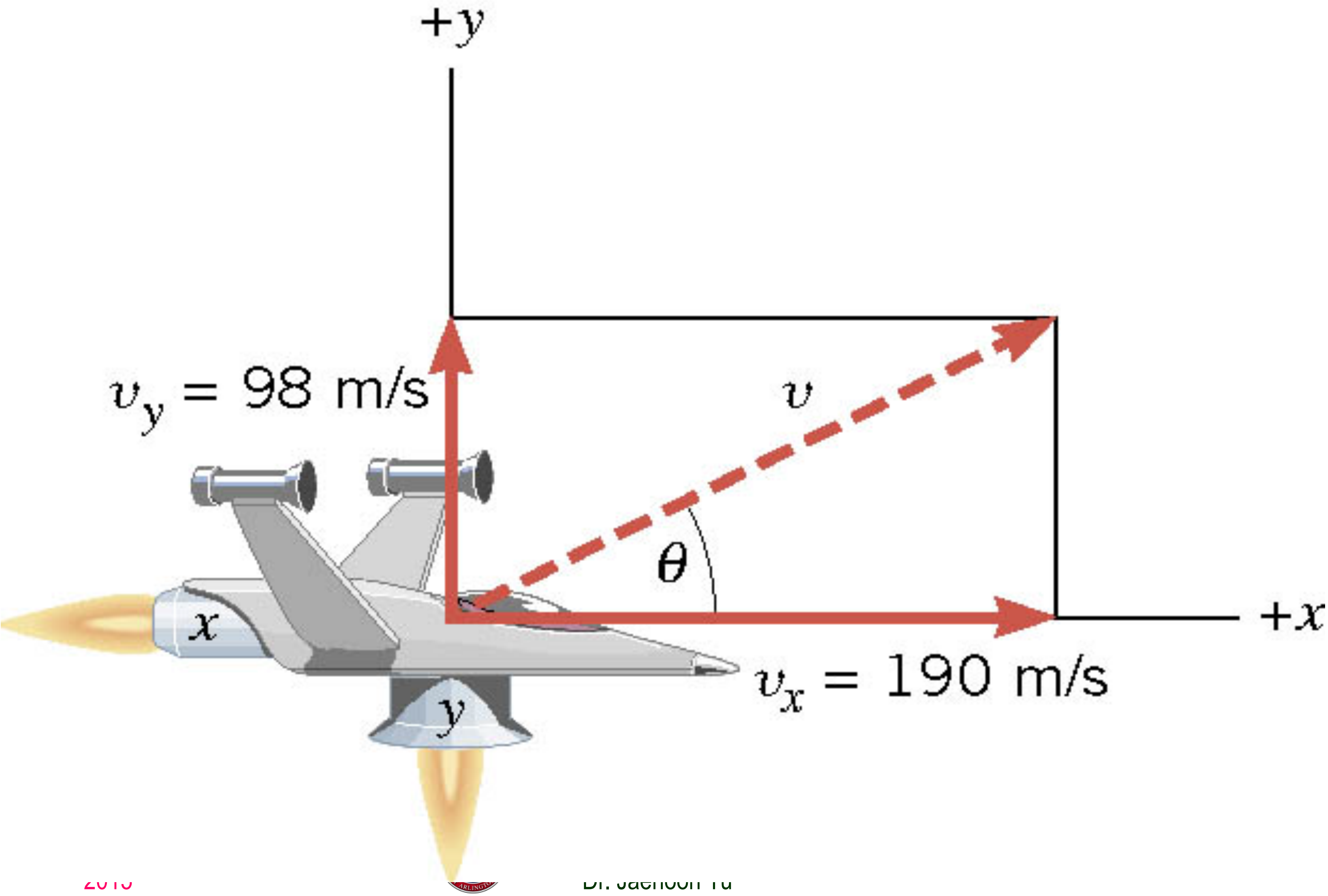
$$\theta = \tan^{-1}(98/190) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = \left(190\vec{i} + 98\vec{j}\right) m/s$$

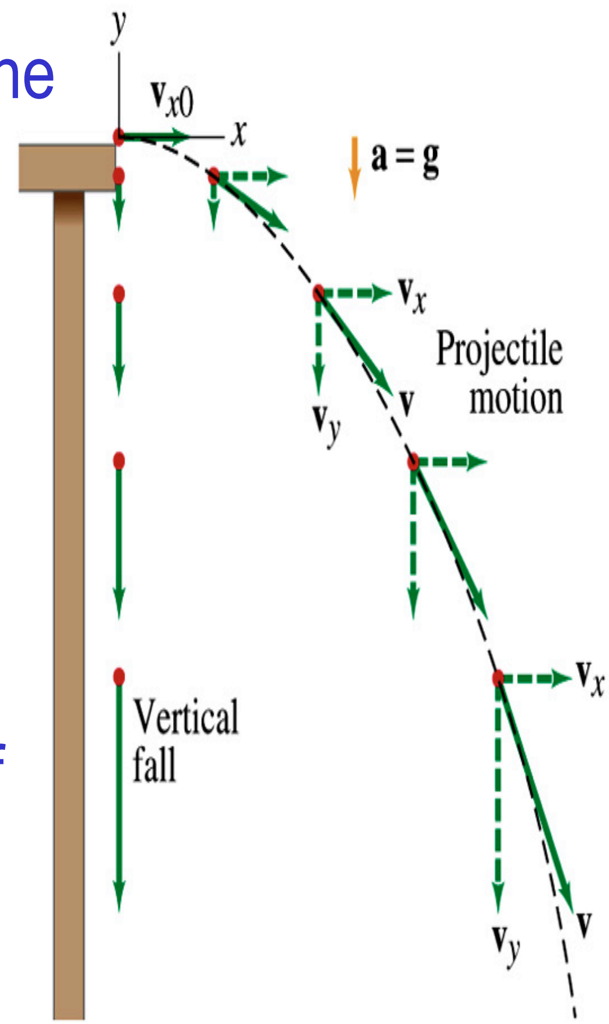
If we visualize the motion…



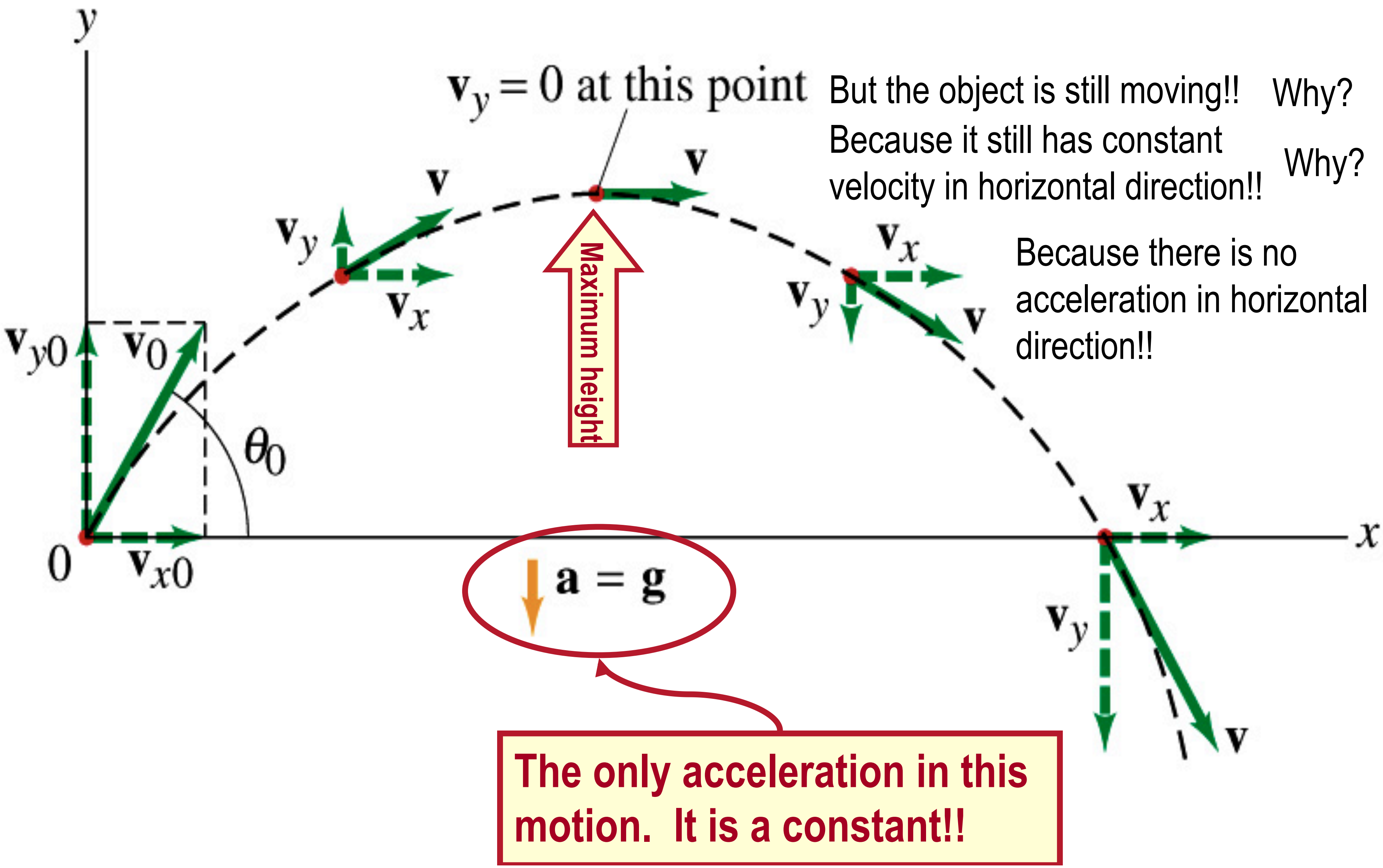
What is the Projectile Motion?

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is **constant** over the range of the motion
    - $\vec{g} = -9.8\vec{j}\ (m/s^2)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (**no acceleration**) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration (g)

$$v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$$



Projectile Motion



Kinematic Equations for a projectile motion

x-component

$$a_x = 0$$

$$v_x = v_{xo}$$

$$\Delta x = v_{xo} t$$

$$v_{x0}^2 = v_{xo}^2$$

$$\Delta x = v_{xo} t$$

y-component

$$a_y = -\left|\vec{g}\right| = -9.8\, m/s^2$$

$$v_y = v_{yo} - gt$$

$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right) t$$

$$v_y^2 = v_{yo}^2 - 2gy$$

$$\Delta y = v_{yo} t - \tfrac{1}{2} g t^2$$

Example for a Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance from the original position when the ball lands.

Which component determines the flight time and the distance?

Flight time is determined by the y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\,\text{sec}$$

$$\therefore t \approx 8\,\text{sec}$$

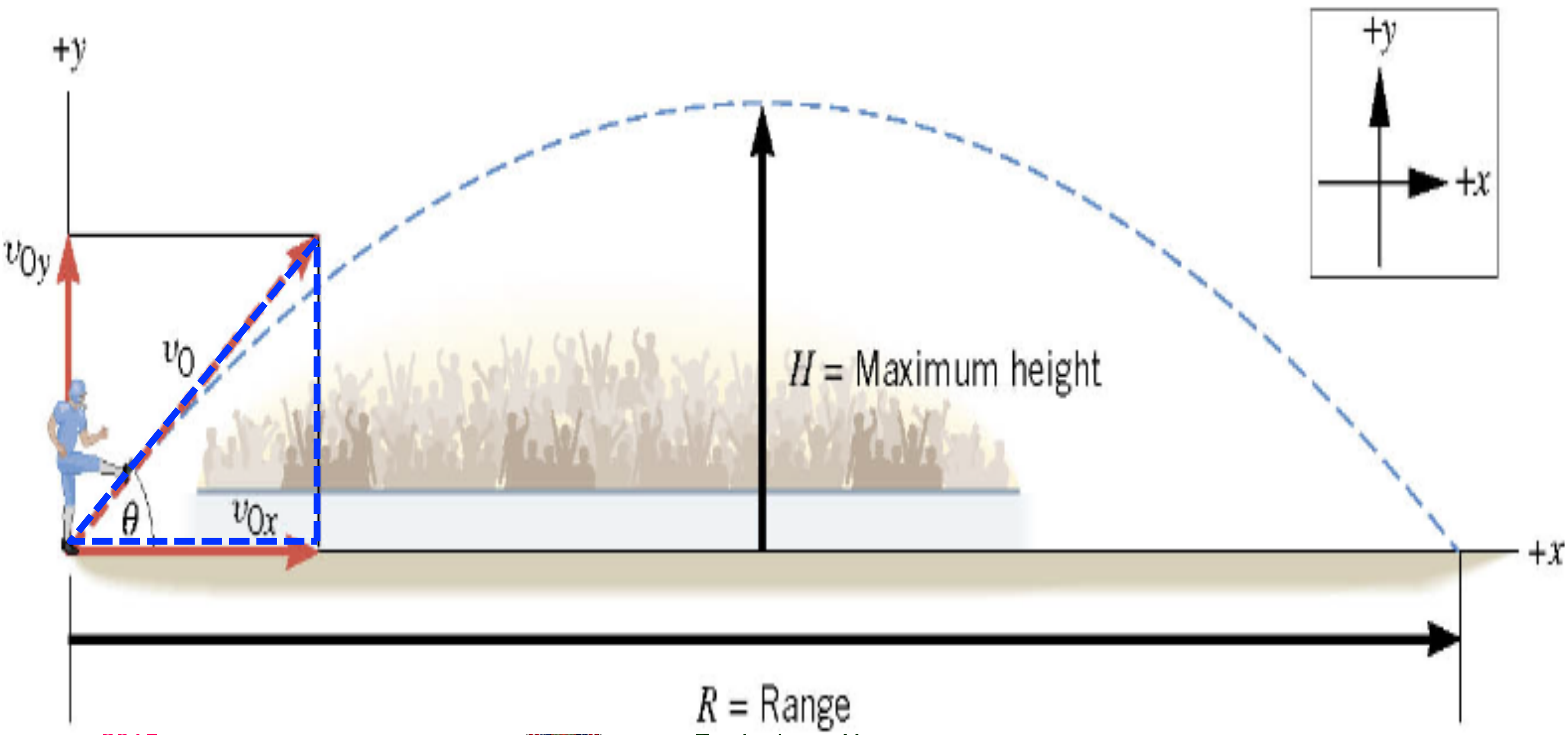
Why isn't 0 the solution?

Distance is determined by the x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

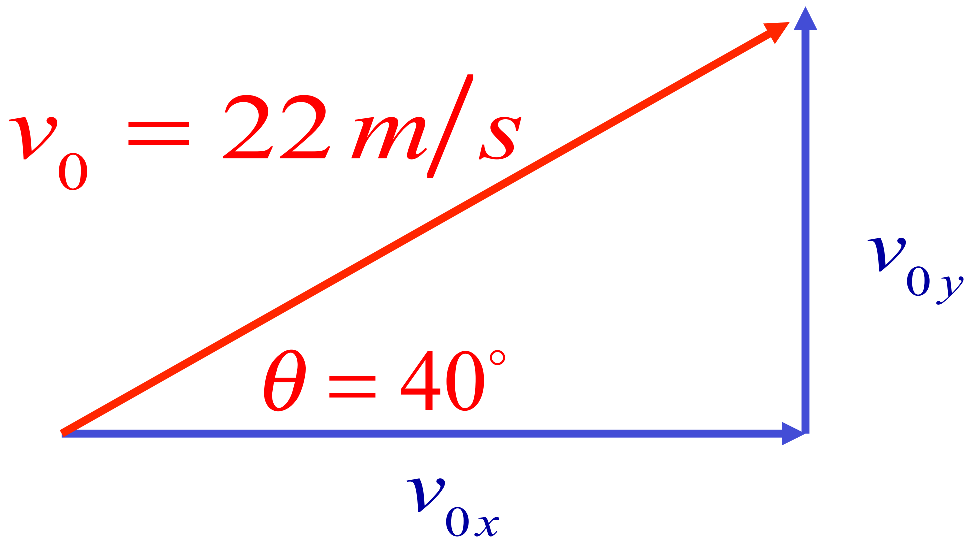
$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$

Ex.3.6 The Height of a Kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



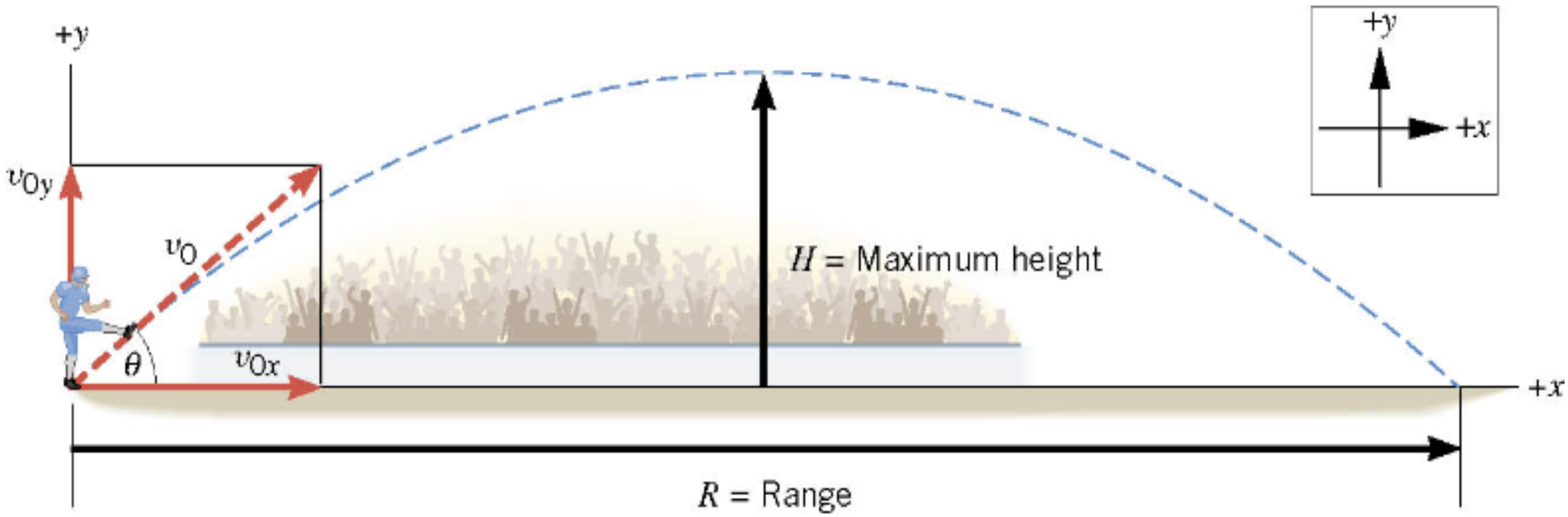
First, the initial velocity components



$$v_{0x} = v_o \cos\theta = (22\,\mathrm{m/s})\cos 40^\circ = 17\,\mathrm{m/s}$$

$$v_{0y} = v_o \sin\theta = (22\,\mathrm{m/s})\sin 40^\circ = 14\,\mathrm{m/s}$$

Motion in y-direction is of the interest..



y	a_y	v_y	v_{0y}	t
?	-9.8 m/s^2	0 m/s	+14 m/s	

Now the nitty, gritty calculations…

y	a_y	v_y	v_{0y}	t
?	-9.80 m/s^2	0	14 m/s	

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

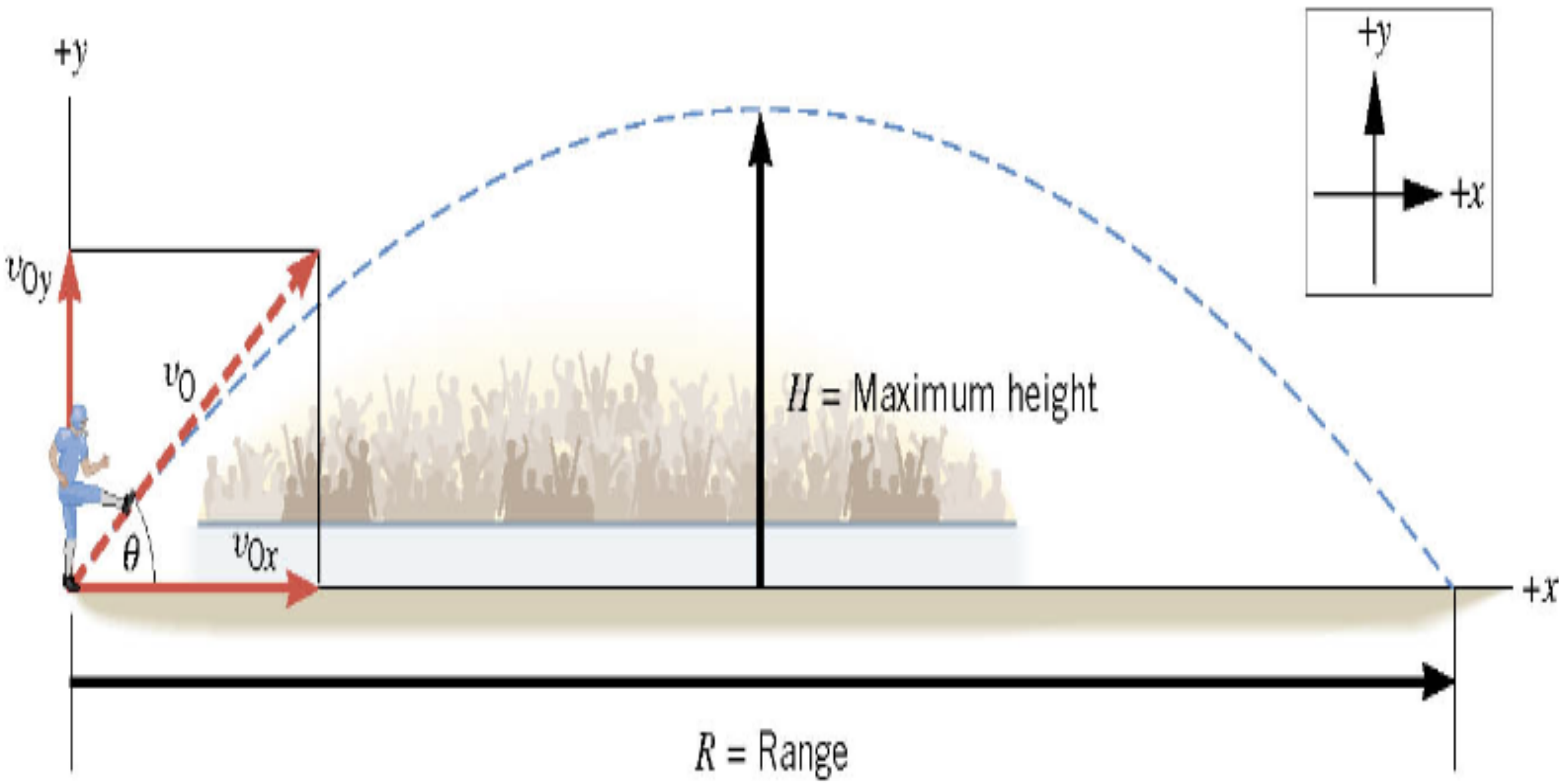
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

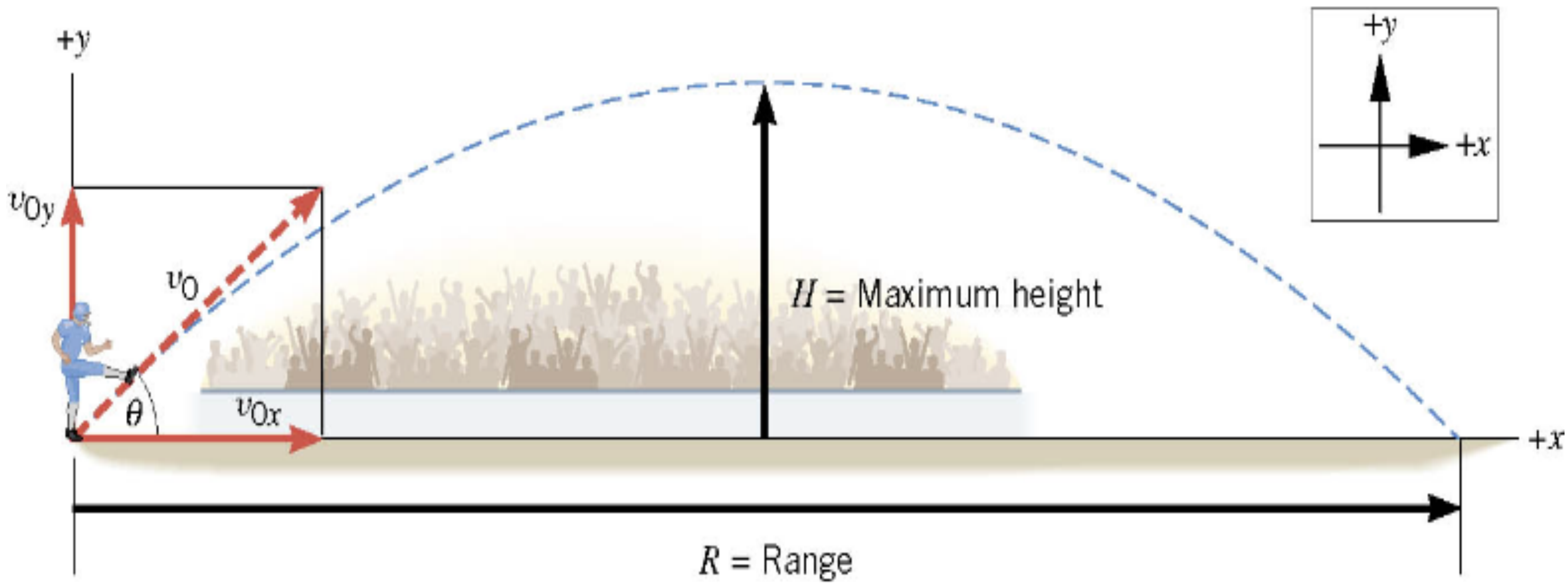
$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\ \mathrm{m}$$

Ex.3.6 extended: The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



What is y when it reached the max range?



y	a_y	v_y	v_{oy}	t
0 m	-9.80 m/s^2		14 m/s	?

Now solve the kinematic equations in y direction!!

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

Since y=0

$$0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

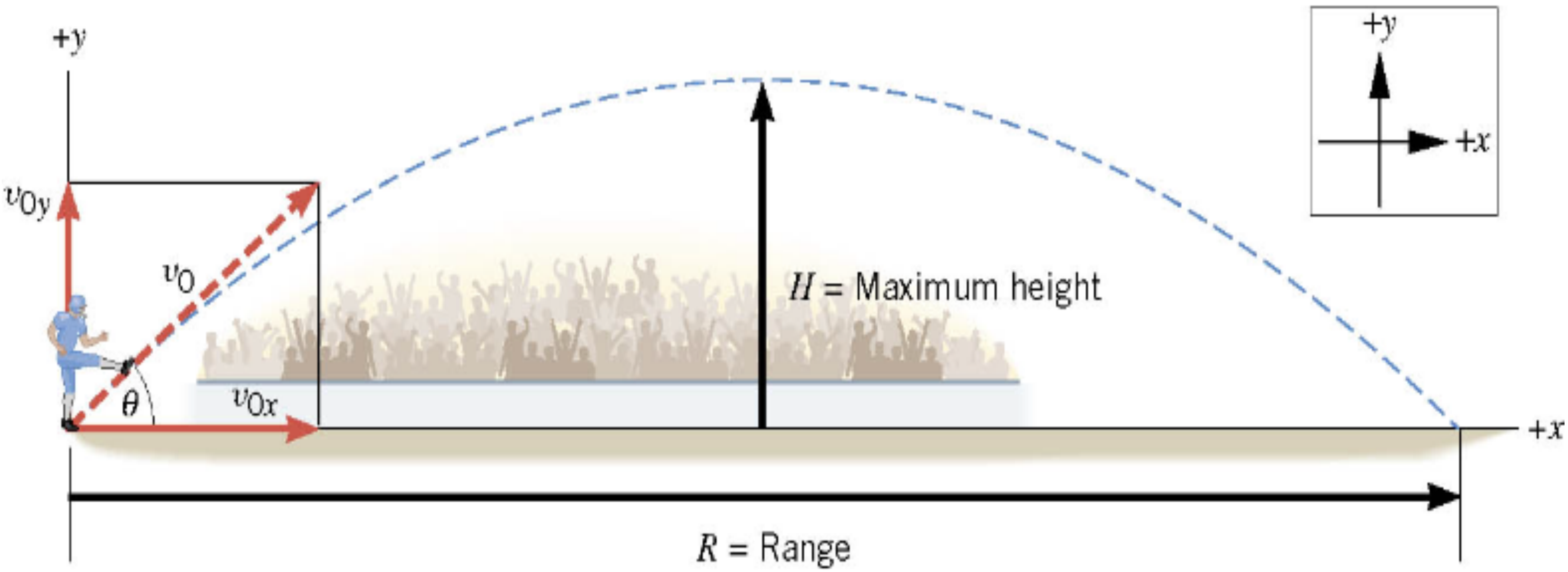
$$v_{oy} + \tfrac{1}{2}a_y t = 0$$

Solve for t

$$t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2 \cdot 14}{-9.8} = 2.9s$$

Ex.3.9 The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = (17\ \text{m/s})(2.9\ \text{s}) = +49\ \text{m}$$

Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi} t - \frac{1}{2} g t^2$$

Becomes

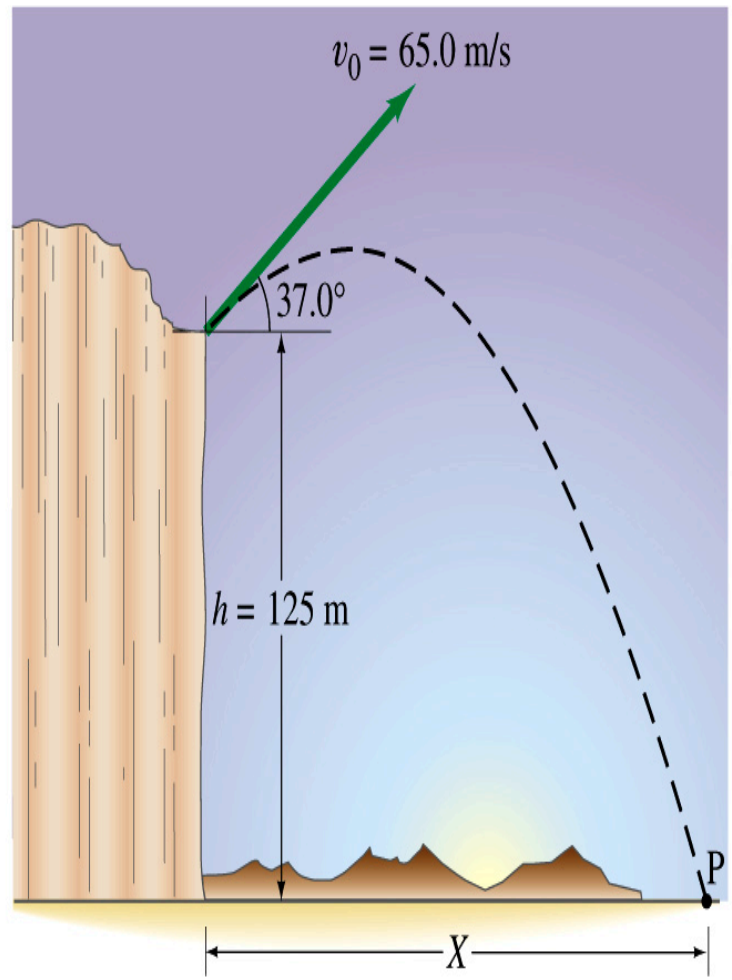
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43s \quad \text{or} \quad t = 10.4s$$

$$t = 10.4s$$

Since negative time does not exist.



Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

Do these yourselves at home for fun!!!