PHYS 1441 – Section 001
Lecture #15

Wednesday, July 8, 2015

Dr. Jaehoon Yu

- Concept of the Center of Mass
- Center of Mass & Center of Gravity
- Fundamentals of the Rotational Motion
- Rotational Kinematics
- Equations of Rotational Kinematics
- Relationship Between Angular and Linear Quantities

Announcements

- Reading Assignments: CH7.9 and 7.10
- Final exam
  - 10:30am – 12:30pm, Monday, July 13, in this room
  - Comprehensive exam, covers from CH1.1 – what we finish this Thursday, July 8, plus appendices A1 – A8
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam → no solutions, derivations or definitions!
    - No additional formulae or values of constants will be provided!

Reminder: Special Project #6

- Make a list of the ***rated power*** of all electric and electronic devices at your home and compiled them in a table. (2 points each for the first 10 items and 1 point for each additional item.)
  - What is an item?
    - Similar electric devices count as one item.
      - All light bulbs make up one item, computers another, refrigerators, TVs, dryers (hair and clothes), electric cooktops, heaters, microwave ovens, electric ovens, dishwashers, etc.
      - All you have to do is to count add all wattages of the light bulbs together as the power of the item
- Estimate the **cost of electricity** for each of the items (taking into account the number of hours you use the device) on the table using the electricity cost per kWh of the power company that serves you and put them in a separate column in the above table for each of the items. (2 points each for the first 10 items and 1 point each additional items). Clearly write down what the unit cost of the power is per kWh above the table.
- Estimate the the total amount of energy in Joules and the total electricity cost ***per month*** and ***per year*** for your home. (5 points)
- Due: Beginning of the class Monday, July 13

Special Project Spread Sheet

PHYS1441-001, Summer 15, Special Project #6

Download this spread sheet from URL: http://www-hep.uta.edu/~yu/teaching/summer15-1441-001/

Just click the file with the name: sp6-spreadsheet.xlsx

Write down at the top your name and the charge per kwh by your electricity company

Item Names	Rated power (W)	Number of devices	Average usage: Number of Hours per day	Daily			Monthly			Yearly		
				Power Consumption (kWh)	Energy Usage (J)	Energy cost ($)	Power Consumption (kWh)	Energy Usage (J)	Energy cost ($)	Power Consumption (kWh)	Energy Usage (J)	Energy cost ($)
Light Bulbs	30, 40, 60, 100, etc	40										
Heaters												
Fans												
Air Conditioner												
Fridgers, Freezers												
Computers												
Game consoles												
Total				0	0	0	0	0	0	0	0	0

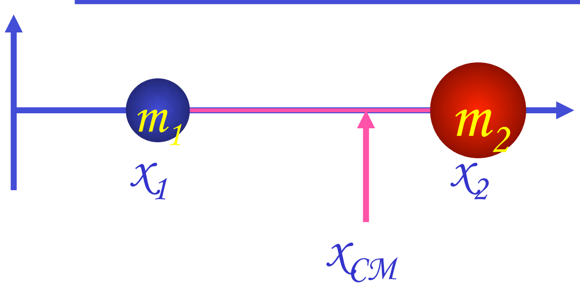
Center of Mass

We've been solving physical problems treating objects as sizeless points with masses, but in realistic situations objects have shapes with masses distributed throughout the body.

Center of mass of a system is the average position of the system's mass and represents the motion of the system as if all the mass is on that point.

What does above statement tell you concerning the forces being exerted on the system?

The total external force exerted on the system of total mass M causes the center of mass to move at an acceleration given by $\vec{a} = \sum \vec{F} / M$ *as if the entire mass of the system is on the center of mass.*



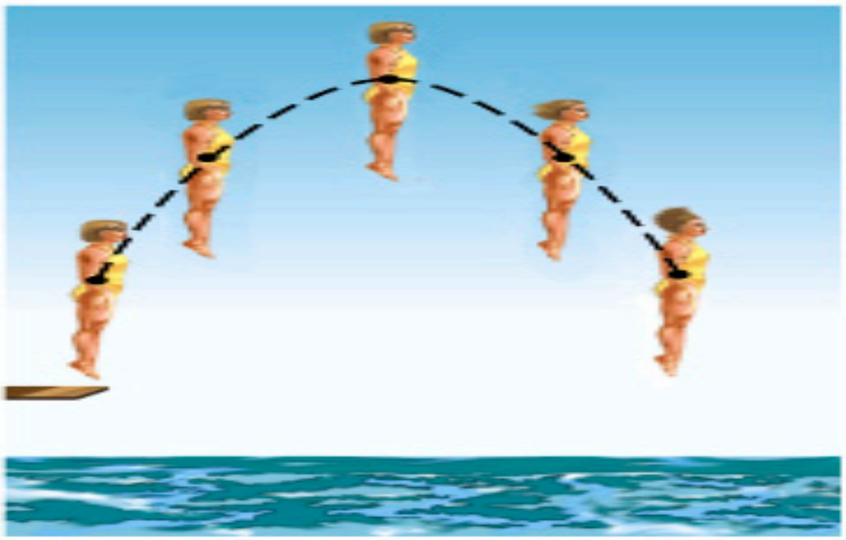
Consider a massless rod with two balls attached at either end.

The position of the center of mass of this system is the mass averaged position of the system

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

CM is closer to the heavier object

Motion of a Diver and the Center of Mass



(a)

Diver performs a simple dive.
The motion of the center of mass follows a parabola since it is a projectile motion.

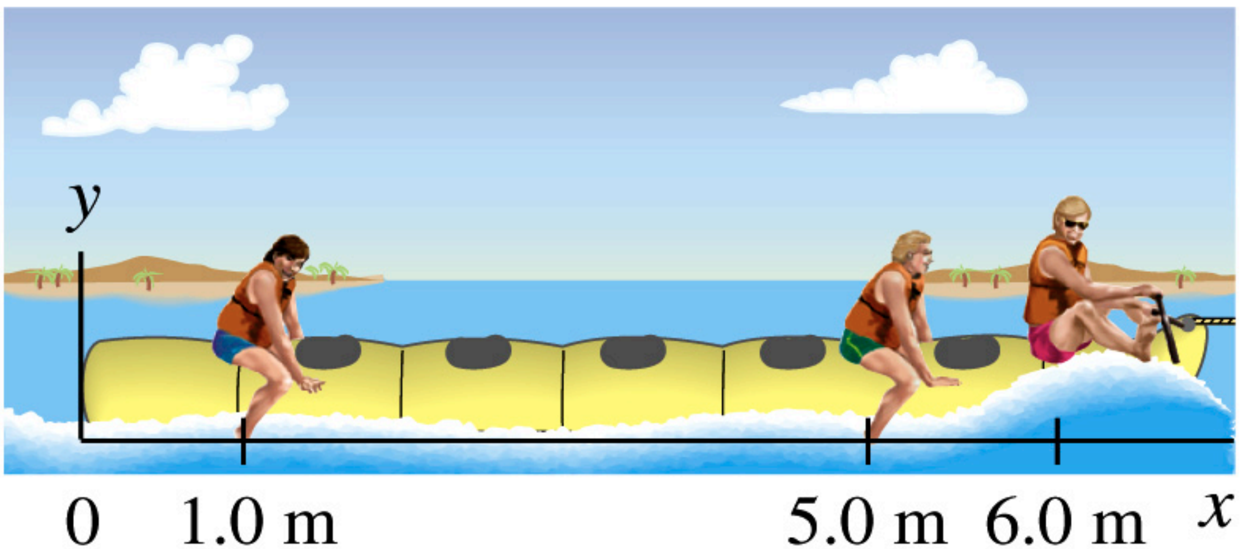
(b)

Diver performs a complicated dive.
The motion of the center of mass still follows the same parabola since it still is a projectile motion.

The motion of the center of mass of the diver is always the same.

Ex. 7 – 12 Center of Mass

Thee people of roughly equivalent mass M on a lightweight (air-filled) banana boat sit along the x axis at positions x_1=1.0m, x_2=5.0m, and x_3=6.0m. Find the position of CM.



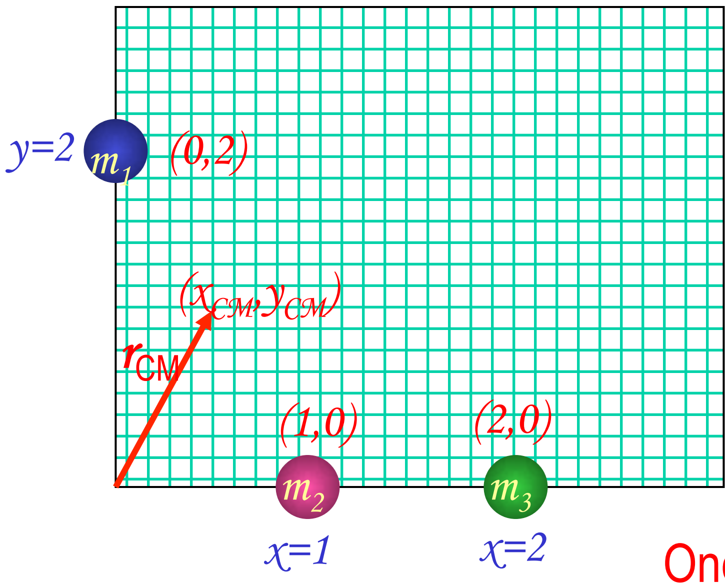
Using the formula for CM

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

$$= \frac{M \cdot 1.0 + M \cdot 5.0 + M \cdot 6.0}{M + M + M} = \frac{12.0M}{3M} = 4.0(m)$$

Example for Center of Mass in 2-D

A system consists of three particles as shown in the figure. Find the position of the center of mass of this system.



Using the formula for CM for each position vector component

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} \qquad y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i}$$

One obtains $\vec{r}_{CM} = x_{CM}\vec{i} + y_{CM}\vec{j} = \frac{(m_2 + 2m_3)\vec{i} + 2m_1\vec{j}}{m_1 + m_2 + m_3}$

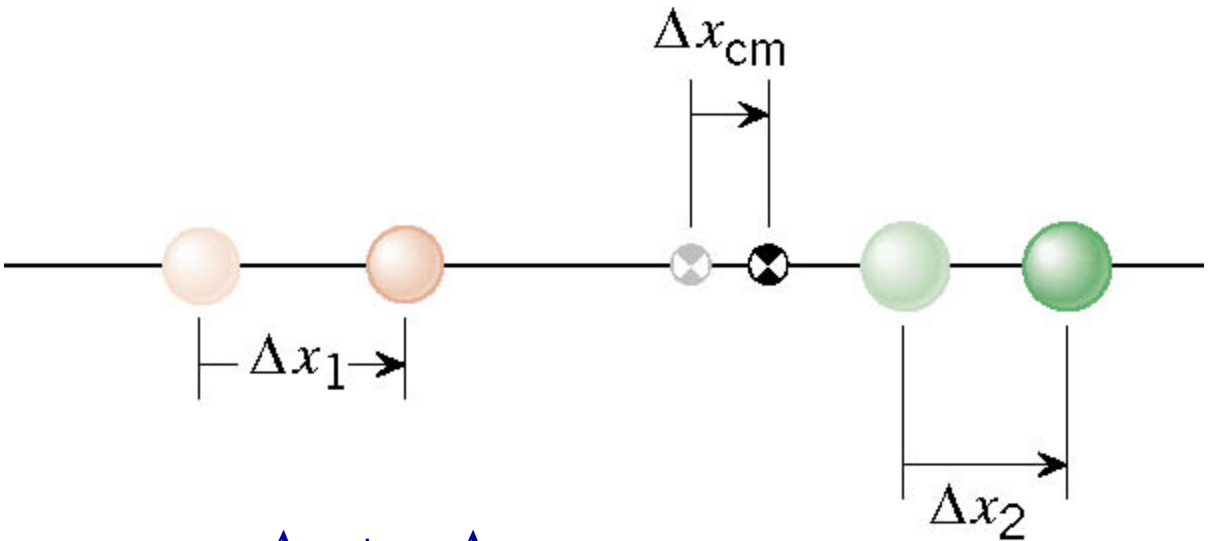
$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m_2 + 2m_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{\sum_i m_i y_i}{\sum_i m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{2m_1}{m_1 + m_2 + m_3}$$

If $m_1 = 2kg; m_2 = m_3 = 1kg$

$$\vec{r}_{CM} = \frac{3\vec{i} + 4\vec{j}}{4} = 0.75\vec{i} + \vec{j}$$

Velocity of the Center of Mass



$$\Delta x_{cm} = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}$$

$$\Rightarrow v_{cm} = \frac{\Delta x_{cm}}{\Delta t} = \frac{m_1 \Delta x_1 / \Delta t + m_2 \Delta x_2 / \Delta t}{m_1 + m_2} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

In an isolated system, the total linear momentum does not change, therefore the velocity of the center of mass does not change.

Another Look at the Ice Skater Problem

Starting from rest, two skaters push off against each other on ice where friction is negligible. One is a 54-kg woman and one is a 88-kg man. The woman moves away with a velocity of +2.5 m/s. What are the man's velocity and that of the CM?

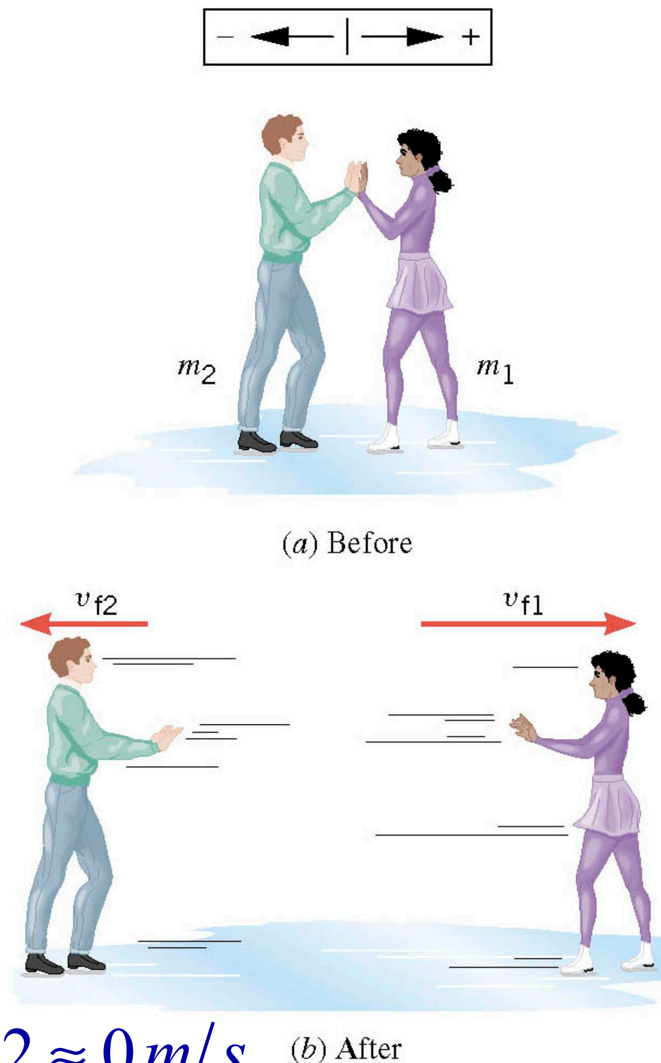
$$v_{10} = 0\,m/s \qquad v_{20} = 0\,m/s$$

$$v_{cm0} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = 0$$

$$v_{1f} = +2.5\,m/s \qquad v_{2f} = -1.5\,m/s$$

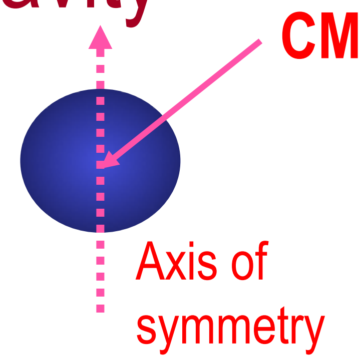
$$v_{cmf} = \frac{m_1 v_{1f} + m_2 v_{2f}}{m_1 + m_2}$$

$$= \frac{54\cdot(+2.5)+88\cdot(-1.5)}{54+88} = \frac{3}{142} = 0.02 \approx 0\,m/s$$



Center of Mass and Center of Gravity

The center of mass of any symmetric object lies on the axis of symmetry and on any plane of symmetry, if the object's mass is evenly distributed throughout the body.

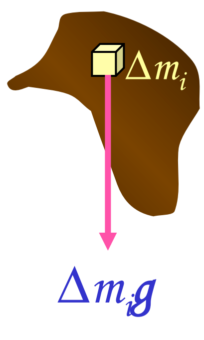


How do you think you can determine the CM of the objects that are not symmetric?

One can use gravity to locate CM.

1. Hang the object by one point and draw a vertical line following a plum-bob.
2. Hang the object by another point and do the same.
3. The point where the two lines meet is the CM.

Center of Gravity



Since a rigid object can be considered as a **collection of small masses**, one can see the total gravitational force exerted on the object as

$$\vec{F}_g = \sum_i \vec{F}_i = \sum_i \Delta m_i \vec{g} = M\vec{g}$$

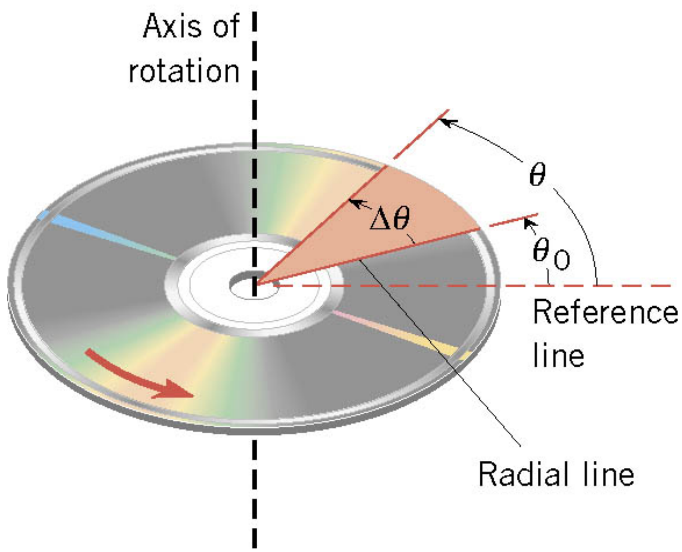
What does this equation tell you?

The net effect of these small gravitational forces is equivalent to a single force acting on a point (Center of Gravity) with mass M.

The CoG is the point in an object as if all the gravitational force is acting on!

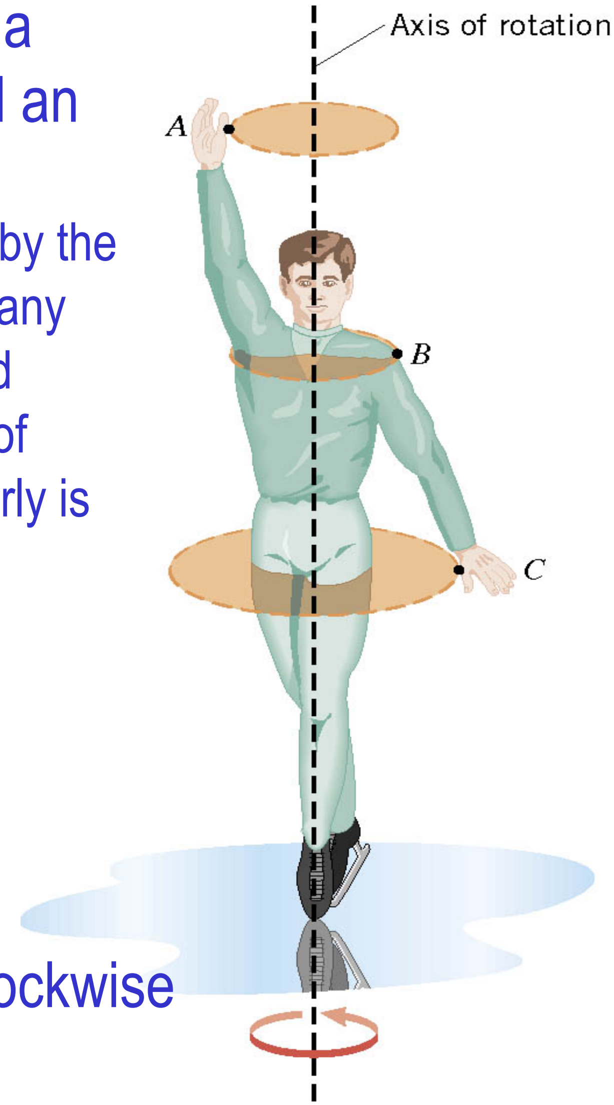
Rotational Motion and Angular Displacement

In the simplest kind of rotation, a point on a rigid object move on circular paths around an ***axis of rotation.***



The angle swept out by the line passing through any point on the body and intersecting the axis of rotation perpendicularly is called the ***angular displacement.***

$$\Delta\theta = \theta - \theta_o$$



It's a vector!! So there must be a direction…

How do we define directions?

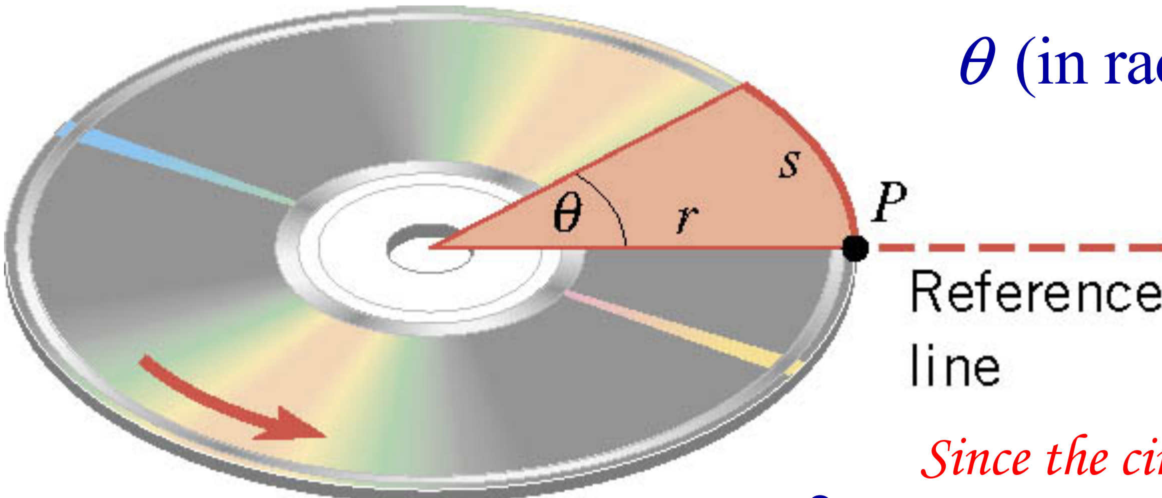
+:if counter-clockwise

-:if clockwise

The direction vector points gets determined based on the right-hand rule.

These are just conventions!!

SI Unit of the Angular Displacement



$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Dimension? None

For one full revolution:

Since the circumference of a circle is 2πr

$$\theta = \frac{2\pi r}{r} = 2\pi \text{ rad} \quad \Rightarrow \quad 2\pi \text{ rad} = 360^{\circ}$$

One radian is an angle subtended by an arc of the same length as the radius!

Unit of the Angular Displacement

How many degrees are in one radian?

1 radian is

$$1\text{ rad}=\frac{360^{\circ}}{2\pi rad}\cdot 1rad=\frac{180^{\circ}}{\pi}\cong\frac{180^{\circ}}{3.14}\cong 57.3^{\text{o}}$$

How radians is one degree?

And one degrees is

$$1^{\text{o}}=\frac{2\pi}{360^{\text{o}}}\cdot 1^{\text{o}}=\frac{\pi}{180^{\text{o}}}\cdot 1^{\text{o}}\cong\frac{3.14}{180^{\text{o}}}\cdot 1^{\text{o}}\cong 0.0175rad$$

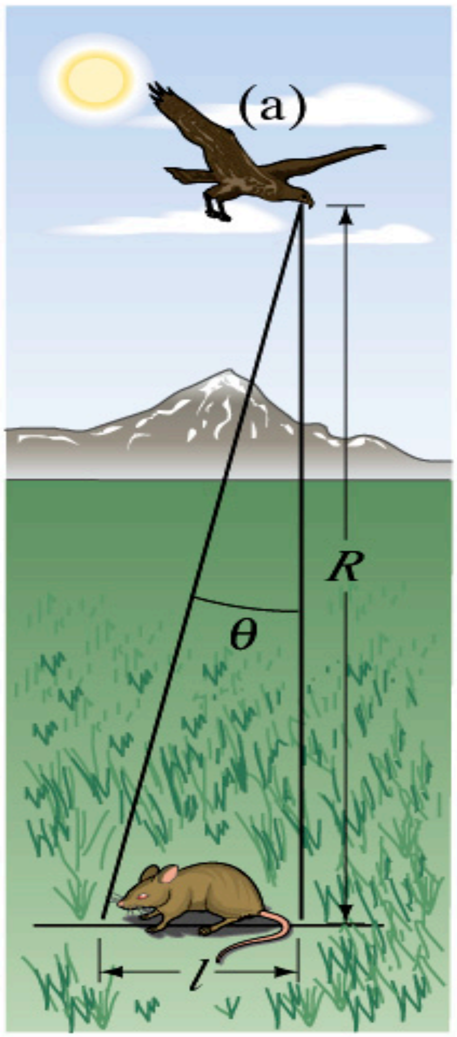
How many radians are in 10.5 revolutions?

$$10.5rev=10.5rev\cdot 2\pi\frac{rad}{rev}=21\pi(rad)$$

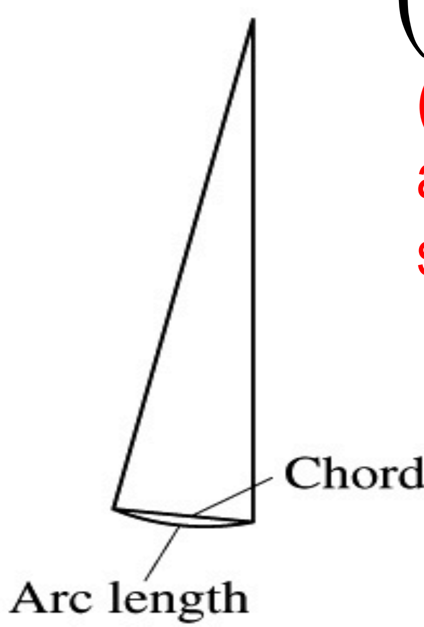
Very important: In solving angular problems, all units, degrees or revolutions, must be converted to radians.

Example 8-2

A particular bird's eyes can just distinguish objects that subtend an angle no smaller than about 3×10^{-4} rad. (a) How many degrees is this? (b) How small an object can the bird just distinguish when flying at a height of 100m?



(b)



(a) One radian is $360^{o}/2\pi$. Thus

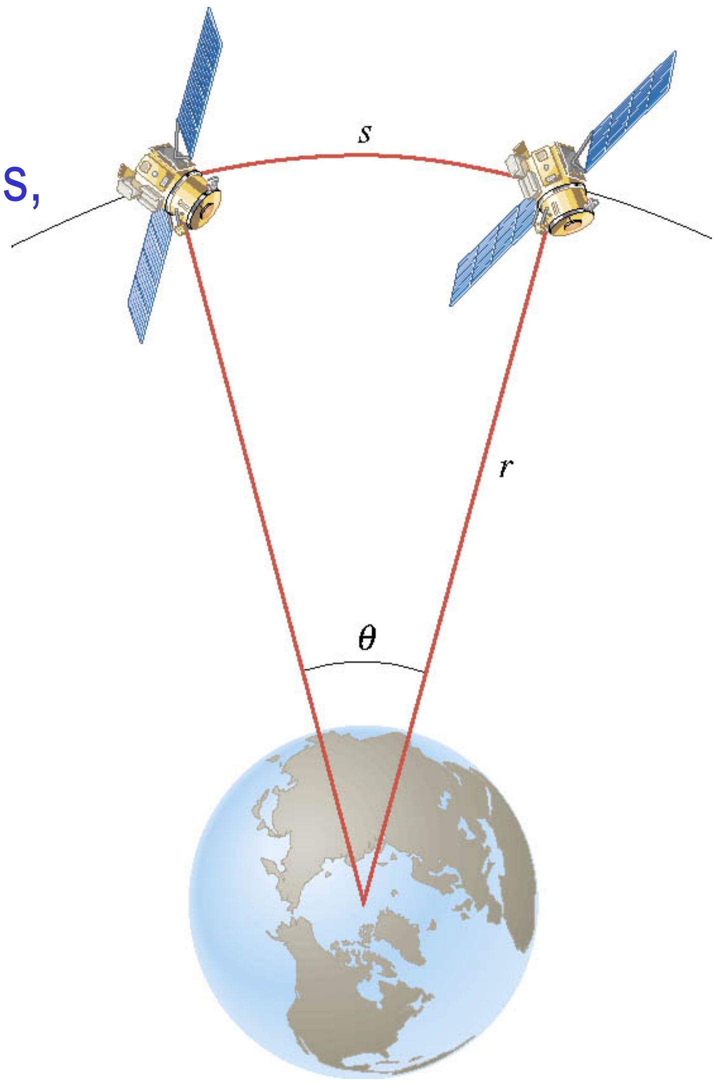
$$3\times10^{-4}\,rad = \left(3\times10^{-4}\,rad\right)\times\left(360^{o}/2\pi\,rad\right) = 0.017^{o}$$

(b) Since $l=r\theta$ and for small angle arc length is approximately the same as the chord length.

$$l = r\theta = 100m\times3\times10^{-4}\,rad = 3\times10^{-2}\,m = 3cm$$

Ex. Adjacent Synchronous Satellites

Synchronous satellites are put into an orbit whose radius is 4.23×10^7m. If the angular separation of the two satellites is 2.00 degrees, find the arc length that separates them.



What do we need to find out? The arc length!!!

$$\theta\text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$

Convert degrees to radians

$$2.00\,\text{deg}\left(\frac{2\pi\text{ rad}}{360\,\text{deg}}\right) = 0.0349\text{ rad}$$

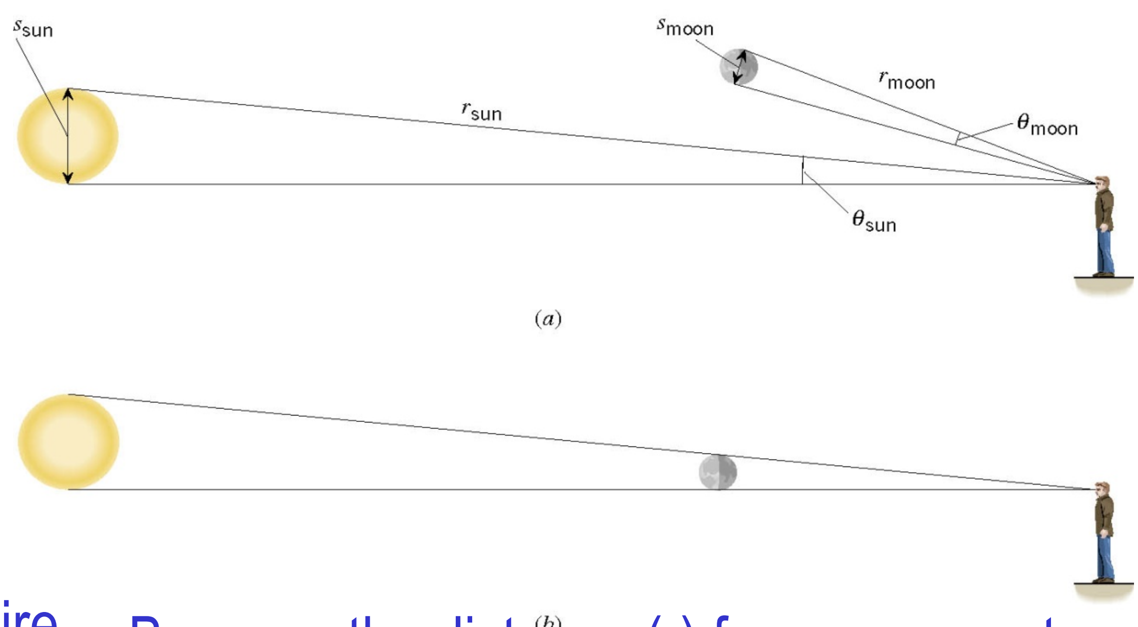
$$s = r\theta = \left(4.23\times10^{7}\,\text{m}\right)\left(0.0349\text{ rad}\right)$$

$$= 1.48\times10^{6}\,\text{m}\ \ (920\text{ miles})$$

Ex. A Total Eclipse of the Sun

The diameter of the sun is about 400 times greater than that of the moon. By coincidence, the sun is also about 400 times farther from the earth than is the moon. For an observer on the earth, compare the angle subtended by the moon to the angle subtended by the sun and explain why this result leads to a total solar eclipse.

$$\theta \text{ (in radians)} = \frac{\text{Arc length}}{\text{Radius}} = \frac{s}{r}$$



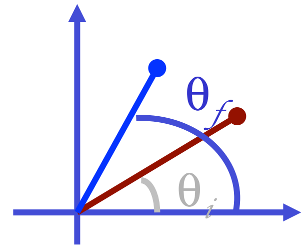
I can even cover the entire sun with my thumb!! Why?

Because the distance (r) from my eyes to my thumb is far shorter than that to the sun.

Angular Displacement, Velocity, and Acceleration

Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$



How about the average angular velocity, the rate of change of angular displacement?

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit? rad/s Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as…

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit? rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Ex. Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s. Find the average angular velocity of the gymnast.

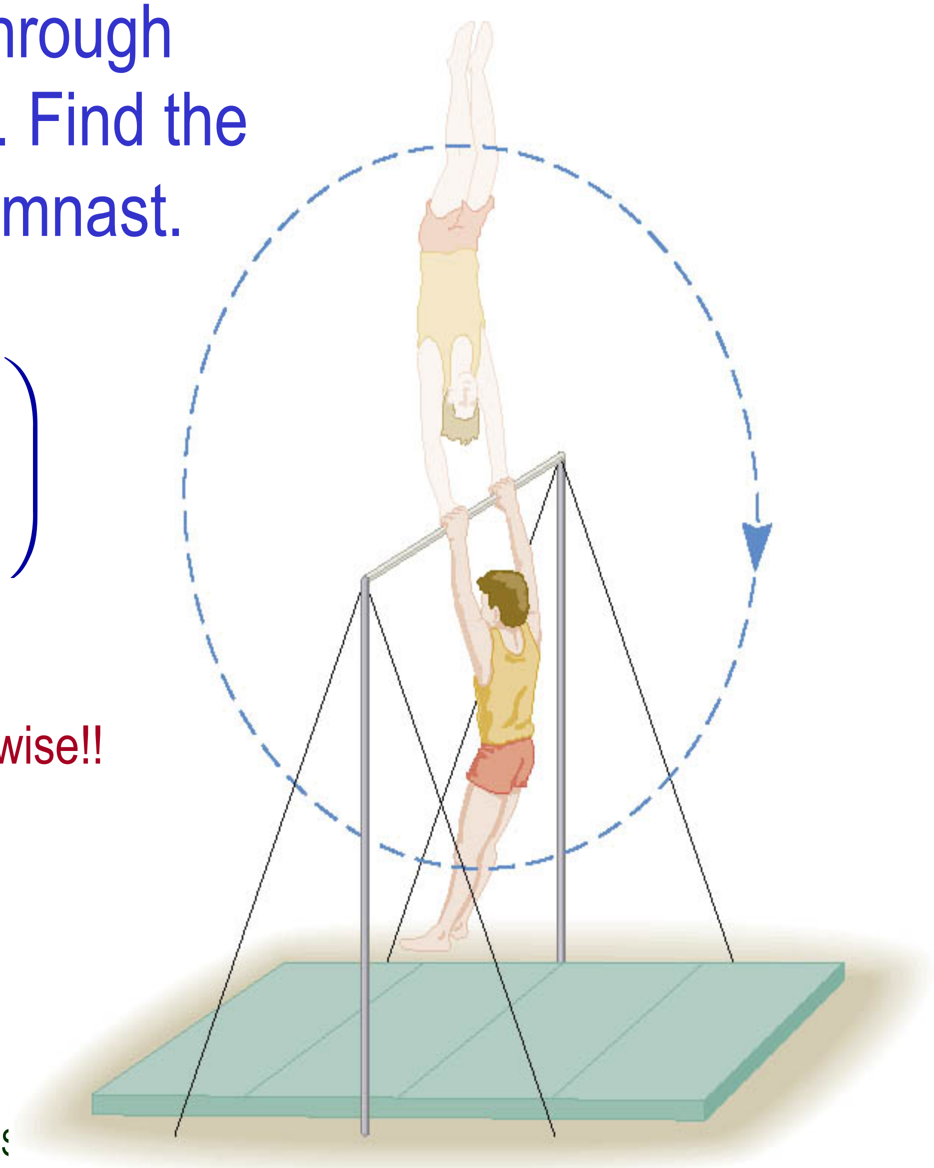
What is the angular displacement?

$$\Delta\theta = -2.00\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)$$

$$= -12.6\text{ rad}$$

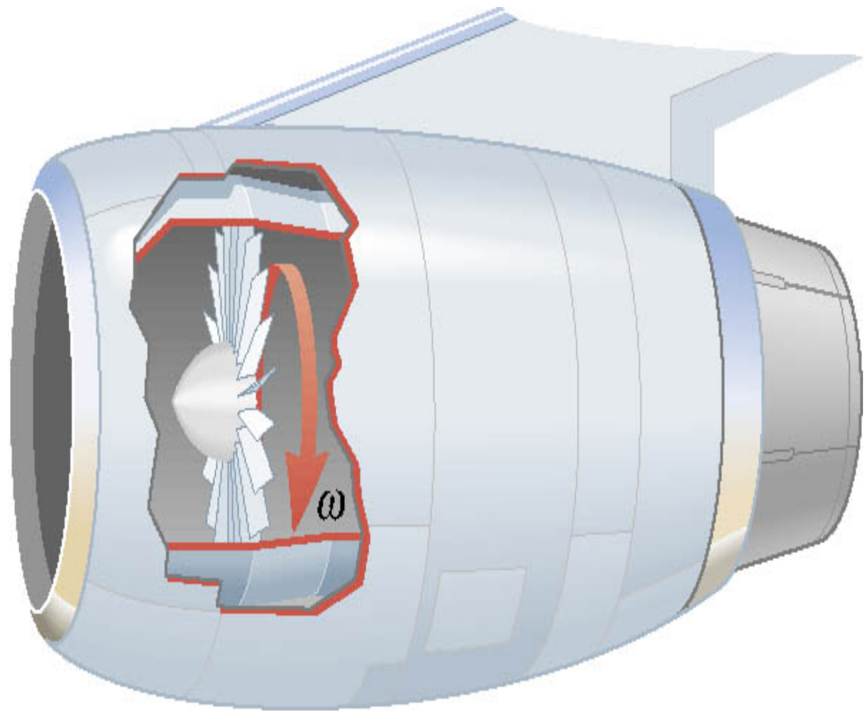
Why negative? Because he is rotating clockwise!!

$$\bar{\omega} = \frac{-12.6\text{ rad}}{1.90\text{ s}} = -6.63\,\text{rad/s}$$



Ex. A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular velocity of -110 rad/s. As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s. Find the angular acceleration, assuming it to be constant.



$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\ \text{s}} = -16\,\text{rad/s}^2$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration (α), because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

$$\omega_f = \omega_0 + \alpha t$$

Translational kinematics $v = v_o + at$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Translational kinematics $x_f = x_0 + v_o t + \frac{1}{2} a t^2$

One can also obtain

$$\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$$

Translational kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

Rotational Kinematics Problem Solving Strategy

- Visualize the problem by drawing a picture.
- Write down the values that are given for any of the five kinematic variables and convert them to SI units.
  - Remember that the unit of the angle must be in radians!!
- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.
- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.
- Keep in mind that there may be two possible answers to a kinematics problem.

Example for Rotational Kinematics

A wheel rotates with a constant angular acceleration of +3.50 rad/s^2. If the angular velocity of the wheel is +2.00 rad/s at t_i=0, a) through what angle does the wheel rotate in 2.00s?

Using the angular displacement formula in the previous slide, one gets

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

$$= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2 = 11.0 rad$$

$$= \frac{11.0}{2\pi} rev. = 1.75 rev.$$

Example for Rotational Kinematics cnt'd

What is the angular velocity at t=2.00s?

Using the angular speed and acceleration relationship

$$\omega_f = \omega_i + \alpha t \quad = 2.00 + 3.50 \times 2.00 = +9.00 rad/s$$

Find the angle through which the wheel rotates between t=2.00 s and t=3.00 s.

Using the angular kinematic formula

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

At t=2.00s

$$\theta_{t=2} = 2.00 \times 2.00 + \frac{1}{2} 3.50 \times 2.00 = 11.0 rad$$

At t=3.00s

$$\theta_{t=3} = 2.00 \times 3.00 + \frac{1}{2} 3.50 \times (3.00)^2 = 21.8 rad$$

Angular displacement

$$\Delta\theta = \theta_3 - \theta_2 = 10.8 rad = \frac{10.8}{2\pi} rev. = 1.72 rev.$$

Ex. Blending with a Blender

The blade is whirling with an angular velocity of +375 rad/s when the "puree" button is pushed in. When the "blend" button is pushed, the blade accelerates and reaches a greater angular velocity after the blade has rotated through an angular displacement of +44.0 rad. The angular acceleration has a constant value of +1740 rad/s^2. Find the final angular velocity of the blade.

θ	α	ω	ω_o	t
+44.0rad	+1740rad/s^2	?	+375rad/s	

Which kinematic eq? $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega = \pm\sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \pm\sqrt{(375\,\text{rad/s})^2 + 2(1740\,\text{rad/s}^2)(44.0\,\text{rad})} = \pm 542\,\text{rad/s}$$

Which sign? $\omega = +542\,\text{rad/s}$ *Why? Because the blade is accelerating in counter-clockwise!*