

PHYS 1441 – Section 001

Lecture #7

Wednesday, June 15, 2016

Dr. Jaehoon Yu

- Chapter 23 Electric Potential
 - Electric Potential due to Point Charges
 - Shape of the Electric Potential
 - V due to Charge Distributions
 - Equi-potential Lines and Surfaces
 - Electric Potential Due to Electric Dipole
- Chapter 24 Capacitance etc..
 - Capacitors

We 20 Today's homework is homework #4, due 11pm, Sunday, June 19!!

Announcements

- Quiz #2
 - Beginning of the class tomorrow, Thursday, June 16
 - Covers CH22.1 through what we cover in class today
 - Bring your calculator but DO NOT input formula into it!
 - Cell phones or any types of computers cannot replace a calculator!
 - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of handwritten formulae and values of constants for the quiz
 - No derivations, word definitions or solutions of any problems!
 - No additional formulae or values of constants will be provided!
- Reading assignments
 - CH23.9



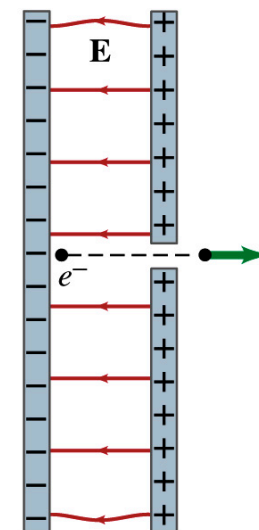
Reminder: Special Project #2 – Angels & Demons

- Compute the total possible energy released from an annihilation of x -grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS#. (20 points)
 - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class tomorrow Thursday, June 16.



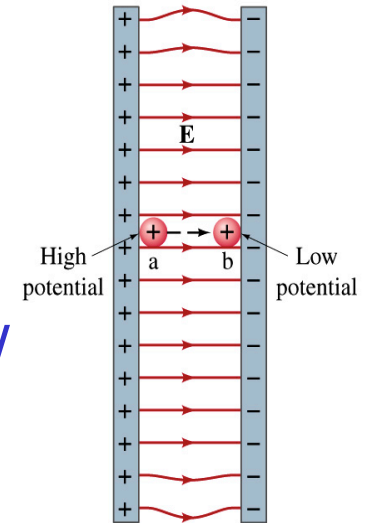
Special Project

- **Particle Accelerator.** A charged particle of mass M with charge $-Q$ is accelerated in the uniform field E between two parallel charged plates whose separation is D as shown in the figure on the right. The charged particle is accelerated from an initial speed v_0 near the negative plate and passes through a tiny hole in the positive plate.
 - Derive the formula for the electric field E to accelerate the charged particle to a fraction f of the speed of light c . Express E in terms of M , Q , D , f , c and v_0 .
 - (a) Using the Coulomb force and kinematic equations. (8 points)
 - (b) Using the work-kinetic energy theorem. (8 points)
 - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Due beginning of the class Monday, June 20



Example 23 – 1

A negative charge: Suppose a negative charge, such as an electron, is placed at point *b* in the figure. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?



- An electron placed at point *b* will move toward the positive plate since it was released at its highest potential energy point.
- It will gain kinetic energy as it moves toward left, decreasing its potential energy.
- The electron, however, moves from the point *b* at a lower potential to point *a* at a higher potential. $\Delta V = V_a - V_b > 0$.
- This is because the potential is generated by the charges on the plates not by the electron.

Electric Potential and Potential Energy

- What is the definition of the electric potential?
 - The potential energy difference per unit charge $V_{ba} = \frac{U_b - U_a}{q}$
- OK, then, how would you express the potential energy that a charge q would obtain when it is moved between point a and b with the potential difference V_{ba} ?

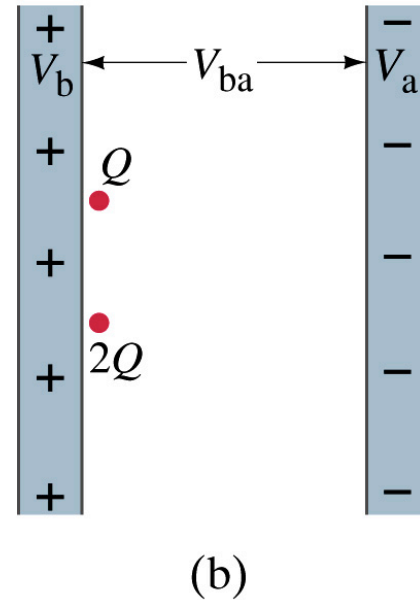
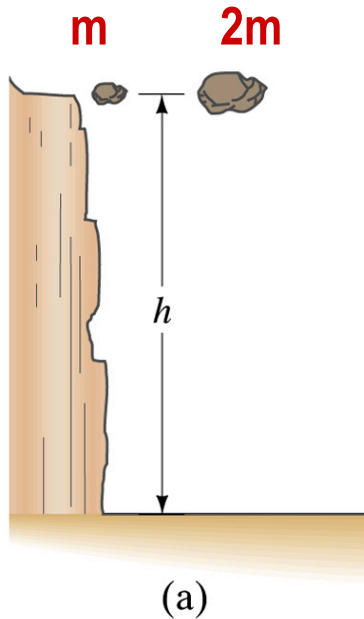
$$U_b - U_a = q(V_b - V_a) = qV_{ba}$$

- In other words, if an object with charge q moves through a potential difference V_{ba} , its potential energy changes by qV_{ba} .
- So based on this, how differently would you describe the electric potential in words?
 - A measure of how much energy an electric charge can acquire in a given situation
 - A measure of how much work a given charge can do.



Comparisons of Potential Energies

- Let's compare gravitational and electric potential energies



- What are the potential energies of the rocks?
 - mgh and $2mgh$
- Which rock has a bigger potential energy?
 - The rock with a larger mass
- Why?
 - It's got a bigger mass.
- What are the potential energies of the charges?
 - QV_{ba} and $2QV_{ba}$
- Which object has a bigger potential energy?
 - The object with a larger charge.
- Why?
 - It's got a bigger charge.

The potential is the same but the heavier rock or larger charge can do a greater work.

Electric Potential and Potential Energy

- The electric potential difference gives potential energy or the possibility to perform work based on the charge of the object.
- So what is happening in batteries or generators?
 - They maintain a potential difference.
 - The actual amount of energy used or transformed depends on how much charge flows.
 - How much is the potential difference maintained by a car's battery?
 - 12Volts
 - If for a given period, 5C charge flows through the headlight lamp, what is the total energy transformed?
 - $E_{\text{tot}} = 5\text{C} \cdot 12\text{V} = 60$ Umm... What is the unit? **Joules**
 - If it is left on twice as long? $E_{\text{tot}} = 10\text{C} \cdot 12\text{V} = 120\text{J}$.



Some Typical Voltages

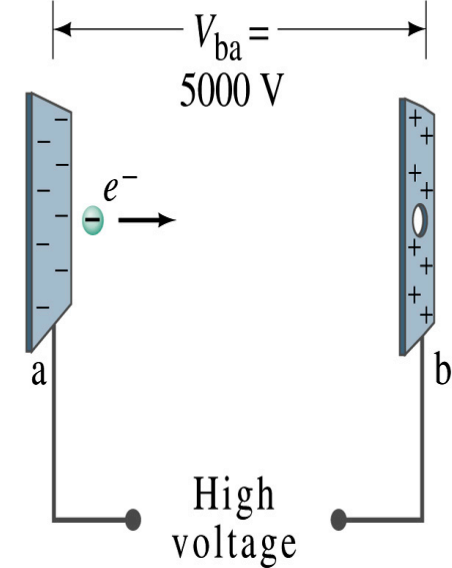
Sources	Approximate Voltage
Thundercloud to ground	10^8 V
High-Voltage Power Lines	10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

In a typical lightening strike, 15C of electrons are released in $500\mu\text{s}$. What is the total kinetic energy of these electrons when they strike ground? What is the power released during this strike? What do you think will happen to a tree hit by this lightening?



Example 23 – 2

Electrons in TV tube: Suppose an electron in the picture tube of a television set is accelerated from rest through a potential difference $V_{ba}=+5000\text{V}$. (a) What is the change in potential energy of the electron? (b) What is the speed of the electron ($m=9.1\times 10^{-31}\text{kg}$) as a result of this acceleration? (c) Repeat for a proton ($m=1.67\times 10^{-27}\text{kg}$) that accelerates through a potential difference of $V_{ba}=-5000\text{V}$.



- (a) What is the charge of an electron?

$$- \quad e = -1.6 \times 10^{-19} \text{ C}$$

- So what is the change of its potential energy?

$$\Delta U = qV_{ba} = eV_{ba} = (-1.6 \times 10^{-19} \text{ C})(+5000 \text{ V}) = -8.0 \times 10^{-16} \text{ J}$$

Example 23 – 2

- (b) Speed of the electron?
 - The entire potential energy of the electron turns to its kinetic energy.

Thus the equation is

$$\Delta K = \frac{1}{2} m_e v_e^2 - 0 = W = -\Delta U = -eV_{ba} = \\ = -(-1.6 \times 10^{-19} \text{ C}) 5000 \text{ V} = 8.0 \times 10^{-16} \text{ J}$$

$$v_e = \sqrt{\frac{2 \times eV_{ba}}{m_e}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{9.1 \times 10^{-31}}} = 4.2 \times 10^7 \text{ m/s}$$

- (C) Speed of a proton?

$$\Delta K = \frac{1}{2} m_p v_p^2 - 0 = W = -\Delta U = -\{(-e)(-V_{ba})\} = -eV_{ba} = 8.0 \times 10^{-16} \text{ J}$$

$$v_p = \sqrt{\frac{2 \times eV_{ba}}{m_p}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{1.67 \times 10^{-27}}} = 9.8 \times 10^5 \text{ m/s}$$



Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
 - What kind of quantities are the electric field and the electric potential?
 - Electric Field: Vector
 - Electric Potential: Scalar
 - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, what are the connections between these two quantities?



Electric Potential and Electric Field

- Potential energy change is expressed in terms of a conservative force (point **a** at a higher potential)

$$U_b - U_a = -\vec{F} \cdot \vec{D} = -W_C$$

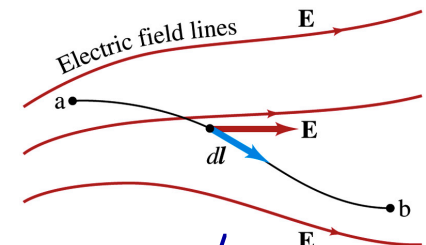
- For the electrical case, we are more interested in the potential difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q} \cdot \vec{D} = -\vec{E} \cdot \vec{D} = -ED \cos \theta$$

- This formula can be used to determine V_{ba} when the electric field is given.

- When the field is uniform

$$V_b - V_a = -\vec{E} \cdot \vec{D} = -ED \cos \theta = -Ed \quad \text{so} \quad E = -V_{ba} / d$$



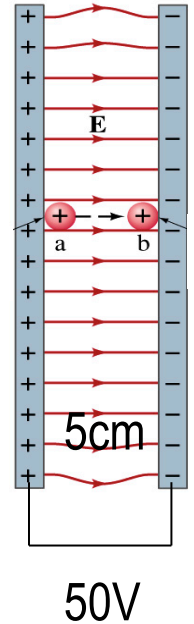
What does “-” sign mean? The direction of \vec{E} is along that of decreasing potential.

Unit of the electric field in terms of potential? V/m Can you derive this from N/C ?

Example

Uniform electric field obtained from voltage:

Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.



What is the relationship between electric field and the potential for a uniform field?

$$V = Ed$$

Solving for E $E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5 \times 10^{-2}m} = 1000V / m$

Which direction is the field? Direction of decreasing potential!

Electric Potential due to Point Charges

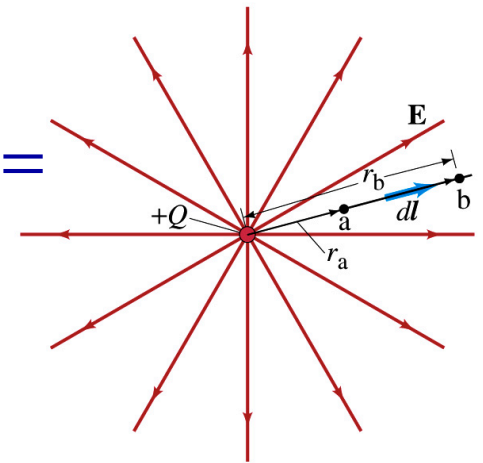
- What is the electric field by a single point charge Q at a distance r ?

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in radial direction away from the charge Q is

$$V_b - V_a = - \int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{\hat{r}}{r^2} \cdot \hat{r} dr =$$

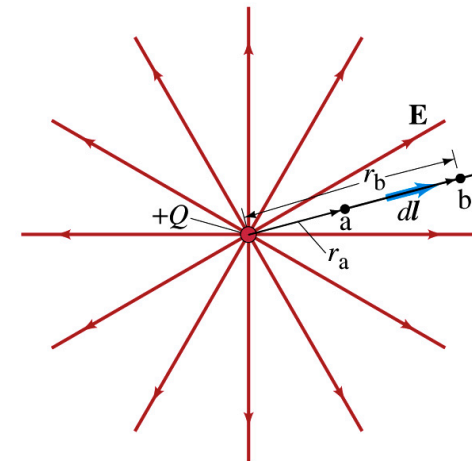
$$= - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{r_b} - \frac{1}{r_a} \right)$$



Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge Q is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of the potential difference by a single point charge between r and infinity

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?

- Electric potential

- Electric potential energy per unit charge
- Inversely proportional to the distance
- Simply add the potential by each of the source charges to obtain the total potential from multiple charges, since potential is a scalar quantity

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

- Electric field

- Electric force per unit charge
- Inversely proportional to the square of the distance
- Need vector sums to obtain the total field from multiple source charges

$$|\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

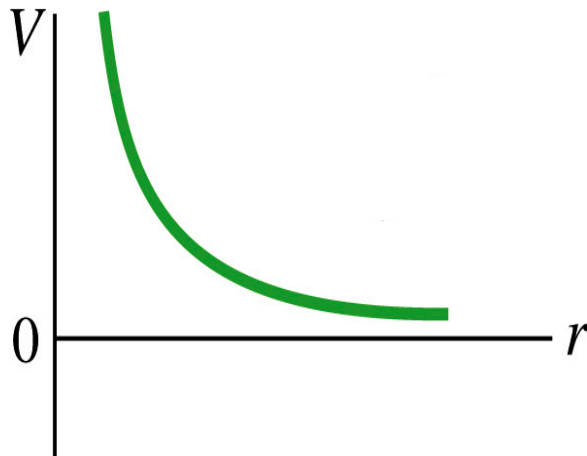
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at the large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at a large distance.

Shape of the Electric Potential

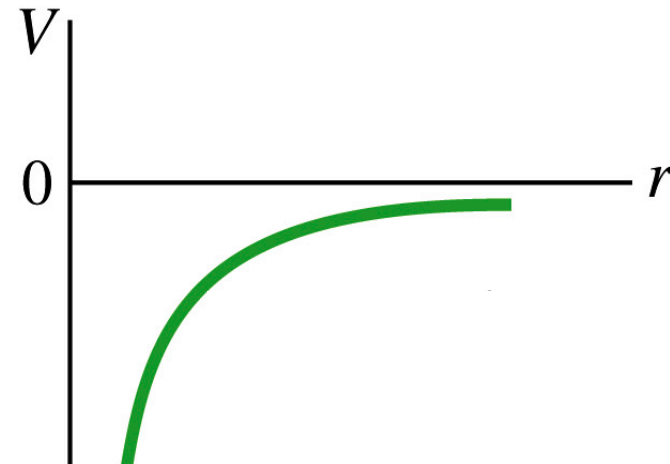
- So, how does the electric potential look like as a function of distance?
 - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean?

Uniformly charged sphere behaves like all the charge is on the single point in the center.

Example 23 – 6

Work to bring two positive charges close together: What minimum work is required by an external force to bring the charge $q=3.00\mu\text{C}$ from a great distance away ($r=\infty$) to a point 0.500m from a charge $Q=20.0\mu\text{C}$?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\epsilon_0} \left(\frac{Q}{r_b} - \frac{Q}{r_a} \right)$$

Since $r_b = 0.500\text{m}$, $r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\epsilon_0} \left(\frac{Q}{r_b} - 0 \right) = -\frac{q}{4\pi\epsilon_0} \frac{Q}{r_b} = -\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \cdot (3.00 \times 10^{-6} \text{ C}) (20.00 \times 10^{-6} \text{ C})}{0.500\text{m}} = -1.08\text{J}$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge $3.00\mu\text{C}$ from infinity to 0.500m to the charge $20.0\mu\text{C}$.

Electric Potential by Charge Distributions

- Let's consider a case of n individual point charges in a given space and $V=0$ at $r=\infty$.
- Then the potential V_{ia} due to the charge Q_i at point a , distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

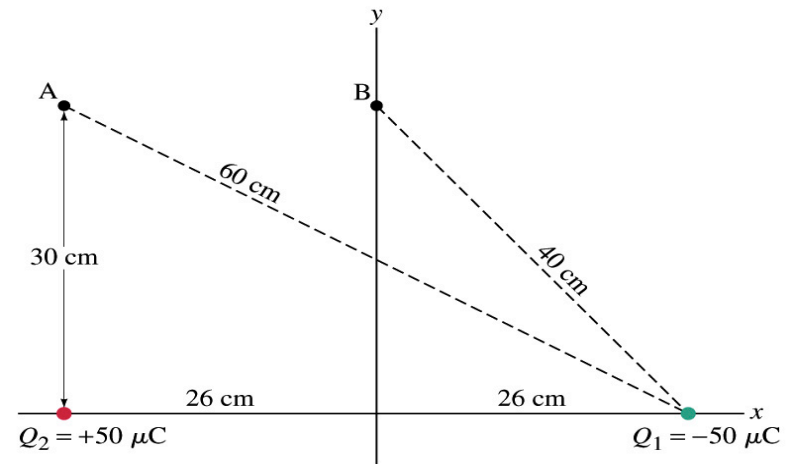
$$V_a = \sum_{i=1}^n V_{ia} = \sum_{i=1}^n \frac{Q_i}{4\pi\epsilon_0} \frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

Example

- **Potential due to two charges:**
Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.
- Potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.



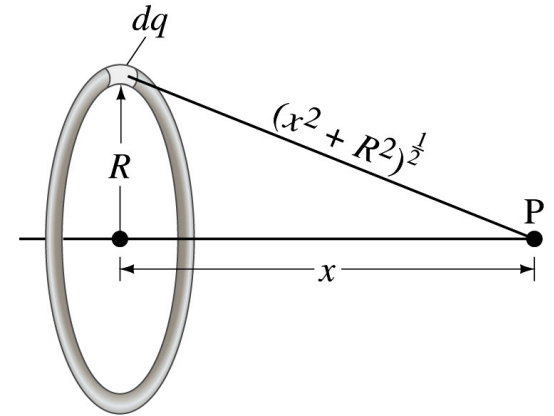
(a) potential at A is

$$\begin{aligned} V_A &= V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\epsilon_0 r_{iA}} = \\ &= \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_{1A}} + \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_{2A}} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}} \right) \\ &= 9.0 \times 10^9 \left(\frac{-50 \times 10^{-6}}{0.60} + \frac{50 \times 10^{-6}}{0.30} \right) = 7.5 \times 10^5 \text{ V} \end{aligned}$$

(b) How about potential at B?

Example 23 – 8

- Potential due to a ring of charge:** A thin circular ring of radius R carries a uniformly distributed charge Q . Determine the electric potential at a point P on the axis of the ring a distance x from its center.



- Each point on the ring is at the same distance from the point P . What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r} \int dq =$$

$$\frac{1}{4\pi\epsilon_0 \sqrt{x^2 + R^2}} \int dq = \frac{Q}{4\pi\epsilon_0 \sqrt{x^2 + R^2}}$$

What's this?