PHYS 1441 – Section 001
Lecture #11

Thursday, June 22, 2016

Dr. Jaehoon Yu

- Chapter 25
  - Microscopic View of Electric Current
  - EMF and Terminal Voltage
- Chapter 26
  - Kirchhoff's Rules
  - EMFs in Series and Parallel
  - RC Circuits
- Chapter 27: Magnetism and Magnetic Field

Today's homework is homework #6, due 11pm, Sunday, June 26!!

Announcements

- One-on-One Mid Term grade discussion
  - Coming Monday after the first 45min
  - My office, CPB342
- Grade scheme reminder
  - Homework: 25%
  - Final exam: 23%
  - Midterm exam: 20%
  - Better of the two term exams: 12%
  - Lab: 10%
  - Quizzes: 10%
  - Extra Credit: 10%

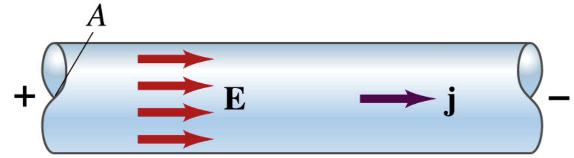
Special Project #4

- Make a list of the power consumption and the resistance of all electric and electronic devices at your home and compiled them in a table. (5 points total for the first 10 items and 0.25 points each additional item.)
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don't find your own, use \$0.12/kWh) and put them in the relevant column. (2 points total for the first 10 items and 0.1 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (6 points)
- Due: Beginning of the class Monday, June 28

Item Name	Rated power (W)	Number of devices	Number of Hours per day	Daily Power Consumption (kWh)	Energy Cost per kWh (cents)	Daily Energy Consumption (J).	Daily Energy Cost ($)	Monthly Energy Consumption (J)	Monthly Energy Cost ($)	Yearly Energy Consumption (J)	Yearly Energy Cost ($)
Light Bulbs	30	4									
	40	6									
	60	15									
Heaters	1000	2									
	1500	1									
	2000	1									
Fans											
Air Conditioners											
Fridgers, Freezers											
Computers (desktop, laptop, ipad)											
Game consoles											
Total				0		0	0	0	0	0	0

Microscopic View of Electric Current

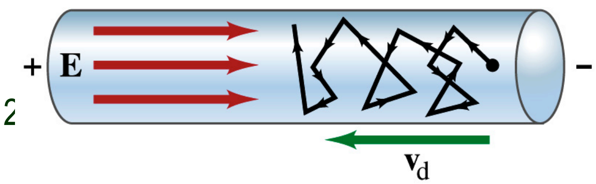
- When a potential difference is applied to the two ends of a wire w/ uniform cross-section, the direction of electric field is parallel to the walls of the wire, this is possible since the charges are moving
- Let's define a microscopic vector quantity, the current density, **j**, the electric current per unit cross-sectional area
  - j=I/A or I= jA if the current density is uniform
  - If not uniform $I = \int \vec{j} \cdot d\vec{A}$
  - The direction of **j** is the direction the positive charge would move when placed at that position, generally the same as E



- The current density exists on any point in space while the current I refers to a conductor as a whole so a macroscopic

Microscopic View of Electric Current

- The direction of j is the direction of a positive charge. So in a conductor, since negatively charged electrons move, the direction is –**j**.
- Let's think about the current in a microscopic view again:
  - When voltage is applied to the end of a wire
  - Electric field is generated by the potential difference
  - Electrons feel force and get accelerated
  - Electrons soon reach to a steady average speed due to collisions with atoms in the wire, called drift velocity, $\mathbf{v}_d$
  - The drift velocity is normally much smaller than electrons' average random speed.

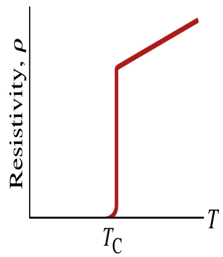


Microscopic View of Electric Current

- The drift velocity of electrons in a wire is only about 0.05mm/s. How could we get light turned on immediately then?
  - While the electrons in a wire travels slow, the electric field travels essentially at the speed of light. Then what is all the talk about electrons flowing through?
    - It is just like water. When you turn on the facet, water flows right off the facet despite the fact that the water travels slow.
    - Electricity is the same. Electrons fill the conductor wire and when the switch is flipped on or a potential difference is applied, the electrons close to the positive terminal flows into the bulb.
    - Interesting, isn't it? Why is the field travel at the speed of light then?

Superconductivity

- At the temperature near absolute 0K, resistivity of certain material becomes 0.
  - This state is called the "superconducting" state.
  - Observed in 1911 by H. K. Onnes when he cooled mercury to 4.2K (-269^{o}C).
    - Resistance of mercury suddenly dropped to 0.
  - In general superconducting materials become superconducting below a transition temperature.
  - The highest temperature superconductivity seen is 160K
    - First observation above the boiling temperature of liquid nitrogen is in 1987 at 90k observed from a compound of yttrium, barium, copper and oxygen.



- Since much smaller amount of material can carry just as much current more efficiently, superconductivity can make electric cars more practical, computers faster, and capacitors store higher energy

Critical Temperature of Superconductors

Critical temperature (T_c), crystal structure and lattice constants of some high-T_c superconductors

Formula	Notation	T_c (K)	No. of Cu-O planes in unit cell	Crystal structure
$YBa_2Cu_3O_7$	123	92	2	Orthorhombic
$Bi_2Sr_2CuO_6$	Bi-2201	20	1	Tetragonal
$Bi_2Sr_2CaCu_2O_8$	Bi-2212	85	2	Tetragonal
$Bi_2Sr_2Ca_2Cu_3O_{10}$	Bi-2223	110	3	Tetragonal
$Tl_2Ba_2CuO_6$	Tl-2201	80	1	Tetragonal
$Tl_2Ba_2CaCu_2O_8$	Tl-2212	108	2	Tetragonal
$Tl_2Ba_2Ca_2Cu_3O_{10}$	Tl-2223	125	3	Tetragonal
$TlBa_2Ca_3Cu_4O_{11}$	Tl-1234	122	4	Tetragonal
$HgBa_2CuO_4$	Hg-1201	94	1	Tetragonal
$HgBa_2CaCu_2O_6$	Hg-1212	128	2	Tetragonal
$HgBa_2Ca_2Cu_3O_8$	Hg-1223	134	3	Tetragonal

Electric Hazards: Leakage Currents

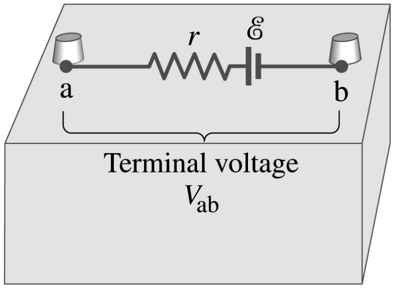
- How does one feel shock by electricity?
  - Electric current stimulates nerves and muscles, and we feel a shock
  - The severity of the shock depends on the amount of current, how long it acts and through what part of the body it passes
  - Electric current heats the tissue and can cause burns
- Currents above 70mA on a torso for a second or more is fatal, causing heart to function irregularly, "ventricular fibrillation"
- A dry human body between two points on opposite side of the body is about 10^4 to 10^6 Ω.
- When wet, it could be $10^3\Omega$.
- A person in good contact with the ground who touches 120V DC line with wet hands can get the current:
  - Could be lethal

$$I = \frac{V}{R} = \frac{120V}{1000\Omega} = 120mA$$

EMF and Terminal Voltage

- What do we need to have current in an electric circuit?
  - A device that provides a potential difference, such as a battery or a generator
    - They normally convert some types of energy into the electric energy
    - These devices are called source of electromotive force (emf)
      - This is does NOT refer to a real "force".
- Potential difference between terminals of an emf source, when no current flows to an external circuit, is called the emf ($\mathcal{E}$) of the source.
- The battery itself has some **internal resistance** (r) due to the flow of charges in the electrolyte
  - Why does the headlight dim when you start the car?
    - The starter needs a large amount of current but the battery cannot provide charge fast enough to supply current to both the starter and the headlight

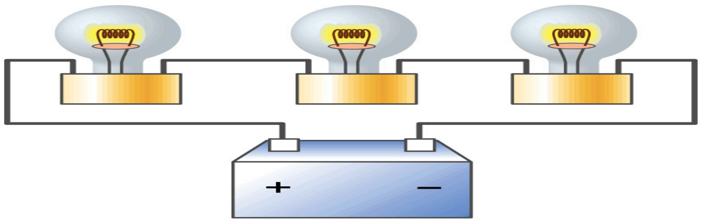
EMF and Terminal Voltage

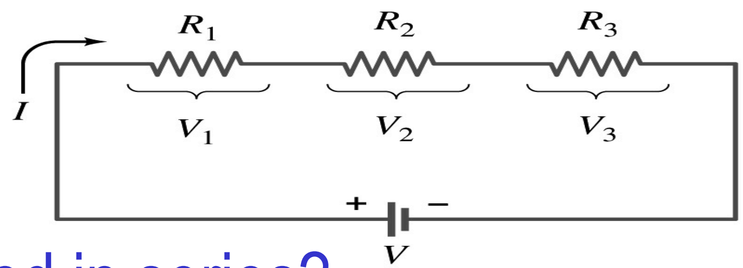


- Since the internal resistance is inside the battery, we can never separate them out.
- So the terminal voltage difference is $V_{ab}=V_a-V_b$.
- When no current is drawn from the battery, the terminal voltage equals the emf which is determined by the chemical reaction; $V_{ab}=\mathcal{E}$.
- However when the current I flows naturally from the battery, there is an internal drop in voltage which is equal to Ir. Thus the actual **delivered** terminal voltage is $\boxed{V_{ab}=\varepsilon-Ir}$

Resisters in Series

- Resisters are in series when two or more resisters are connected end to end
  - These resisters represent simple resisters in circuit or electrical devices, such as light bulbs, heaters, dryers, etc





- What is common in a circuit connected in series?
  - Current is the same through all the elements in series
- Potential difference across every element in the circuit is
  - $V_1=IR_1$, $V_2=IR_2$ and $V_3=IR_3$
- Since the total potential difference is V, we obtain
  - $V=IR_{eq}=V_1+V_2+V_3=I(R_1+R_2+R_3)$
  - Thus, $R_{eq}=R_1+R_2+R_3$

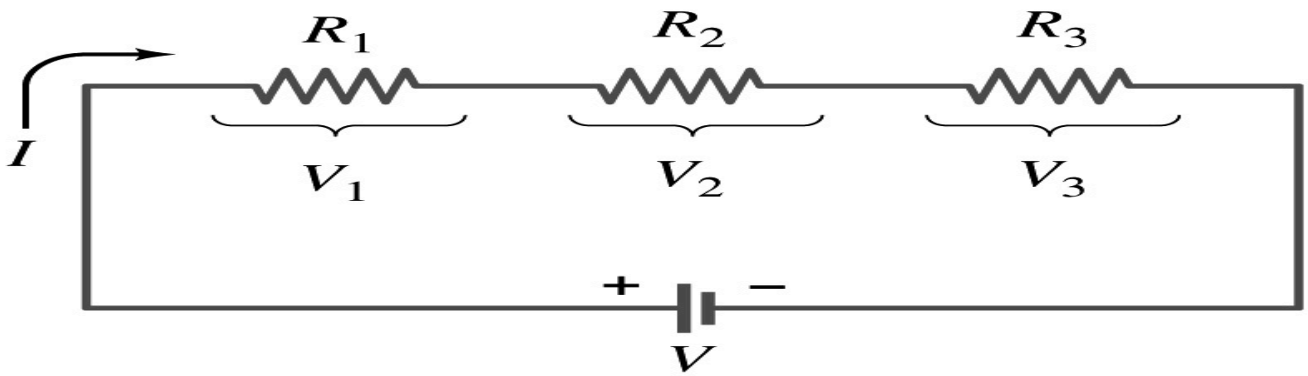
$$R_{eq} = \sum_i R_i$$

Resisters in series

When resisters are connected in series, the total resistance increases and the current decreases.

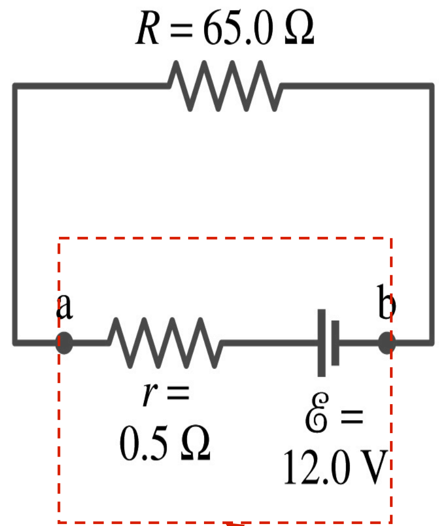
Energy Losses in Resisters

- Why is it true that $V=V_1+V_2+V_3$?



- What is the potential energy loss when charge q passes through resisters R_1, R_2 and R_3
  - $\Delta U_1=qV_1$, $\Delta U_2=qV_2$, $\Delta U_3=qV_3$
- Since the total energy loss should be the same as the total energy provided to the system, we obtain
  - $\Delta U=qV=\Delta U_1+\Delta U_2+\Delta U_3=q(V_1+V_2+V_3)$
  - Thus, $V=V_1+V_2+V_3$

Example 26 – 1



What is this?

A battery or a source of emf.

Battery with internal resistance. A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0V and whose internal resistance is 0.5-Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistor.

(a) Since $V_{ab} = \varepsilon - Ir$ We obtain $V_{ab} = IR = \varepsilon - Ir$

Solve for I

$$I = \frac{\varepsilon}{R + r} = \frac{12.0V}{65.0\Omega + 0.5\Omega} = 0.183A$$

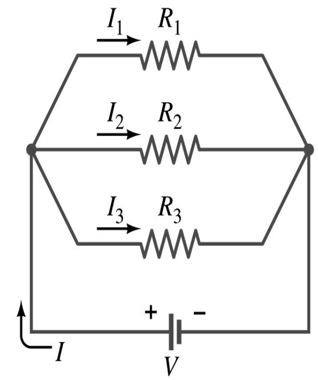
(b) The terminal voltage V_{ab} is $V_{ab} = \varepsilon - Ir = 12.0V - 0.183A \cdot 0.5\Omega = 11.9V$

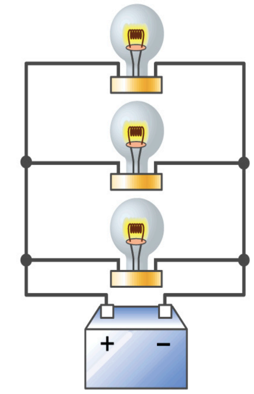
(c) The power dissipated in R and r are

$$P = I^2 R = (0.183A)^2 \cdot 65.0\Omega = 2.18W$$

$$P = I^2 r = (0.183A)^2 \cdot 0.5\Omega = 0.02W$$

Resisters in Parallel





- Resisters are in parallel when two or more resisters are connected in separate branches
  - Most the house and building wirings are arranged this way.
- What is common in a circuit connected in parallel?
  - The voltage is the same across all the resisters.
  - The total current that leaves the battery, is however, split.
- The current that passes through every element is
  - $I_1=V/R_1$, $I_2=V/R_2$, $I_3=V/R_3$
- Since the total current is I, we obtain
  - $I=V/R_{eq}=I_1+I_2+I_3=V(1/R_1+1/R_2+1/R_3)$
  - Thus, $1/R_{eq}=1/R_1+1/R_2+1/R_3$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Resisters in parallel

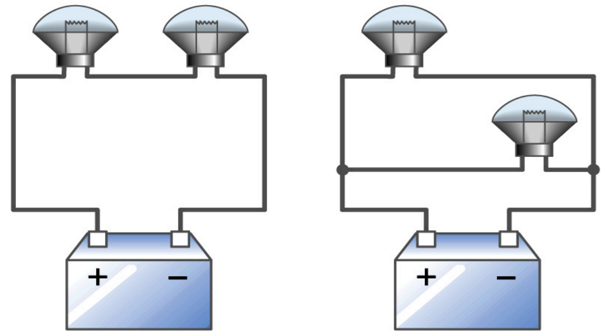
When resisters are connected in parallel, the total resistance decreases and the current increases.

Resister and Capacitor Arrangements

- Parallel Capacitor arrangements $$C_{eq} = \sum_i C_i$$
- Parallel Resister arrangements $$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$
- Series Capacitor arrangements $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$
- Series Resister arrangements $$R_{eq} = \sum_i R_i$$

Example 26 – 2

Series or parallel? (a) The light bulbs in the figure are identical and have identical resistance R. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired?



(1) Series (2) Parallel

(a) What are the equivalent resistances for the two cases?

Series $R_{eq} = 2R$ Parallel $\frac{1}{R_{eq}} = \frac{2}{R}$ So $R_{eq} = \frac{R}{2}$

The bulbs get brighter when the total power transformed is larger.

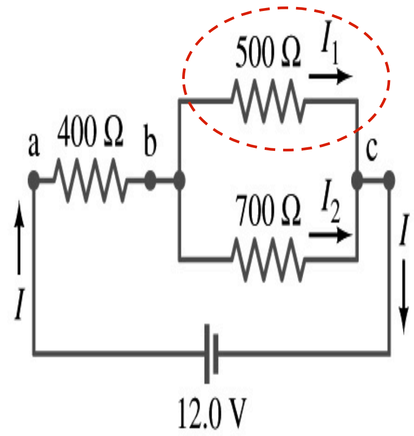
series $P_S = IV = \frac{V^2}{R_{eq}} = \frac{V^2}{2R}$ parallel $P_P = IV = \frac{V^2}{R_{eq}} = \frac{2V^2}{R} = 4P_S$

So parallel circuit provides brighter lighting.

(b) Car's headlights are in parallel to provide brighter lighting and also to prevent both lights going out at the same time when one burns out.

So what is bad about parallel circuits? Uses more energy in a given time.

Example 26 – 5



Current in one branch. What is the current flowing through the 500-Ω resister in the figure?

What do we need to find first?

We need to find the total current.

To do that we need to compute the equivalent resistance.

R_{eq} of the small parallel branch is: $\frac{1}{R_P} = \frac{1}{500} + \frac{1}{700} = \frac{12}{3500}$ $\quad R_P = \frac{3500}{12}$

R_{eq} of the circuit is: $R_{eq} = 400 + \frac{3500}{12} = 400 + 292 = 692\Omega$

Thus the total current in the circuit is $I = \frac{V}{R_{eq}} = \frac{12}{692} = 17mA$

The voltage drop across the parallel branch is $V_{bc} = IR_P = 17\times10^{-3}\cdot 292 = 4.96V$

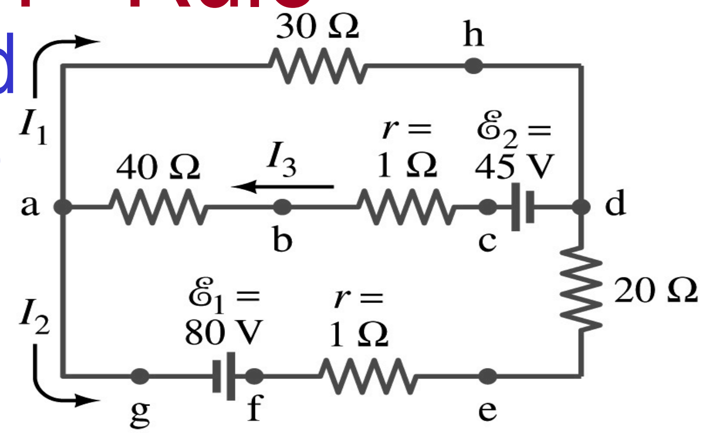
The current flowing across 500-Ω resister is therefore

$$V_{bc} I_{500} = \frac{V_{bc}}{R} = \frac{4.96}{500} = 9.92\times10^{-3} = 9.92mA$$

What is the current flowing 700-Ω resister? $I_{700} = I - I_{500} = 17 - 9.92 = 7.08mA$

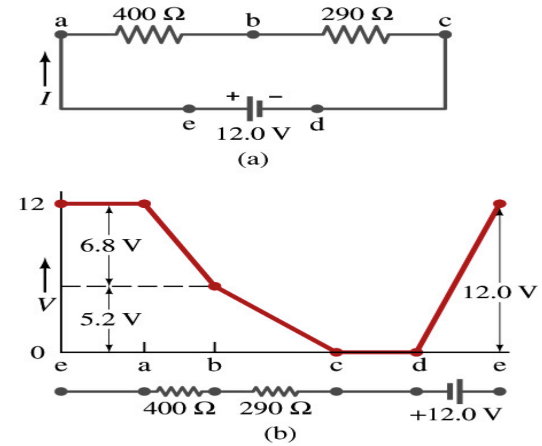
Kirchhoff's Rules – 1st Rule

- Some circuits are very complicated to do the analysis using the simple combinations of resisters
  - G. R. Kirchhoff devised two rules to deal with complicated circuits.



- Kirchhoff's rules are based on **conservation of charge and energy**
  - Kirchhoff's 1st rule: The junction rule, charge conservation.
    - At any junction point, the sum of all currents entering the junction must equal to the sum of all currents leaving the junction.
    - In other words, what goes in must come out.
    - At junction *a* in the figure, I_3 comes into the junction while I_1 and I_2 leaves: $I_3 = I_1 + I_2$

Kirchhoff's Rules – 2nd Rule

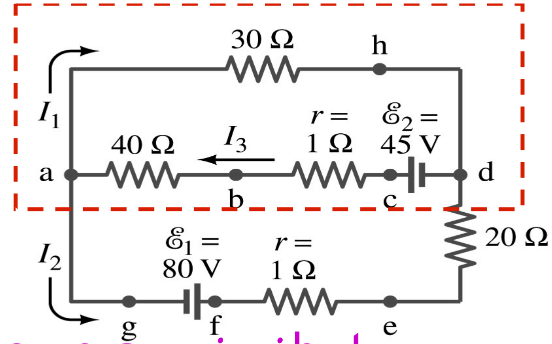


- Kirchoff's 2nd rule: The loop rule, uses **conservation of energy**.
  - The sum of the changes in potential in any closed path of a circuit must be zero.
- The current in the circuit in the figure is I=12/690=0.017A.
  - Point e is the high potential point while point d is the lowest potential.
  - When the test charge starts at e and returns to e, the total potential change is 0.
  - Between point e and a, no potential change since there is no source of potential nor any resistance.
  - Between a and b, there is a 400Ω resistance, causing IR=0.017*400 =6.8V drop.
  - Between b and c, there is a 290Ω resistance, causing IR=0.017*290 =5.2V drop.
  - Since these are voltage drops, we use negative sign for these, -6.8V and -5.2V.
  - No change between c and d while from d to e there is +12V change.
  - Thus the total change of the voltage through the loop is: -6.8V-5.2V+12V=0V.

Using Kirchhoff's Rules

1. Determine the flow of currents at the junctions and label each and everyone of the currents.
   - It does not matter which direction, you decide but keep it!
   - If the value of the current after completing the calculations are negative, you just need to flip the direction of the current flow.
2. Write down the current equation based on Kirchhoff's 1st rule at various junctions.
   - Be sure to see if any of them are the same.
3. Choose closed loops in the circuit
4. Write down the potential in each interval of the junctions, keeping the sign properly.
5. Write down the potential equations for each loop.
6. Solve the equations for unknowns.

Example 26 – 9



Use Kirchhoff's rules. Calculate the currents I_1, I_2 and I_3 in each of the branches of the circuit in the figure.

The directions of the current through the circuit is not known a *priori* but since the current tends to move away from the positive terminal of a battery, we arbitrarily choose the direction of the currents as shown.

We have three unknowns so we need three equations.

Using Kirchhoff's junction rule at point *a*, we obtain $I_3 = I_1 + I_2$

This is the same for junction *d* as well, so no additional information.

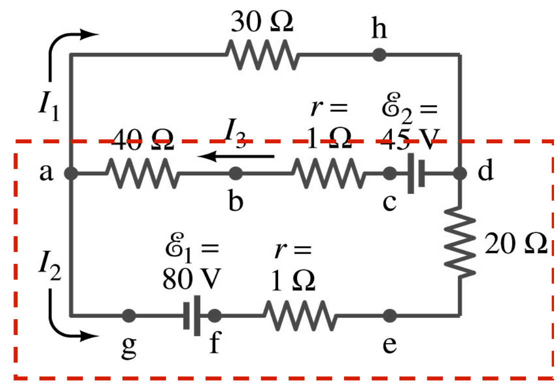
Now the second rule on the loop *ahdcba*.

$$V_{ah} = -I_1 30 \qquad V_{hd} = 0 \qquad V_{dc} = +45 \qquad V_{cb} = -I_3 \qquad V_{ba} = -40I_3$$

The total voltage change in the loop *ahdcba* is.

$$V_{ahdcba} = -30I_1 + 45 - 1 \cdot I_3 - 40I_3 = 45 - 30I_1 - 41I_3 = 0$$

Example 26 – 9, cnt'd



Now the second rule on the other loop *agfedcba*.

$$V_{ag} = 0 \quad V_{gf} = +80 \quad V_{fe} = -I_2 \cdot 1 \quad V_{ed} = -I_2 \cdot 20$$

$$V_{dc} = +45 \quad V_{cb} = -I_3 \cdot 1 \quad V_{ba} = -40 \cdot I_3$$

The total voltage change in loop *agfedcba* is. $V_{agfedcba} = -21I_2 + 125 - 41I_3 = 0$

So the three equations become

$$I_3 = I_1 + I_2$$

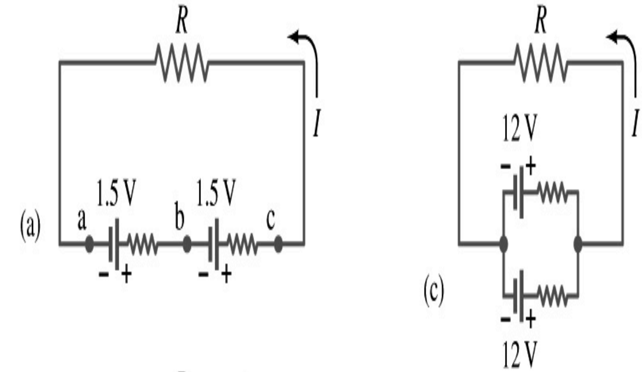
$$45 - 30I_1 - 41I_3 = 0$$

$$125 - 21I_2 - 41I_3 = 0$$

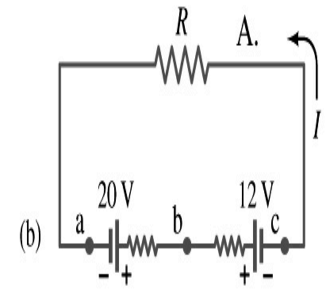
We can obtain the three current by solving these equations for I_1, I_2 and I_3.

Do this yourselves!!

EMFs in Series and Parallel: Charging a Battery



- When two or more sources of emfs, such as batteries, are connected in series
  - The total voltage is the algebraic sum of their voltages, if their direction is the same
    - V_{ab}=1.5 + 1.5=3.0V in figure (a).
  - If the batteries are arranged in an opposite direction, the total voltage is the difference between them

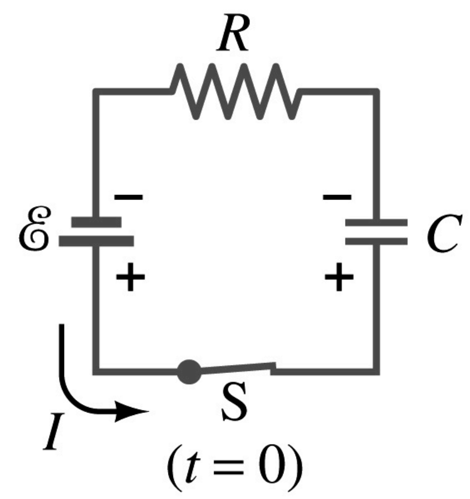


Parallel arrangements (c) are used only to increase currents. An example?

    - V_{ac}=20 – 12=8.0V in figure (b)
    - Connecting batteries in opposite direction is wasteful.
    - This, however, is the way a battery charger works.
    - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
    - Some battery are rechargeable since their chemical reactions are reversible but most the batteries do not reverse their chemical reactions

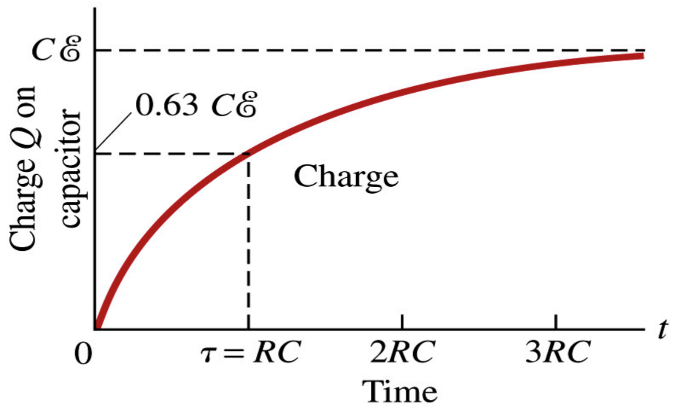
RC Circuits

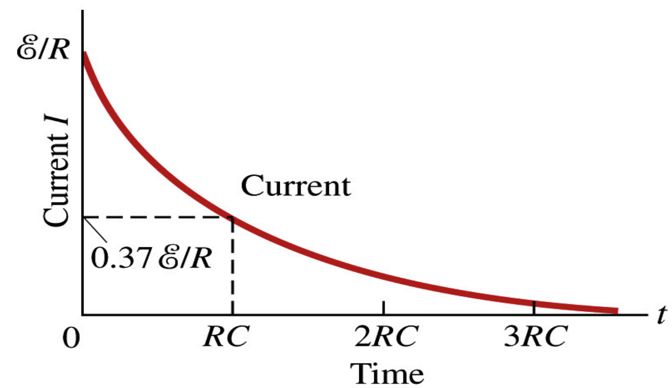
- Circuits containing both resisters and capacitors
  - RC circuits are used commonly in everyday life
    - Control windshield wiper
    - Timing of traffic light from red to green
    - Camera flashes and heart pacemakers
- How does an RC circuit look?
  - There should be a source of emf, capacitors and resisters
- What happens when the switch S is closed?
  - Current immediately starts flowing through the circuit.
  - Electrons flow out of negative terminal of the emf source, through the resister R and accumulates on the upper plate of the capacitor.
  - The electrons from the bottom plate of the capacitor will flow into the positive terminal of the battery, leaving only positive charge on the bottom plate.
  - As the charge accumulates on the capacitor, the potential difference across it increases
  - The current reduces gradually to 0 till the voltage across the capacitor is the same as emf.
  - The charge on the capacitor increases till it reaches to its maximum $C\mathscr{E}$.



RC Circuits

- How does all this look like in graphs?
  - The charge and the current on the capacitor as a function of time





  - From energy conservation (Kirchhoff's 2nd rule), the emf $\mathcal{E}$ must be equal to the voltage drop across the capacitor and the resister
    - $\mathcal{E} = IR + Q/C$
    - R includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instance, and Q is the charge of the capacitor at that same instance.

Analysis of RC Circuits

- From the energy conservation, we obtain $\mathscr{E}=I\mathcal{R}+Q/C$
- Which ones are constant in the above equation?
  - $\mathscr{E}$, $\mathcal{R}$ and C are constant
  - Q and I are functions of time
- How do we write the rate at which the charge is accumulated on the capacitor?
  - We can rewrite the above equation as $\varepsilon = R\frac{dQ}{dt} + \frac{1}{C}Q$
  - This equation can be solved by rearranging the terms as $\frac{dQ}{C\varepsilon - Q} = \frac{dt}{RC}$

Analysis of RC Circuits

- Now integrating from $t=0$ when there was no charge on the capacitor to t when the capacitor is fully charged, we obtain
- $\int_0^Q \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC}\int_0^t dt$ $\rightarrow$
- $-\ln\left(C\varepsilon - Q\right)\Big|_0^Q = -\ln\left(C\varepsilon - Q\right) - \left(-\ln C\varepsilon\right) = \frac{t}{RC}\Big|_0^t = \frac{t}{RC}$
- So, we obtain $\ln\left(1 - \frac{Q}{C\varepsilon}\right) = -\frac{t}{RC}$ $\rightarrow$ $1 - \frac{Q}{C\varepsilon} = e^{-t/RC}$
- Or $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$
- The potential difference across the capacitor is V=Q/C, so $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$