PHYS 1441 – Section 001
Lecture #12

Monday, June 27, 2016

Dr. Jaehoon Yu

- Chapter 26
  - Resisters in Series and Parallel
  - EMFs in Series and Parallel
  - RC Circuits
- Chapter 27: Magnetism and Magnetic Field
  - Electric Current and Magnetism
  - Magnetic Forces on Electric Current
  - About Magnetic Field

Today's homework is homework #7, due 11pm, Wednesday, June 29!!

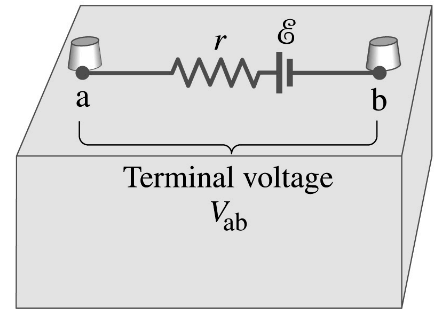
Announcements

- One-on-One Mid Term grade discussion
  – Coming today bottom
  – My office, CPB342
- Quiz #3
  – Beginning of the class Wednesday, June 29
  – Covers from CH26.1 to what we cover tomorrow
  – BYOF
- Term 2
  – In class, Thursday, June 30
  – Covers from CH 26.1 through what we cover Wednesday
  – BYOF
- Mid-term results
  – Class average: 75/100
    - Previous exam: 62/100
  – Top score:91/100

EMF and Terminal Voltage

- What do we need to have current in an electric circuit?
  - A device that provides a potential difference, such as a battery or a generator
    - They normally convert some types of energy into the electric energy
    - These devices are called source of electromotive force (emf)
      - This is does NOT refer to a real "force".
- Potential difference between terminals of an emf source, when no current flows to an external circuit, is called the emf ($\mathscr{E}$) of the source.
- The battery itself has some **internal resistance** (r) due to the flow of charges in the electrolyte
  - Why does the headlight dim when you start the car?
    - The starter needs a large amount of current but the battery cannot provide charge fast enough to supply current to both the starter and the headlight

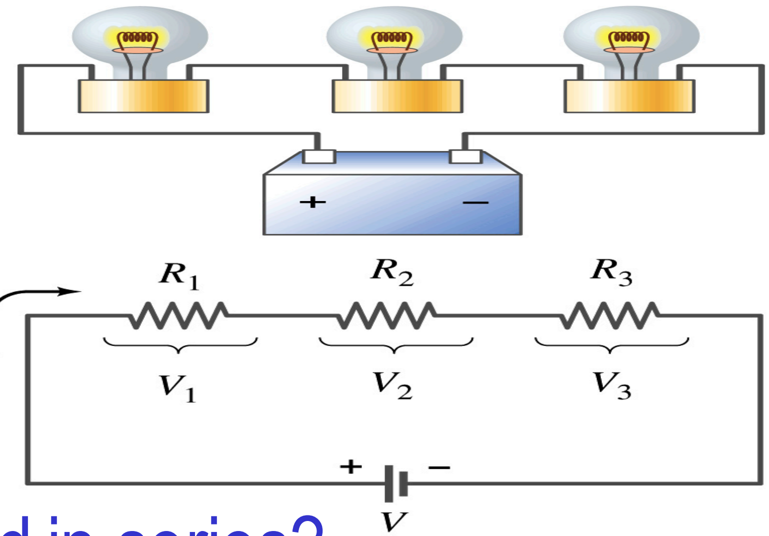
EMF and Terminal Voltage



- Since the internal resistance is inside the battery, we can never separate them out.
- So the terminal voltage difference is $V_{ab}=V_a-V_b$.
- When no current is drawn from the battery, the terminal voltage equals the emf which is determined by the chemical reaction; $V_{ab}= \mathcal{E}$.
- However when the current I flows naturally from the battery, there is an internal drop in voltage which is equal to Ir. Thus the actual **delivered** terminal voltage is $\boxed{V_{ab} = \varepsilon - Ir}$

Resisters in Series

- Resisters are in series when two or more resisters are connected end to end
  - These resisters represent simple resisters in circuit or electrical devices, such as light bulbs, heaters, dryers, etc



- What is common in a circuit connected in series?
  - Current is the same through all the elements in series
- Potential difference across every element in the circuit is
  - $V_1=IR_1$, $V_2=IR_2$ and $V_3=IR_3$
- Since the total potential difference is V, we obtain
  - $V=IR_{eq}=V_1+V_2+V_3=I(R_1+R_2+R_3)$
  - Thus, $R_{eq}=R_1+R_2+R_3$

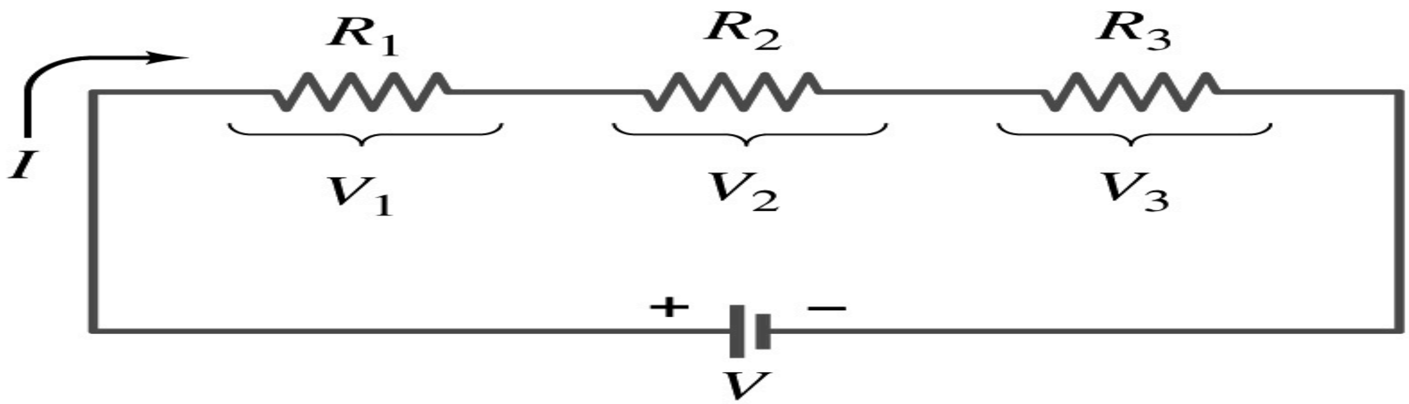
$$R_{eq} = \sum_i R_i$$

Resisters in series

When resisters are connected in series, the total resistance increases and the current decreases.

Energy Losses in Resisters

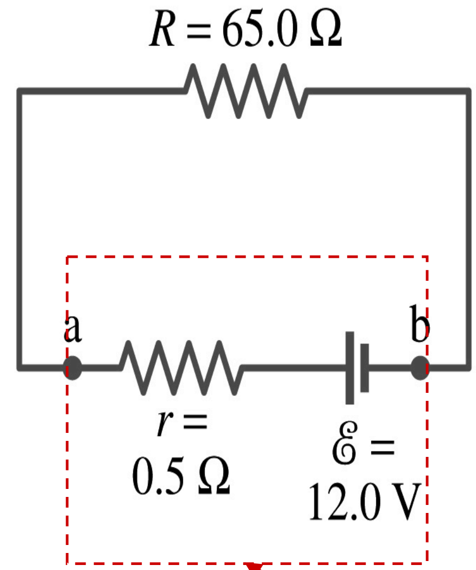
- Why is it true that $V=V_1+V_2+V_3$?



- What is the potential energy loss when charge q passes through resisters R_1, R_2 and R_3
  - $\Delta U_1=qV_1$, $\Delta U_2=qV_2$, $\Delta U_3=qV_3$
- Since the total energy loss should be the same as the total energy provided to the system, we obtain
  - $\Delta U=qV=\Delta U_1+\Delta U_2+\Delta U_3=q(V_1+V_2+V_3)$
  - Thus, $V=V_1+V_2+V_3$

Example 26 – 1

Battery with internal resistance. A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0V and whose internal resistance is 0.5-Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistor.



What is this?

A battery or a source of emf.

(a) Since $V_{ab} = \varepsilon - Ir$ We obtain $V_{ab} = IR = \varepsilon - Ir$

Solve for I

$$I = \frac{\varepsilon}{R + r} = \frac{12.0V}{65.0\Omega + 0.5\Omega} = 0.183A$$

(b) The terminal voltage V_{ab} is $V_{ab} = \varepsilon - Ir = 12.0V - 0.183A \cdot 0.5\Omega = 11.9V$

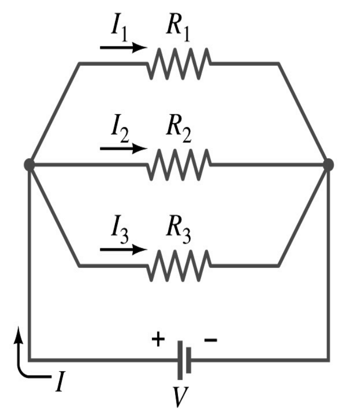
(c) The power dissipated in R and r are

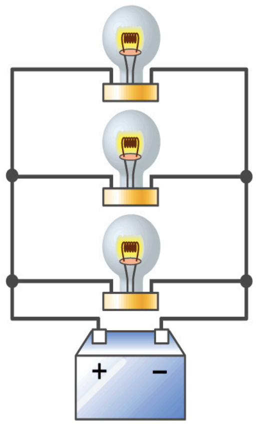
$$P = I^2 R = (0.183A)^2 \cdot 65.0\Omega = 2.18W$$

$$P = I^2 r = (0.183A)^2 \cdot 0.5\Omega = 0.02W$$

Resisters in Parallel

- Resisters are in parallel when two or more resisters are connected in separate branches
  - Most the house and building wirings are arranged this way.





- What is common in a circuit connected in parallel?
  - The voltage is the same across all the resisters.
  - The total current that leaves the battery, is however, split.
- The current that passes through every element is
  - $I_1=V/R_1$, $I_2=V/R_2$, $I_3=V/R_3$
- Since the total current is I, we obtain
  - $I=V/R_{eq}=I_1+I_2+I_3=V(1/R_1+1/R_2+1/R_3)$
  - Thus, $1/R_{eq}=1/R_1+1/R_2+1/R_3$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Resisters in parallel

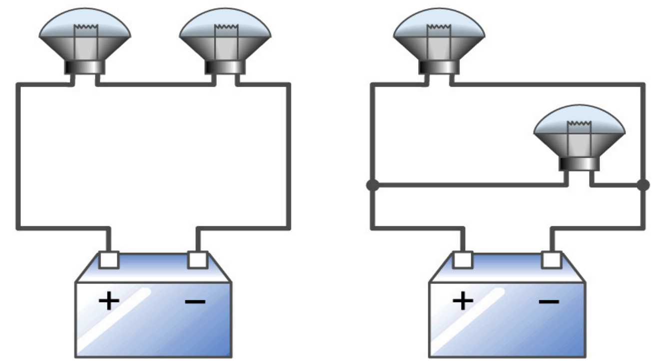
When resisters are connected in parallel, the total resistance decreases and the current increases.

Resister and Capacitor Arrangements

- Parallel Capacitor arrangements $$C_{eq} = \sum_i C_i$$
- Parallel Resister arrangements $$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$
- Series Capacitor arrangements $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$
- Series Resister arrangements $$R_{eq} = \sum_i R_i$$

Example 26 – 2

Series or parallel? (a) The light bulbs in the figure are identical and have identical resistance R. Which configuration produces more light? (b) Which way do you think the headlights of a car are wired?



(1) Series (2) Parallel

(a) What are the equivalent resistances for the two cases?

Series $\Rightarrow$ $R_{eq} = 2R$ Parallel $\Rightarrow$ $\dfrac{1}{R_{eq}} = \dfrac{2}{R}$ So $\Rightarrow$ $R_{eq} = \dfrac{R}{2}$

The bulbs get brighter when the total power transformed is larger.

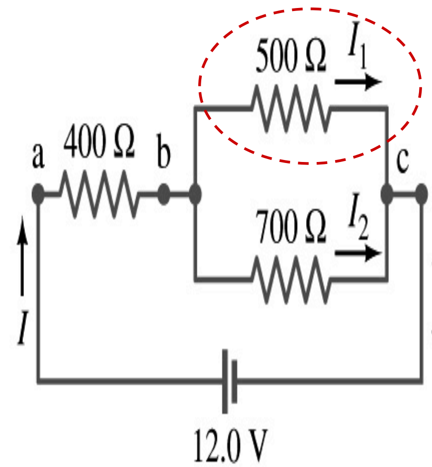
series $P_S = IV = \dfrac{V^2}{R_{eq}} = \dfrac{V^2}{2R}$ parallel $P_P = IV = \dfrac{V^2}{R_{eq}} = \dfrac{2V^2}{R} = 4P_S$

So parallel circuit provides brighter lighting.

(b) Car's headlights are in parallel to provide brighter lighting and also to prevent both lights going out at the same time when one burns out.

So what is bad about parallel circuits? Uses more energy in a given time.

Example 26 – 5



Current in one branch. What is the current flowing through the 500-Ω resister in the figure?

What do we need to find first?

We need to find the total current.

To do that we need to compute the equivalent resistance.

R_{eq} of the small parallel branch is: $\frac{1}{R_P} = \frac{1}{500} + \frac{1}{700} = \frac{12}{3500}$ $\quad R_P = \frac{3500}{12}$

R_{eq} of the circuit is: $R_{eq} = 400 + \frac{3500}{12} = 400 + 292 = 692\Omega$

Thus the total current in the circuit is $I = \frac{V}{R_{eq}} = \frac{12}{692} = 17mA$

The voltage drop across the parallel branch is $V_{bc} = IR_P = 17\times10^{-3}\cdot 292 = 4.96V$

The current flowing across 500-Ω resister is therefore

$$V_{bc} I_{500} = \frac{V_{bc}}{R} = \frac{4.96}{500} = 9.92\times10^{-3} = 9.92mA$$

What is the current flowing 700-Ω resister?

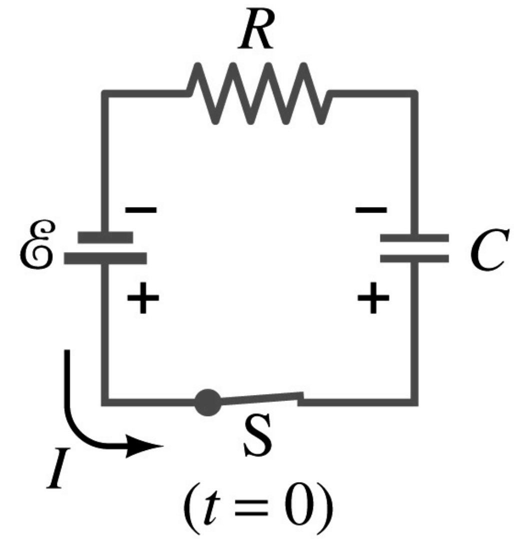
$I_{700} = I - I_{500} = 17 - 9.92 = 7.08mA$

Analysis of RC Circuits

- Since $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$ and $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$
- What can we see from the above equations?
  - Q and V_C increase from 0 at t=0 to maximum value $Q_{max} = C\mathscr{E}$ and $V_C = \mathscr{E}$.
- In how much time?
  - The quantity RC is called the time constant of the circuit, τ
    - τ=RC, What is the unit? Sec.
  - What is the physical meaning?
    - The time required for the capacitor to reach $(1-e^{-1})=0.63$ or 63% of the full charge
- The current is $I = \frac{dQ}{dt} = \frac{\varepsilon}{R}e^{-t/RC}$

Example 26 – 12

RC circuit, with emf. The capacitance in the circuit of the figure is C=0.30μF, the total resistance is 20kΩ, and the battery emf is 12V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when, the current I is 0.20 its maximum value.



(a) Since $\tau = RC$ We obtain $\tau = 20\times10^{3}\cdot 0.30\times10^{-6} = 6.0\times10^{-3}\ \text{sec}$

(b) Maximum charge is $Q_{\max} = C\varepsilon = 0.30\times10^{-6}\cdot 12 = 3.6\times10^{-6}\ C$

(c) Since $Q = C\varepsilon\left(1-e^{-t/RC}\right)$ For 99% we obtain $0.99C\varepsilon = C\varepsilon\left(1-e^{-t/RC}\right)$

$$e^{-t/RC} = 0.01;\quad -t/RC = -2\ln 10;\quad t = RC\cdot 2\ln 10 = 4.6RC = 28\times10^{-3}\ \text{sec}$$

(d) Since $\varepsilon = IR + Q/C$ We obtain $I = \left(\varepsilon - Q/C\right)/R$

The current when Q is $0.5Q_{\max}$ $I = \left(12 - 1.8\times10^{-6}/0.30\times10^{-6}\right)/20\times10^{3} = 3\times10^{-4}\ A$

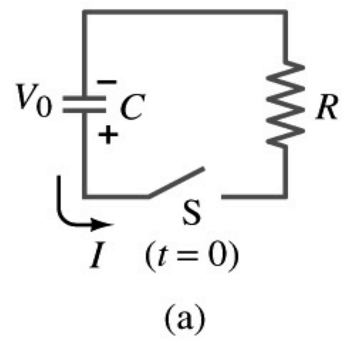
(e) When is I maximum? when Q=0: $I = 12/20\times10^{3} = 6\times10^{-4}\ A$

(f) What is Q when I=120mA? $Q = C\left(\varepsilon - IR\right) =$

$$= 0.30\times10^{-6}\left(12 - 1.2\times10^{-4}\cdot 2\times10^{4}\right) = 2.9\times10^{-6}\ C$$

Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance R.
  - When the switch S is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus IR=Q/C.
  - The rate at which the charge leaves the capacitor equals the negative the current flows through the resistor
    - I= - dQ/dt
    - Since the current is leaving the capacitor
  - Thus the voltage equation becomes a differential equation



$$-\frac{dQ}{dt}R=\frac{Q}{C} \quad \text{Rearrange terms} \quad \frac{dQ}{Q}=-\frac{dt}{RC}$$

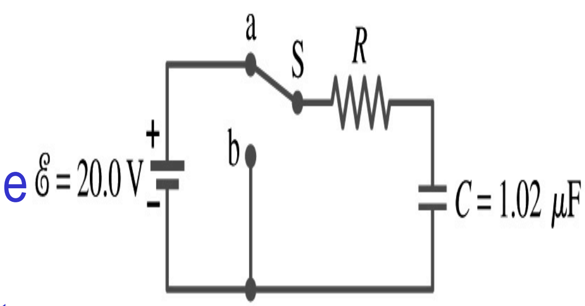
Discharging RC Circuits

- Now, let's integrate from t=0 when the charge is Q_0 to t when the charge is Q $$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC}$$
- The result is $$\ln Q\Big|_{Q_0}^{Q} = \ln \frac{Q}{Q_0} = -\frac{t}{RC}$$
- Thus, we obtain $$\boxed{Q(t) = Q_0 e^{-t/RC}}$$
- What does this tell you about the charge on the capacitor?
  - It decreases exponentially w/ time at the time constant RC
  - Just like the case of charging

What is this?

- The current is: $$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}$$ $$\boxed{I(t) = I_0 e^{-t/RC}}$$
  - The current also decreases exponentially w/ time w/ the constant RC

Example 26 – 13



Discharging RC circuit. In the RC circuit shown in the figure the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at t=0, the switch is thrown from position a to b. The battery emf is 20.0V, and the capacitance C=1.02μF. The current I is observed to decrease to 0.50 of its initial value in 40μs. (a) what is the value of R? (b) What is the value of Q, the charge on the capacitor, at t=0? (c) What is Q at t=60μs?

(a) Since the current reaches to 0.5 of its initial value in 40μs, we can obtain

$$I(t) = I_0 e^{-t/RC}$$

For 0.5I_0

$$0.5 I_0 = I_0 e^{-t/RC}$$

Rearrange terms

$$-t/RC = \ln 0.5 = -\ln 2$$

Solve for R

$$R = t/(C \ln 2) = 40\times 10^{-6} / \left(1.02\times 10^{-6} \cdot \ln 2\right) = 56.6\Omega$$

(b) The value of Q at t=0 is

$$Q_0 = Q_{\max} = C\varepsilon = 1.02\times 10^{-6} \cdot 20.0 = 20.4\mu C$$

(c) What do we need to know first for the value of Q at t=60μs?

The RC time $\tau = RC = 56.6 \cdot 1.02\times 10^{-6} = 57.7\mu s$

Thus $Q(t = 60\mu s) = Q_0 e^{-t/RC} = 20.4\times 10^{-6} \cdot e^{-60\mu s/57.7\mu s} = 7.2\mu C$

Application of RC Circuits

- What do you think the charging and discharging characteristics of RC circuits can be used for?
  - To produce voltage pulses at a regular frequency
  - How?
    - The capacitor charges up to a particular voltage and discharges
    - A simple way of doing this is to use breakdown of voltage in a gas filled tube
      - The discharge occurs when the voltage breaks down at V_0
      - After the completion of discharge, the tube no longer conducts
      - Then the voltage is at V_0' and it starts charging up
      - How do you think the voltage as a function of time look?
        - » A sawtooth shape
    - Pace maker, intermittent windshield wiper, etc