PHYS 1441 – Section 001
Lecture #13

Tuesday, June 28, 2016

Dr. Jaehoon Yu

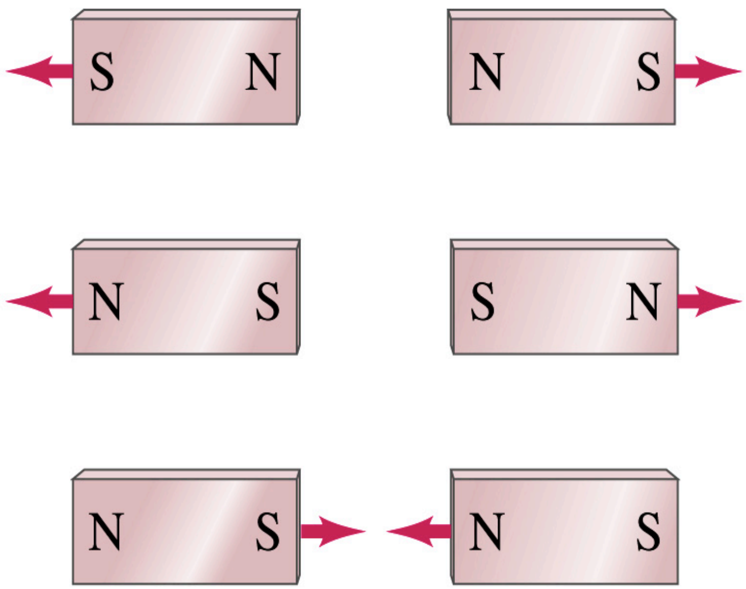
- Chapter 27: Magnetism and Magnetic Field
  - Electric Current and Magnetism
  - Magnetic Forces on Electric Current
  - About Magnetic Field
  - Magnetic Forces on a Moving Charge
  - Charged Particle Path in a Magnetic Field
  - Cyclotron Frequency
  - Torque on a Current Loop
  - Magnetic Dipole Moment

Announcements

- Quiz #3
  - Beginning of the class tomorrow, Wednesday, June 29
  - Covers from CH26.1 to what we cover today
  - BYOF
- Term 2
  - In class, Thursday, June 30
  - Covers from CH 26.1 through what we cover tomorrow
  - BYOF

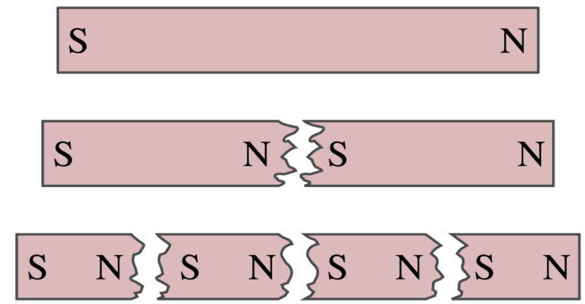
Magnetism

- What are magnets?
  - Objects with two poles, North and South poles
    - The pole that points to the geographical North is the North pole and the other is the South pole
      - Principle of compass
  - These are called magnets due to the name of the region, Magnesia, where rocks that attract each other were found
- What happens when two magnets are brought to each other?
  - They exert force onto each other
  - What kind?
  - Both repulsive and attractive forces depending on the configurations
    - Like poles repel each other while the unlike poles attract



Magnetism

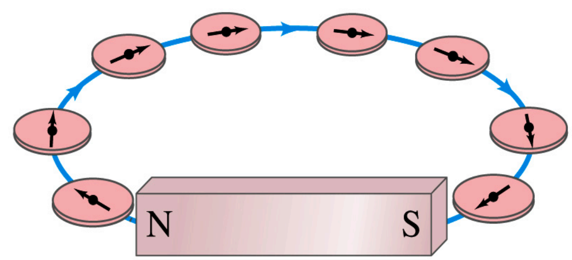
- So the magnetic poles are the same as the electric charge?
  - No. Why not?
  - While the electric charges (positive and negative) can be isolated, the magnetic poles cannot be isolated.
  - So what happens when a magnet is cut?
    - If a magnet is cut, two magnets are made.
    - The more they get cut, the more magnets are made

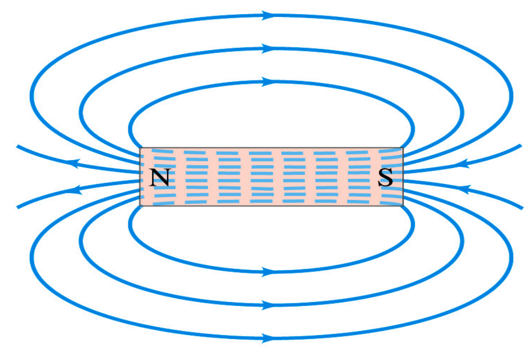


  - Single pole magnets are called the monopole but it has not been seen yet
- Ferromagnetic materials: Materials that show strong magnetic effects
  - Iron, cobalt, nickel, gadolinium and certain alloys
- Other materials show very weak magnetic effects

Magnetic Field

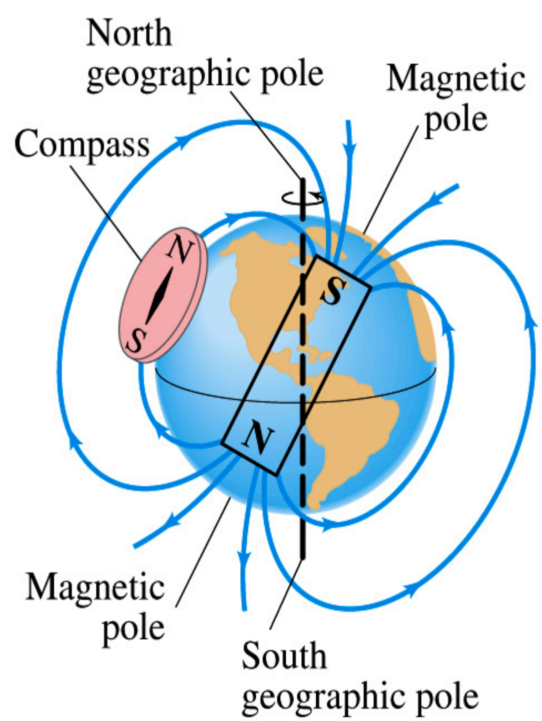
- Just like the electric field that surrounds electric charge, a magnetic field surrounds a magnet
- What does this mean?
  - Magnetic force is also a field force
  - The force one magnet exerts onto another can be viewed as the interaction between the magnet and the magnetic field produced by the other magnet
  - What kind of quantity is the magnetic field? Vector or Scalar? **Vector**
- So one can draw magnetic field lines, too.
  - The direction of the magnetic field is tangential to a line at any point
  - The direction of the field is the direction the north pole of a compass would point to
  - The number of lines per unit area is proportional to the strength of the magnetic field
  - Magnetic field lines continue inside the magnet
  - Since magnets always have both the poles, magnetic field lines form closed loops unlike electric field lines





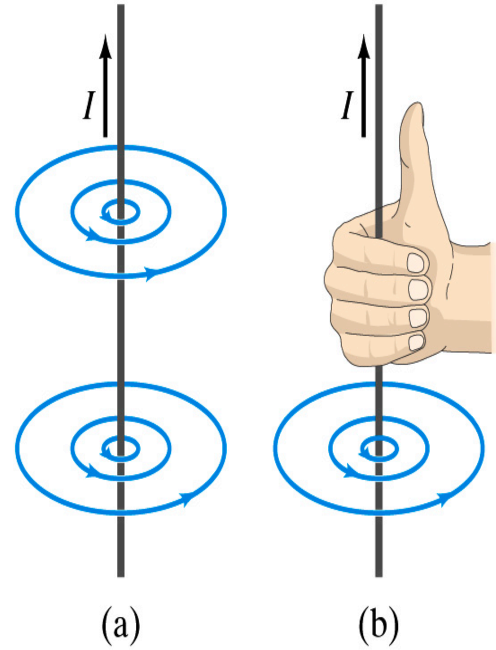
Earth's Magnetic Field

- What magnetic pole does the geographic North pole has to have?
  - Magnetic South pole. What? How do you know that?
  - Since the magnetic North pole points to the geographic North, the geographic north must have magnetic south pole
    - The pole in the North is still called geomagnetic North pole just because it is in the North
  - Similarly, South pole has magnetic North pole
- The Earth's magnetic poles do not coincide with the geographic poles ➔ magnetic declination
  - Geomagnetic North pole is in Northern Canada, some 900km off the true North pole
- Earth's magnetic field line is not tangent to the earth's surface at all points
  - The angle the Earth's field makes to the horizontal line is called the angle dip



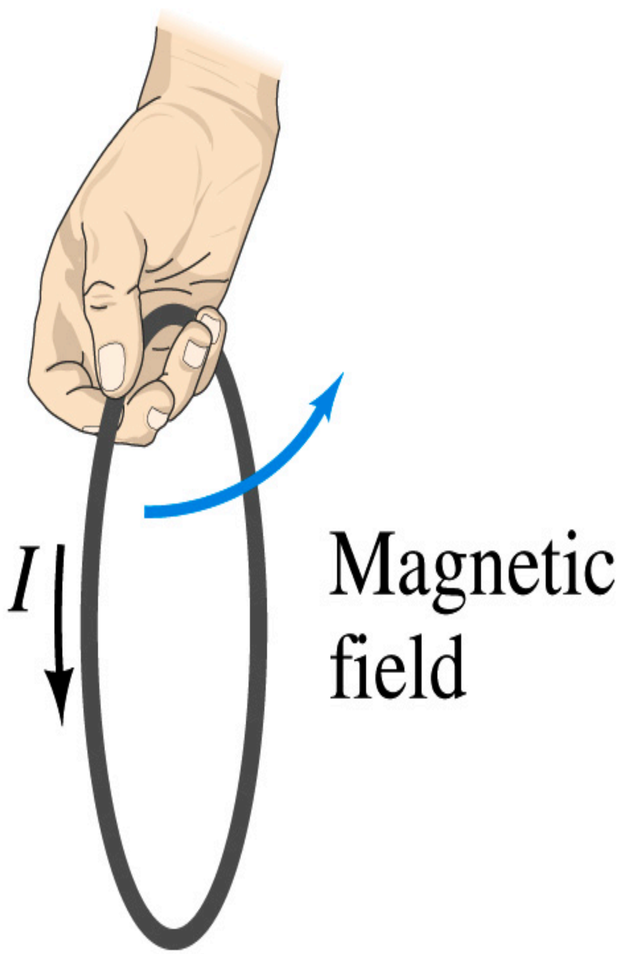
Electric Current and Magnetism

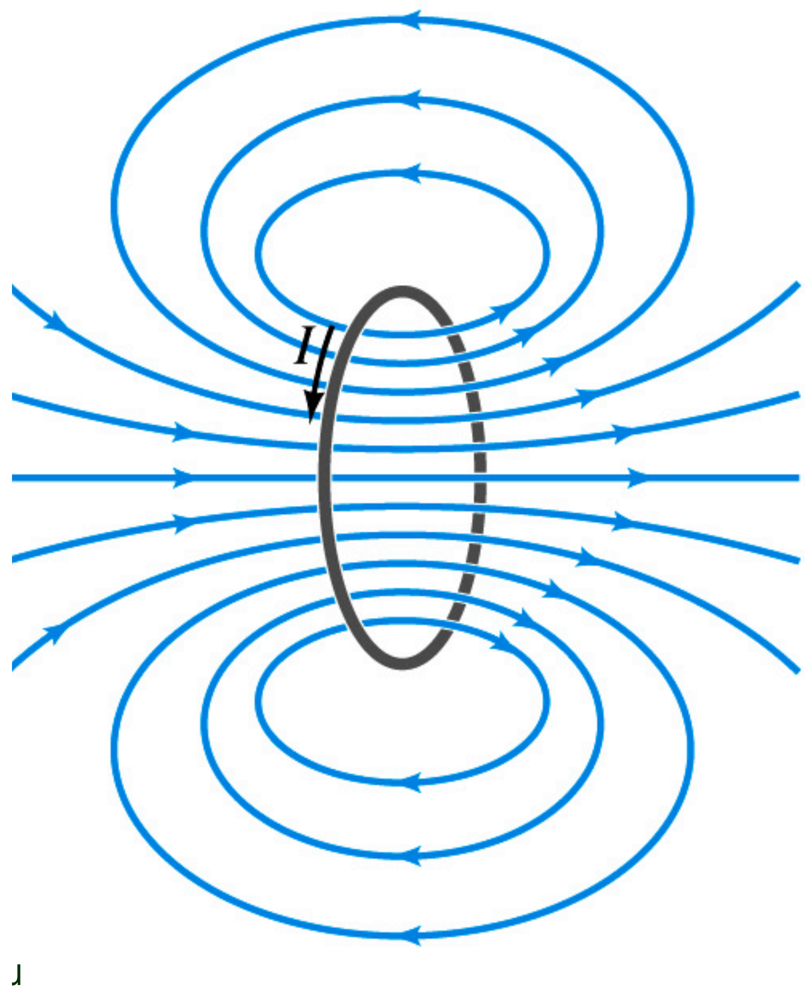
- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
  - Electric current produces a magnetic field
    - The first indication that electricity and magnetism are of the same origin
  - What about a stationary electric charge and magnet?
    - They don't affect each other.
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the "right-hand" rule
  - The field lines follow right-hand fingers wrapped around the wire when the thumb points to the direction of the electric current



Directions in a Circular Wire?

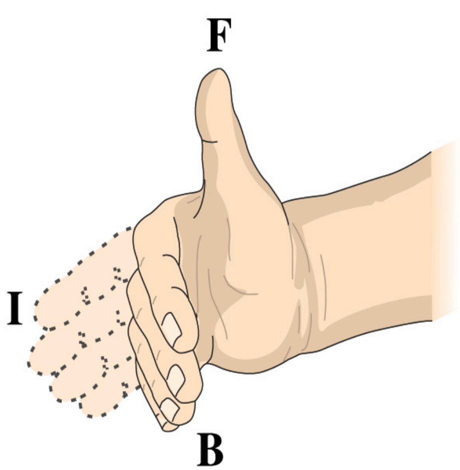
- OK, then what is the direction of the magnetic field generated by the current flowing through a circular loop?



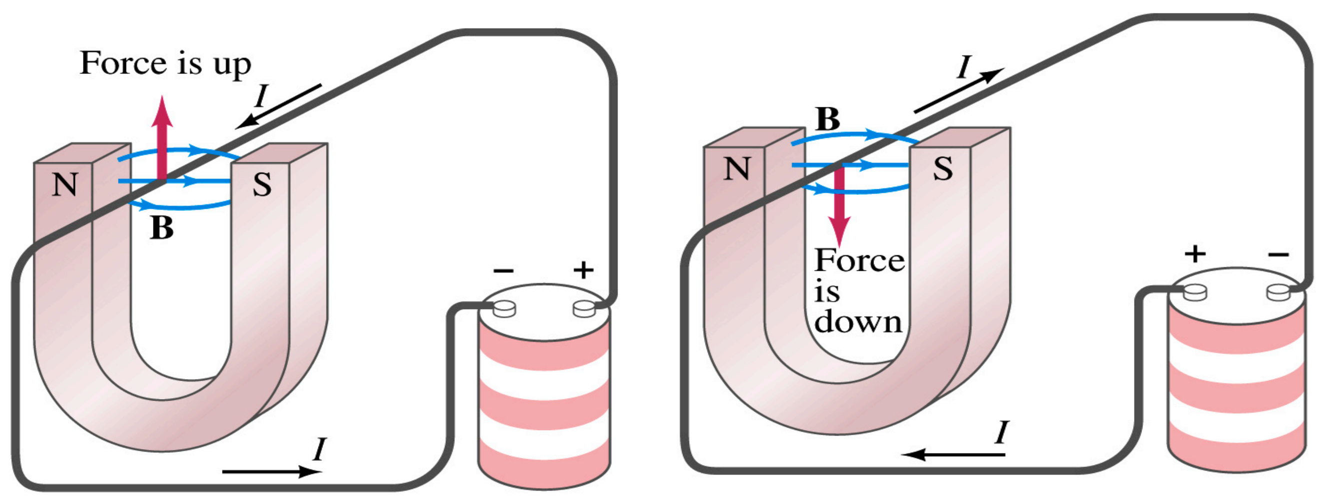


Magnetic Forces on Electric Current

- Since the electric current exerts force on a magnet, the magnet should also exert force on the electric current
  - Which law justifies this?
    - Newton's 3rd law
  - This was also discovered by Oersted
- Direction of the force is always
  - perpendicular to the direction of the current
  - perpendicular to the direction of the magnetic field, **B**
- Experimentally the direction of the force is given by another right-hand rule ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to the direction of the force



Right-hand rule



Magnetic Forces on Electric Current

- OK, we are set for the direction but what about the magnitude?
- It is found that the magnitude of the force is directly proportional
  - To the current in the wire
  - To the length of the wire in the magnetic field (if the field is uniform)
  - To the strength of the magnetic field
- The force also depends on the angle θ between the directions of the current and the magnetic field
  - When the wire is perpendicular to the field, the force is the strongest
  - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in the wire w/ length ℓ in a uniform field $\mathcal{B}$ is

$$F \propto IlB \sin\theta$$

Magnetic Forces on Electric Current

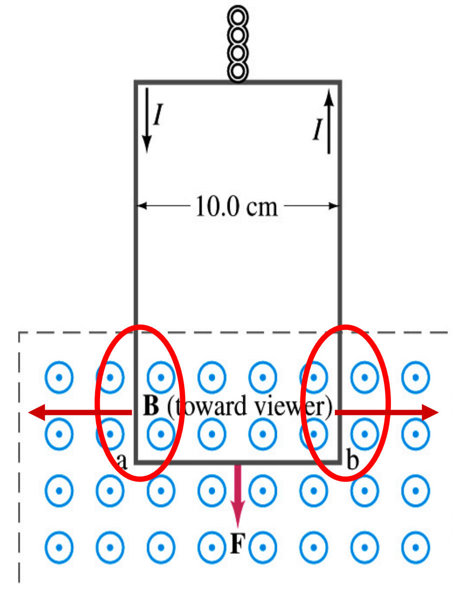
- Magnetic field strength B can be defined using the previous proportionality relationship w/ the constant 1: $F = IlB\sin\theta$
- if θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$
- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of the wire carrying the current I when the wire is perpendicular to **B**
- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - $\vec{l}$ is the vector whose magnitude is the length of the wire and its direction is along the wire in the direction of the conventional current
  - This formula works if **B** is uniform.
- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

Fundamentals on the Magnetic Field, B

- The magnetic field is a vector quantity
- The SI unit for B is tesla (T)
  - What is the definition of 1 Tesla in terms of other known units?
  - 1T=1N/Am
  - In older names, tesla is the same as weber per meter-squared
    - $1Wb/m^2=1T$
- The cgs unit for B is gauss (G)
  - How many T is one G?
    - $1G=10^{-4}$ T
  - For computation, one MUST convert G to T at all times
- Magnetic field on the Earth's surface is about $0.5G=0.5x10^{-4}T$
- On a diagram, e for field coming out and ⊗ for going in.

Example 27 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire *ab* (length *l*=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of F=3.48x10^{-2}N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is $\vec{F} = I\vec{l} \times \vec{B}$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

Solving for B

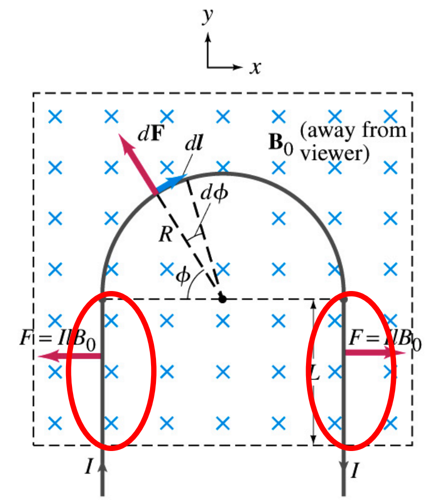
$$B = \frac{F}{Il} = \frac{3.48\times10^{-2}\,N}{0.245A\cdot 0.10m} = 1.42T$$

Something is not right! What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.

Example 27 – 3

Magnetic force on a semi-circular wire. A rigid wire, carrying the current I, consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field $\mathbf{B_0}$. The straight portions each have length ℓ within the field. Determine the net force on the wire due to the magnetic field $\mathbf{B_0}$.



As in the previous example, the forces on the straight sections of the wire is equal and in opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle? $d\vec{F} = Id\vec{l} \times \vec{B}$

We divide the semicircle into infinitesimal straight sections. $dl = Rd\phi$

What is the net x component of the force exerting on the circular section? 0 **Why?**

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = d\left(F\sin\phi\right) = IRB_0 d\phi$

Integrating over φ=0 - π

$$F = \int_0^{\pi} d\left(F\sin\phi\right) = IB_0R\int_0^{\pi}\sin\phi d\phi = -IB_0R\left[\cos\phi\right]_0^{\pi} = 2RIB_0$$

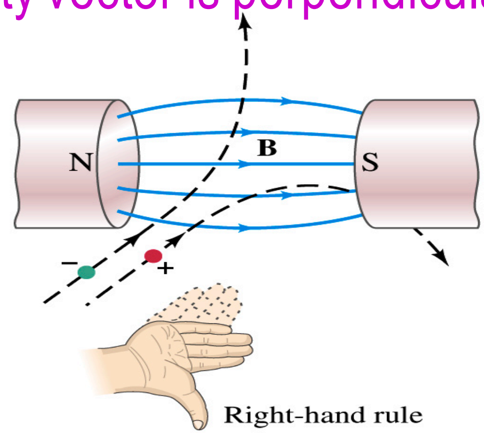
Which direction? Vertically upward direction. The wire will be pulled deeper into the field.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since the wire carrying a current (moving charge) experience force in a magnetic field, a free moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by a given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for a charge q to travel a distance L in a magnetic field **B**
    - Then, the length vector ℓ becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\vec{F} = q\vec{v} \times \vec{B}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity v in a field B is
    - $F = qvB\sin\theta$
    - What is θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the angle between the field and the velocity vector is perpendicular.
    - $F_{max} = qvB$ ➔ $B = \frac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 27 – 5

Magnetic force on a proton. A proton having a speed of $5x10^6$m/s in a magnetic field feels a force of F=$8.0x10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6\times10^{-19}C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward West, the field should be pointing to …. North

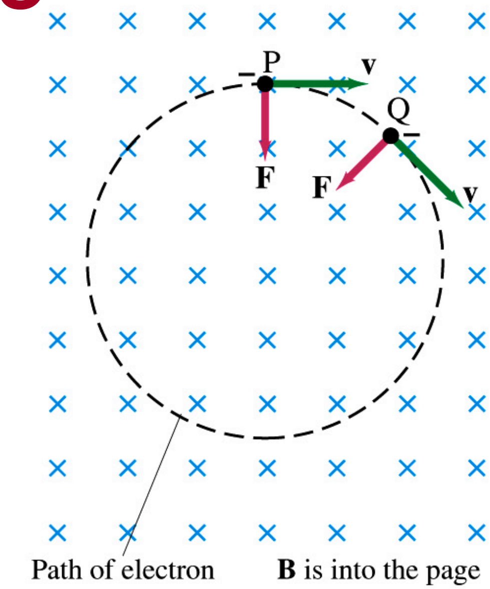
Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0\times10^{-14}N}{1.6\times10^{-19}C\cdot5.0\times10^{6}m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field

- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward



  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0x10^{7}m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\frac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is

$$F = evB$$

Since the magnetic force provides the centripetal force, we can establish an equation with the two forces

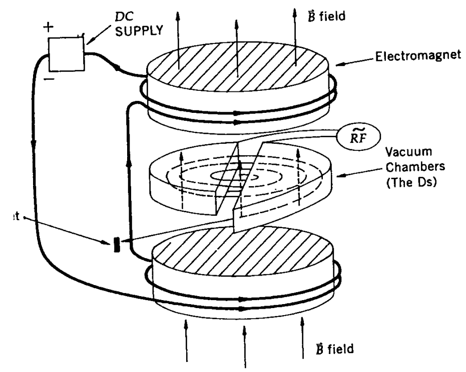
$$F = evB = m\frac{v^2}{r}$$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



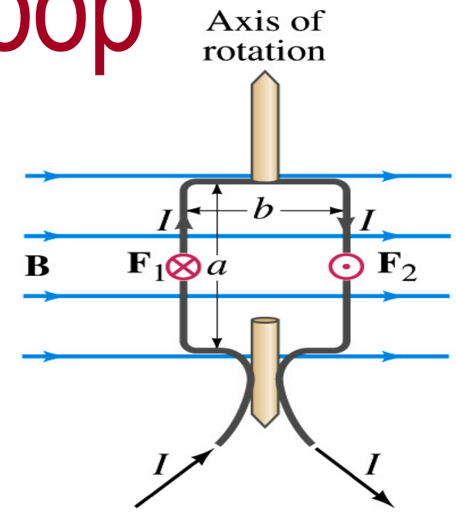
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

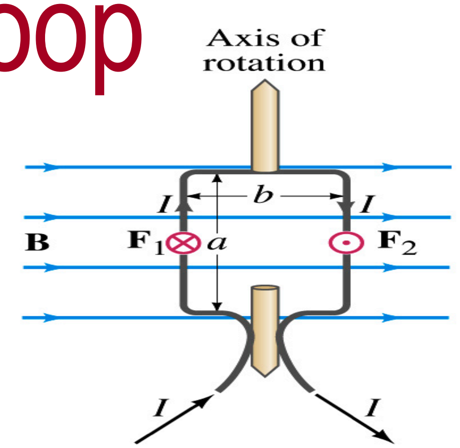
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop

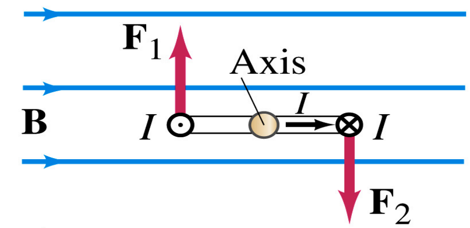


- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces



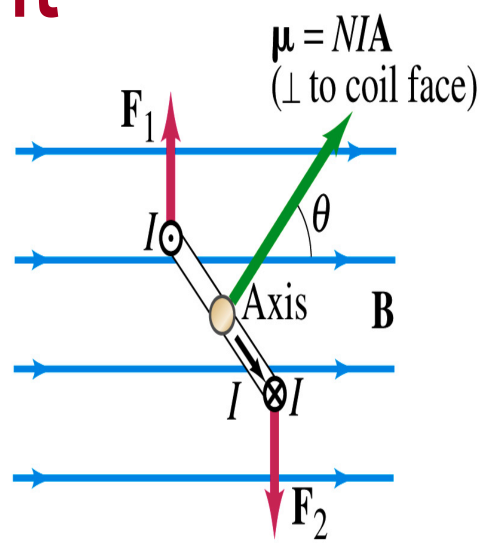
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A = ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity N$I\mathcal{A}$ is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\overrightarrow{A}$
    - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the direction of the current
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\overrightarrow{A} \times \vec{B} = \vec{\mu} \times \vec{B}$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at $\theta=\pi/2$, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from $F=\dfrac{e^2}{4\pi\varepsilon_0 r^2}=\dfrac{m_e v^2}{r}$

Solving for v

$$v=\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}}=2.19\times10^6\,m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current $I=\dfrac{e}{T}=\dfrac{ev}{2\pi r}$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu=IA=\frac{ev}{2\pi r}\pi r^2=\frac{evr}{2}=\frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$