PHYS 1441 – Section 001
Lecture #14

Tuesday, June 29, 2016

Dr. Jaehoon Yu

- Chapter 28:Sources of Magnetic Field
  - Sources of Magnetic Field
  - Magnetic Field Due to Straight Wire
  - Forces Between Two Parallel Wires
  - Ampére's Law and Its Verification
  - Solenoid and Toroidal Magnetic Field
  - Biot-Savart Law
  - Magnetic Materials

Today's homework is #8, due 11pm, Saturday, July 2!!

Announcements

- Term 2
  - In class, tomorrow, Thursday, June 30
  - Covers from CH 26.1 through what we cover today
  - BYOF
- Reading Assignment
  - CH27.8, CH28.8, 9 and 10

Special Project #5

B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that

(a) the Ampere's law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$. (10 points)

(b) That Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$. (10 points)

- Must be your OWN work. No credit will be given for for copying straight out of the book, lecture notes or from your friends' work.
- Due is at the beginning of the exam on Tuesday, July 5

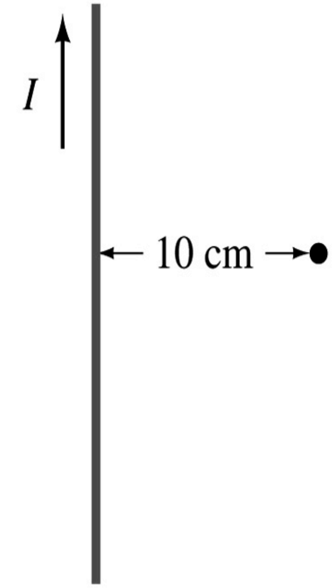
Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
  - How do we determine magnetic field strengths in certain situations?
  - How do two wires with electric current interact?
  - What is the general approach to finding the connection between current and magnetic field?

Magnetic Field due to a Straight Wire

- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes $$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$
  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Example 28 – 1



Calculation of B near wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due north of this wire?

Using the formula for the magnetic field near a straight wire

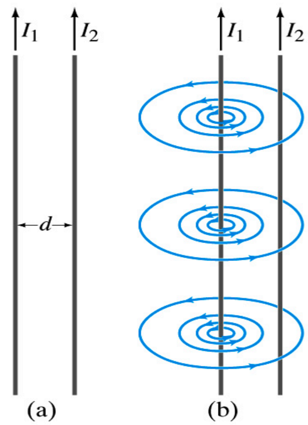
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ T \cdot m/A\right) \cdot \left(25A\right)}{\left(2\pi\right) \cdot \left(0.01m\right)} = 5.0 \times 10^{-5}\ T$$

Force Between Two Parallel Wires

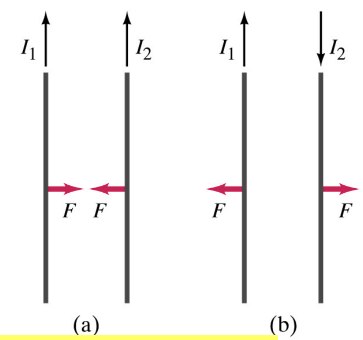
- We have learned that a wire carrying the electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or att
  - Depending on the direction of the currents



- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance d, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

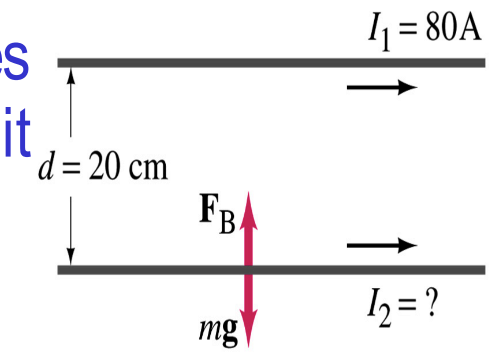
- The force F by a magnetic field B_1 on a wire of length l, carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  – So the force per unit length is $$\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi} \frac{I_1}{d}$$
  – This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in opposite direction
- So the force per unit length is $$\boxed{\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}}$$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times10^{-3}\,kg\right)\cdot(0.20m)}{\left(4\pi\times10^{-7}\;T\cdot m/A\right)\cdot(80A)} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

  – So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly 2×10^{-7}N/m.

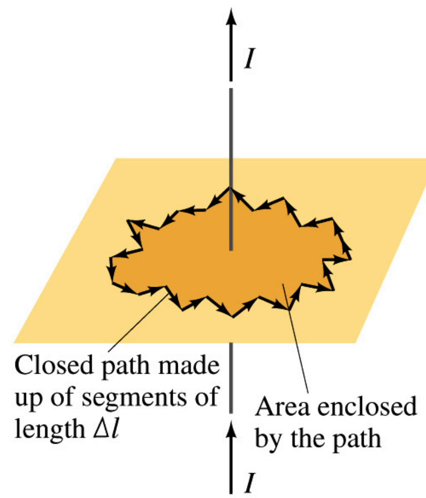
- Coulomb is then defined as exactly 1C=1A s.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.

Ampére's Law

- What is the relationship between the magnetic field strength and the current? $\boxed{B = \frac{\mu_0 I}{2\pi r}}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shapes of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path with small segments each of Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $\boxed{\sum B_{P} \Delta l = \mu_0 I_{encl}}$
  - In the limit $\Delta l \rightarrow 0$, this relation becomes
  - $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$

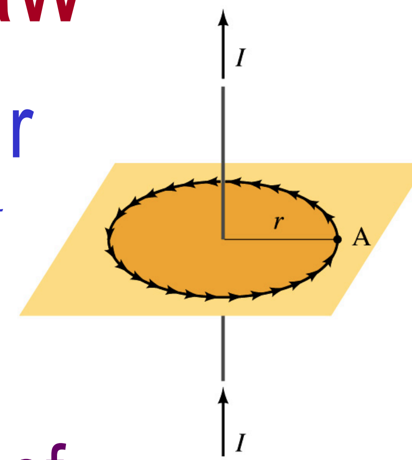


Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

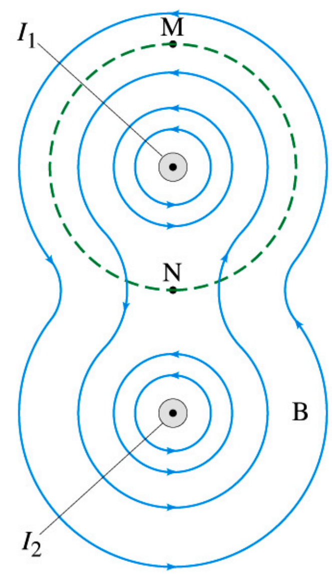
Solving for $\mathcal{B}$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi} \frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance, however, is that it provides means to relate magnetic field to current

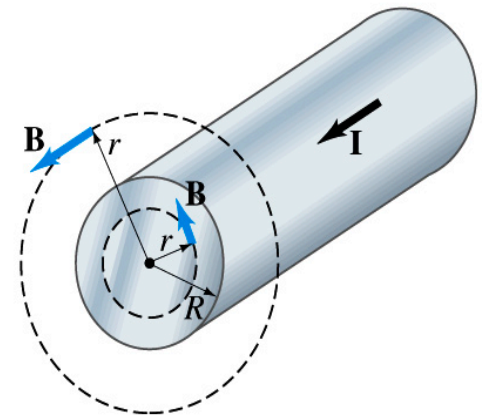
Verification of Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl}.
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

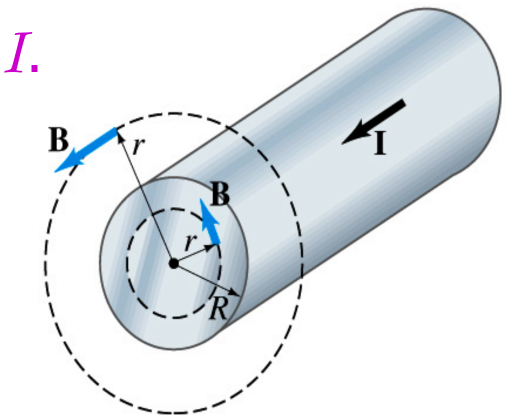
$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Example 28 – 6 cont'd

For r<R, the current inside the closed path is less than I. How much is it?



$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$

So using Ampere's law

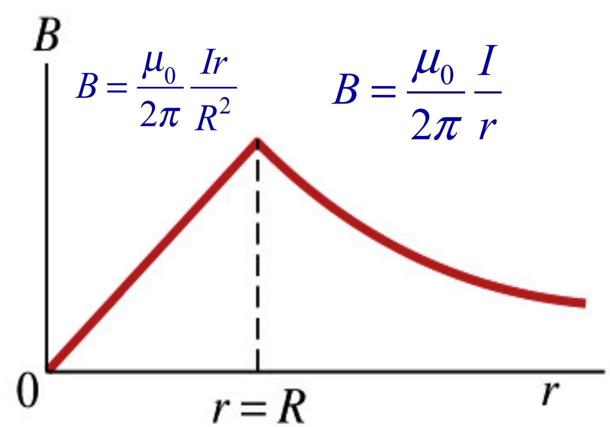
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

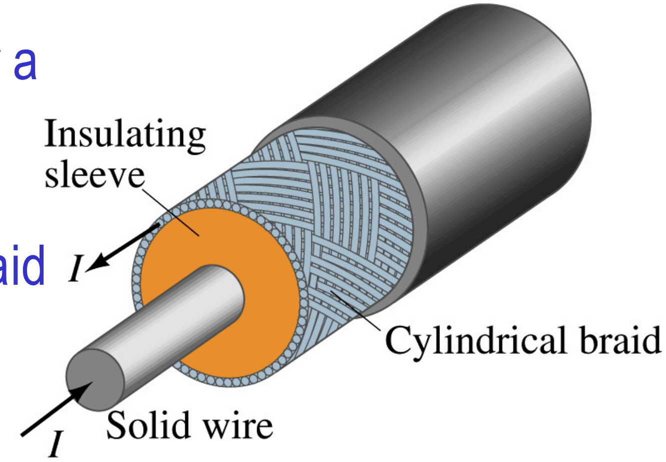
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

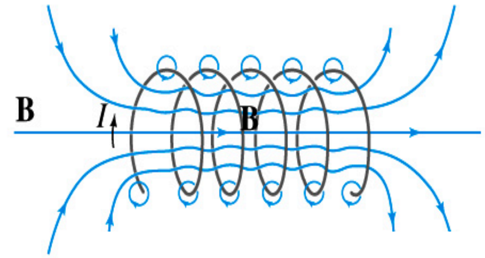
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

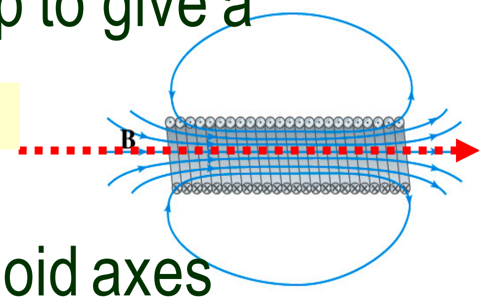
So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field

- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops are wide
    - The field near the wires are nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform
  - For a long, densely packed loops
    - The field is nearly uniform and parallel to the solenoid axes within the entire cross section
    - The field outside the solenoid is very small compared to the field inside, except the ends
      - The same number of field lines spread out to an open space

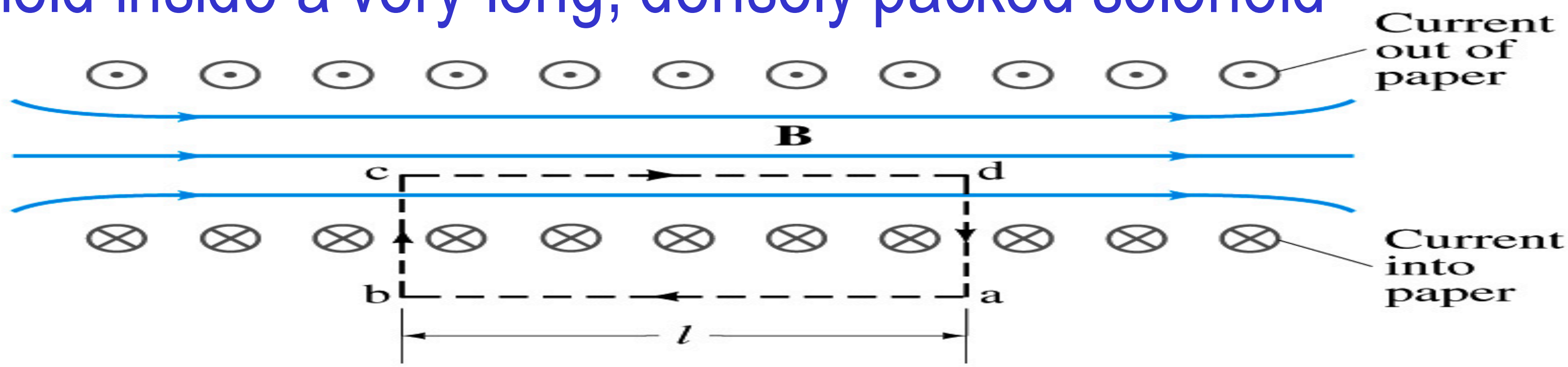


Solenoid Axis



Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} + \int_d^a \vec{B}\cdot d\vec{l}$$
  - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
  - Now the field B is perpendicular to the *bc* and *da* segments. So these integrals become 0, also.

Solenoid Magnetic Field

– So the sum becomes: $\oint \vec{B}\cdot d\vec{l} = \int_c^d \vec{B}\cdot d\vec{l} = Bl$

– If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$

  • Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed

– Thus Ampere's law gives us $Bl = \mu_0 NI$

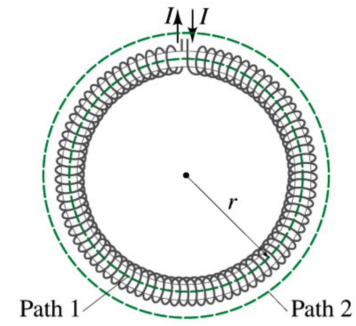
– If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes

– $$\boxed{B = \mu_0 nI}$$

  • B depends on the number of loops per unit length, n, and the current I

    – Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

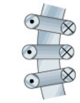
Example 28 – 10



(a)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.

(a) How do you think the magnetic field lines inside the toroid look?


(b)

Since it is a bent solenoid, it should be a circle concentric with the toroid. If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

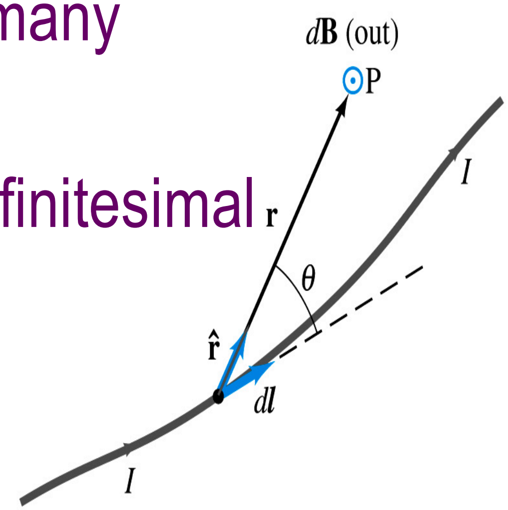
$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI \quad \xrightarrow{\text{Solving for B}} \quad B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The filed in this case is $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Biot-Savart Law

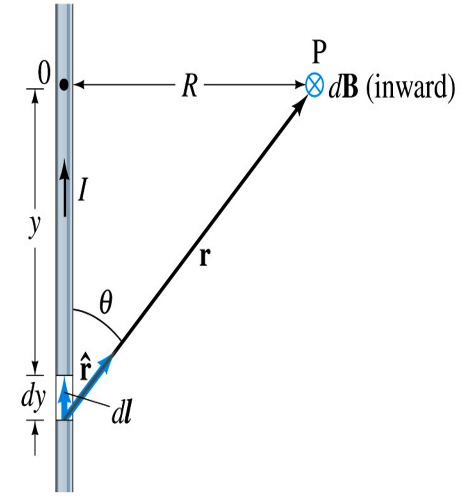
- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method of using infinitesimal current segments for B field
  - Jean Baptiste Biot and Feilx Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
  - The infinitesimal magnetic field d**B** caused by the infinitesimal length $d\ell$ that carries current I is
  - $$\boxed{d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l} \times \hat{r}}{r^2}}$$ **Biot-Savart Law**
    - **r** is the displacement vector from the element $d\ell$ to the point P
    - Biot-Savart law is the magnetic equivalent to Coulomb's law



B field in Biot-Savart law is only that by the current, nothing else.

Example 28 – 11

B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that the Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$.



What is the direction of the field **B** at point P? Going into the page.

All d**B** at point P has the same direction based on right-hand rule.

The magnitude of **B** using Biot-Savart law is

$$B=\int dB=\frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{\left|d\vec{l}\times\hat{r}\right|}{r^2}=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}$$

Where $dy=dl$ and $r^2=R^2+y^2$ and since $y=-R\cot\theta$ we obtain

$$dy=+R\csc^2\theta d\theta=\frac{Rd\theta}{\sin^2\theta}=\frac{Rd\theta}{\left(R/r\right)^2}=\frac{r^2 d\theta}{R}$$

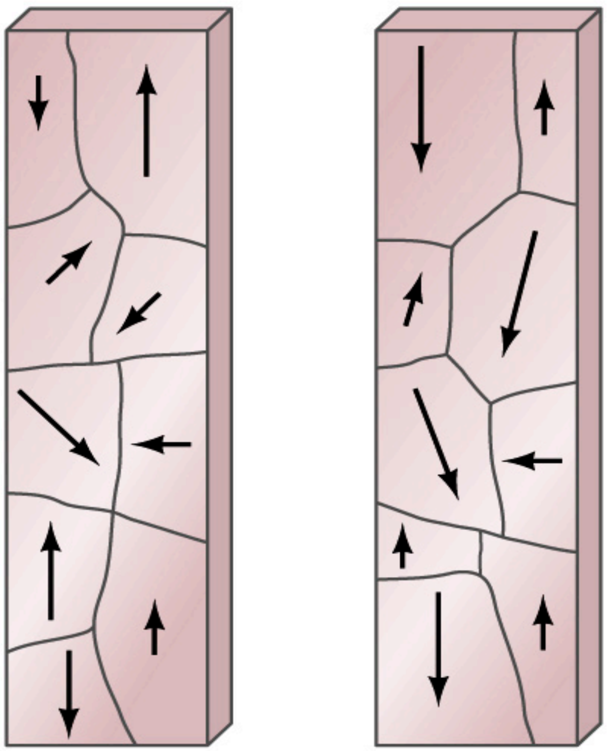
Integral becomes

$$B=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}=\frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi}\sin\theta d\theta=-\frac{\mu_0 I}{4\pi}\frac{1}{R}\cos\theta\Big|_0^{\pi}=\frac{\mu_0 I}{2\pi}\frac{1}{R}$$

The same as the simple, long straight wire!! It works!!

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
  - This kind of material is called **ferromagnetic** material
- In microscopic sense, ferromagnetic materials consist of many tiny regions called **domains**
  - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
  - Randomly arranged
- What if they are magnetized?
  - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
  - This gives magnetization to the material
- How do we demagnetize a bar magnet?
  - Hit the magnet hard or heat it over the Curie temperature


(b)

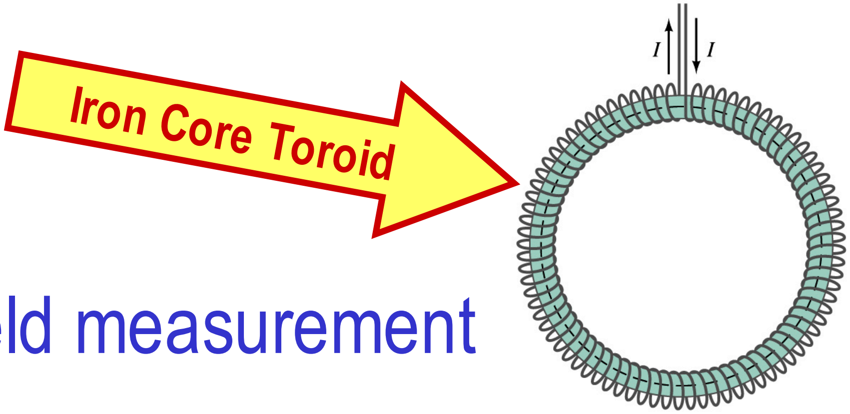
B in Magnetic Materials

- What is the magnetic field inside a solenoid?
- $$B_0 = \mu_0 n I$$
  - Magnetic field in a long solenoid is directly proportional to the current.
  - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
  - It will be increased dramatically, when the current flows
    - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
  - Since the domains in the iron aligns permanently by the external field.
  - The resulting magnetic field is the sum of that due to current and due to the iron

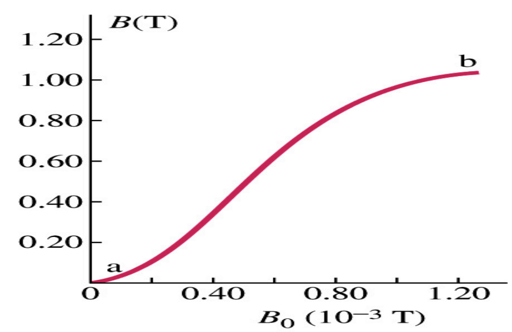
B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $\vec{B} = \vec{B}_0 + \vec{B}_M$
  - **B_0** is the field due only to the current in the wire, namely the external field
    - The field that would be present without a ferromagnetic material
  - **B_M** is the additional field due to the ferromagnetic material itself; often **B_M**>>**B_0**
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ, the magnetic permeability of the material $\boxed{B = \mu n I}$
  - μ is a property of a magnetic material
  - μ is not a constant but varies with the external field

Hysteresis

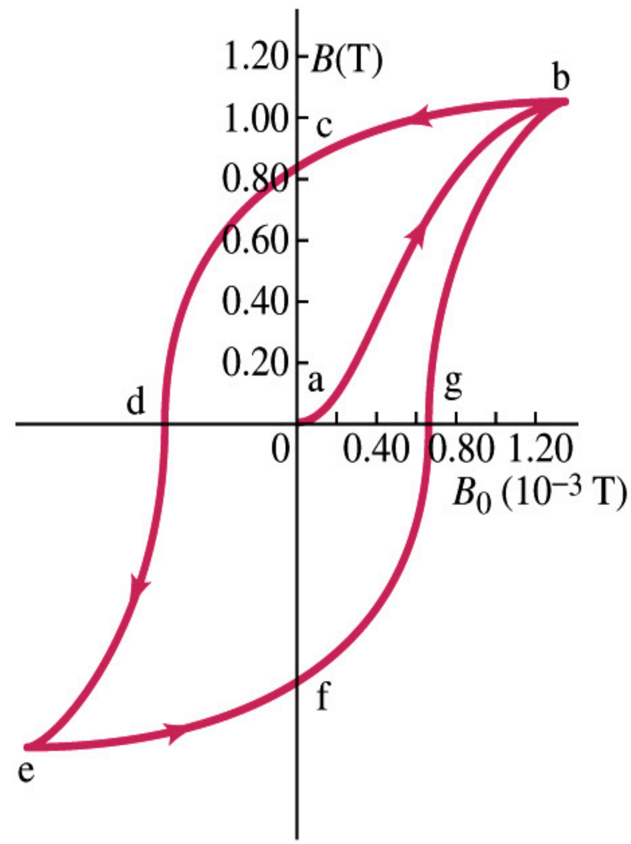


- What is a toroid?
  - A solenoid bent into a shape
- Toroid can be used for magnetic field measurement
  - Why?
  - Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.
- Consider an un-magnetized iron core toroid, without any current flowing in the wire
  - What do you think will happen if the current slowly increases?
  - B_0 increases linearly with the current.
  - And B increases also but follows the curved line shown in the graph
  - As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)
    - The iron is said to be approaching saturation
    - Point b is typically at 70% of the max



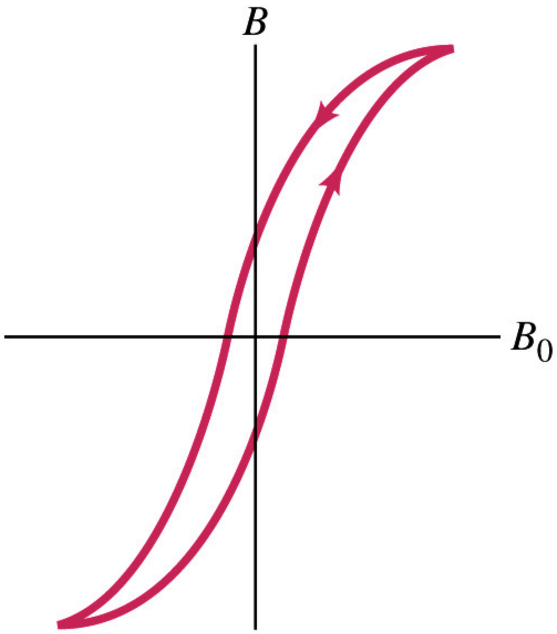
Hysteresis

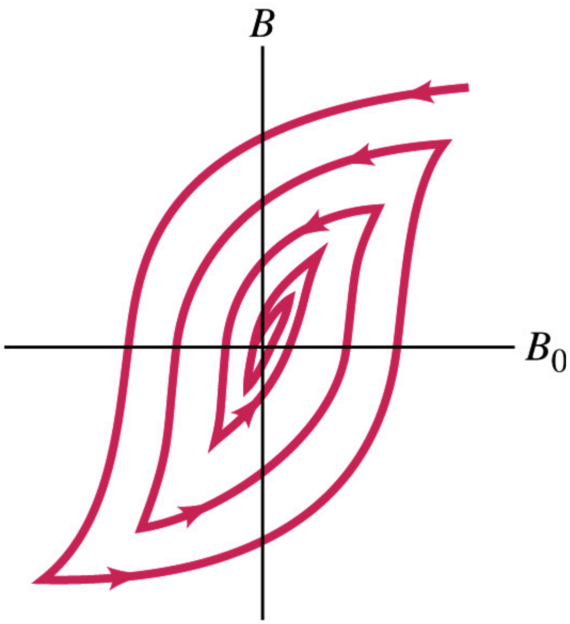
- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
  - ~~Of course it goes to 0!!~~
  - Wrong! Wrong! Wrong! They do not go to 0. Why not?
  - The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
  - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
  - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
  - Which one do you think are preferred in electromagnets or transformers?
    - Soft. Why?
    - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
  - Keep repeating the Hysteresis loop, reducing the range of B_0.



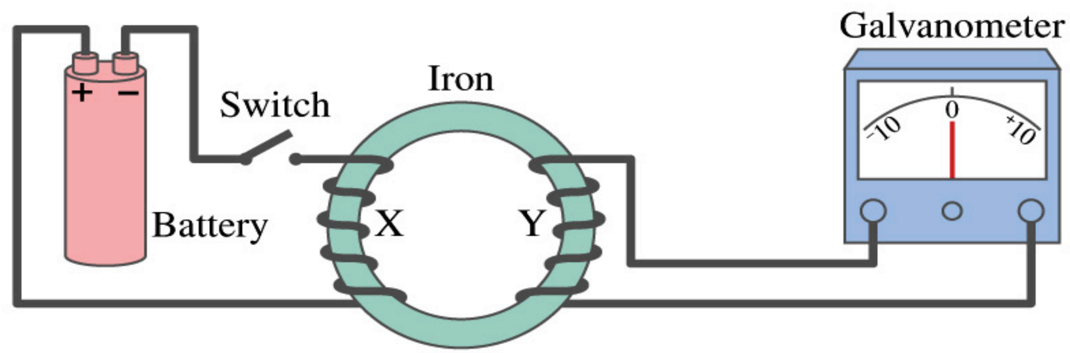


Induced EMF

- It has been discovered by Oersted and company in early 19th century that
  - Magnetic field can be produced by the electric current
  - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
  - Yes, you are absolutely right! You would wonder if the magnetic field can create the electric current.
  - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
    - Though, Faraday was given the credit since he published his work before Henry did
      - He also did a lot of detailed studies on magnetic induction

Electromagnetic Induction

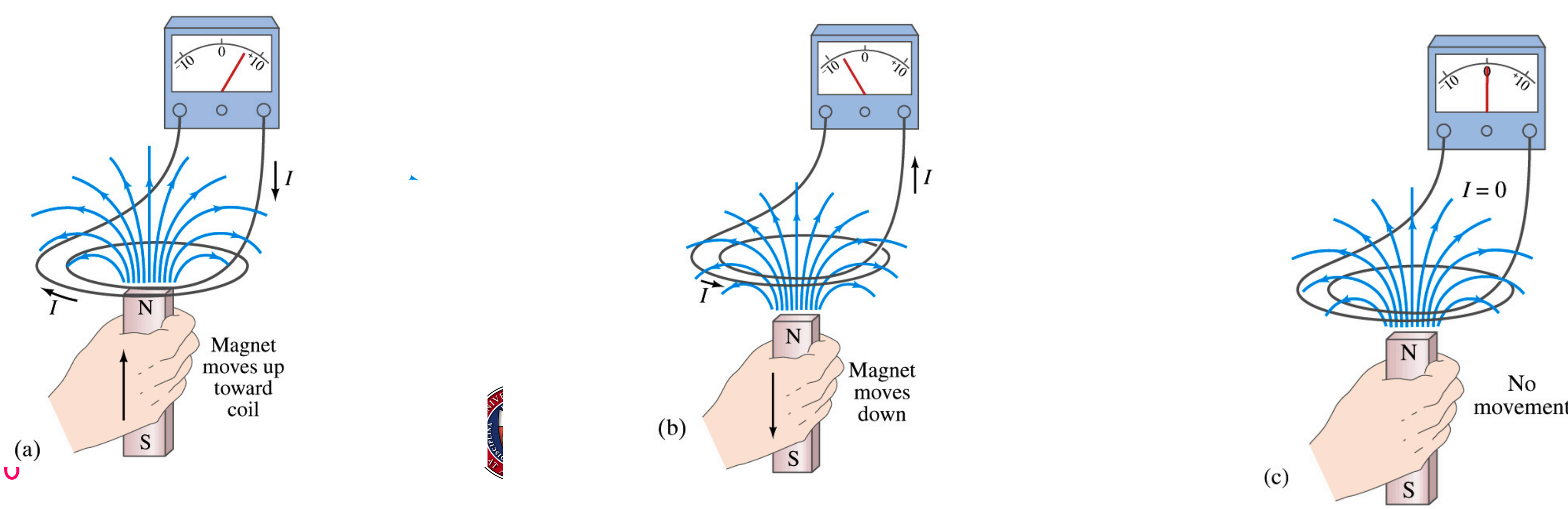
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
  - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field ➔ Electromagnetic Induction**

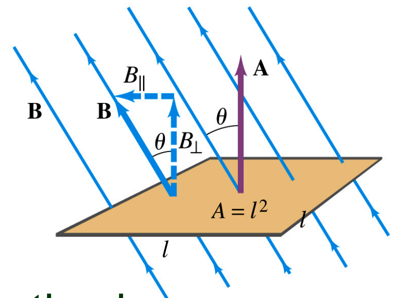
Electromagnetic Induction

- Further studies on electromagnetic induction taught
  - If a magnet is moved quickly into a coil of wire, a current is induced in the wire.
  - If a magnet is removed from the coil, a current is induced in the wire in the opposite direction
  - By the same token, the current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
  - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
  - The rate of changes of the magnetic field?
    - the higher the changes the higher the induction
  - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B.
  - Magnetic flux is defined as (just like the electric flux)
  - $$\boxed{\Phi_B = B_\perp A = BA\cos\theta = \vec{B}\cdot\vec{A}}$$



    - θ is the angle between **B** and the area vector **A** whose direction is perpendicular to the face of the loop based on the right-hand rule
  - What kind of quantity is the magnetic flux?
    - Scalar. Unit?
    - $T \cdot m^2$ or weber $\boxed{1Wb = 1T \cdot m^2}$
- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as $$\boxed{\Phi_B = \int \vec{B}\cdot d\vec{A}}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

Faraday's Law of Induction

- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\boxed{\varepsilon = -N\frac{d\Phi_B}{dt}}$$

  - Why negative?
    - Has got a lot to do with the direction of induced emf…

Lenz's Law

- It is experimentally found that
  - An induced emf gives rise to a current whose magnetic field opposes the original change in flux ➔ This is known as **Lenz's Law**
  - In other words, an induced emf is always in a direction that opposes the original change in flux that caused it.
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise
- Which law is Lenz's law result of?
  - Energy conservation. Why?