

# PHYS 1441 – Section 001

## Lecture #16

*Tuesday, July 5, 2016*

*Dr. Jaehoon Yu*

- Chapter 29: EM Induction & Faraday's Law
  - Faraday's Law of Induction
  - Lenz's Law
  - Generation of Electricity
  - Transformer
  - Mutual and Self Inductance
  - Energy Stored in Magnetic Field
  - Alternating Current and AC Circuits
- Chapter 30: Inductance

Today's homework is #9, due 11pm, Thursday, July 7!!

Tuesday, July 5 Last homework #10, due 11pm, Saturday, July 9!!

# Announcements

- Quiz #4 Thursday, July 7
  - Beginning of the class
  - Covers 29.1 through what we finish tomorrow, Wednesday, July 6
  - BYOF
- Final exam
  - In class (10:30am – 12:30pm) Monday, July 11
  - Comprehensive exam
    - Covers CH21.1 through what we finish Thursday, July 7
  - BYOF
- Reading Assignments
  - CH30.9 – CH30.11



# Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
  - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

**Faraday's Law of Induction**

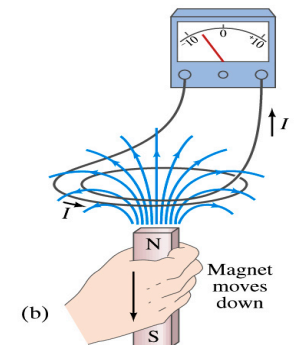
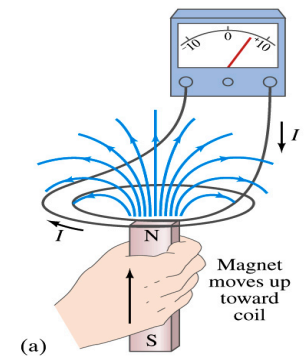
- If the circuit contains  $N$  closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- Why negative?
  - Has got a lot to do with the direction of induced emf...

# Lenz's Law

- It is experimentally found that
  - An induced emf gives rise to a current whose magnetic field opposes the original change in flux → This is known as **Lenz's Law**
  - In other words, an induced emf is always in a direction that opposes the original change in flux that caused it.
  - We can use Lenz's law to explain the following cases in the figures
    - When the magnet is moving into the coil
      - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
    - When the magnet is moving out
      - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise
- Which law is Lenz's law result of?
  - Energy conservation. Why?

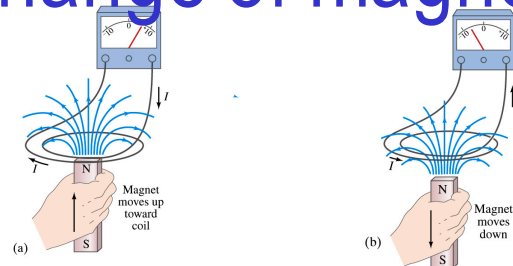




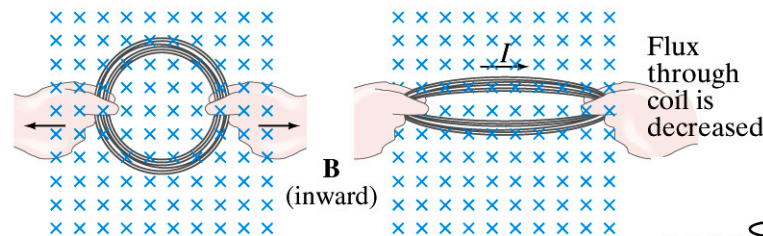
# Induction of EMF

- How can we induce emf?
- Let's look at the formula for the magnetic flux
- $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \theta dA$
- What do you see? What are the things that can change with time to result in a change of magnetic flux?

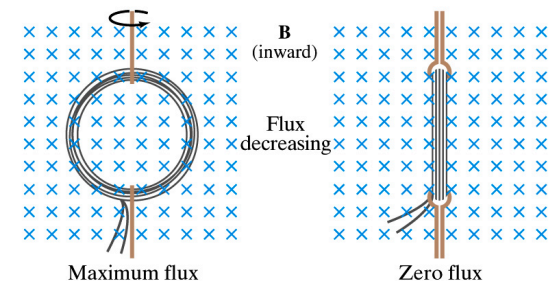
– Magnetic field



– The area of the loop



– The angle  $\theta$  between the field and the area vector



Tuesday, July 5, 2016

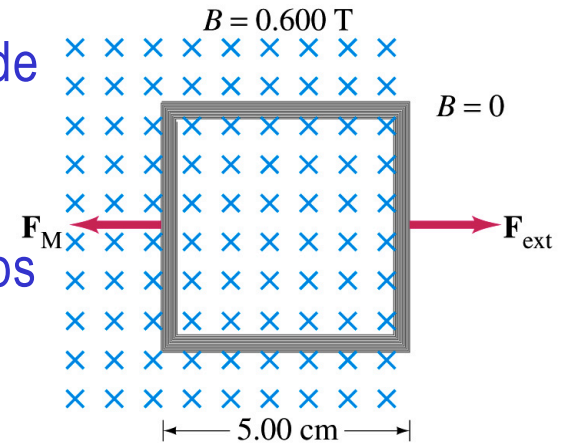


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# Example 29 – 5

**Pulling a coil from a magnetic field.** A square coil of wire with side 5.00cm contains 100 loops and is positioned perpendicular to a uniform 0.600-T magnetic field. It is quickly and uniformly pulled from the field (moving perpendicular to  $B$ ) to a region where  $B$  drops abruptly to zero. At  $t=0$ , the right edge of the coil is at the edge of the field. It takes 0.100s for the whole coil to reach the field-free region. Find (a) the rate of change in flux through the coil, (b) the emf and current induced, and (c) how much energy is dissipated in the coil if its resistance is  $100\Omega$ . (d) what was the average force required?



What should be computed first? The initial flux at  $t=0$ .

The flux at  $t=0$  is  $\Phi_B = \vec{B} \cdot \vec{A} = BA = 0.600T \cdot (5 \times 10^{-2}m)^2 = 1.50 \times 10^{-3} Wb$

The change of flux is  $\Delta\Phi_B = 0 - 1.50 \times 10^{-3} Wb = -1.50 \times 10^{-3} Wb$

Thus the rate of change of the flux is

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{-1.50 \times 10^{-3} Wb}{0.100s} = -1.50 \times 10^{-2} Wb/s$$

# Example 29 – 5, cnt'd

Thus the total emf induced in this period is

$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = -100 \cdot (-1.50 \times 10^{-2} \text{ Wb/s}) = 1.5V$$

The induced current in this period is

$$I = \frac{\mathcal{E}}{R} = \frac{1.5V}{100\Omega} = 1.50 \times 10^{-2} \text{ A} = 15.0 \text{ mA}$$

Which direction would the induced current flow?    Clockwise

The total energy dissipated is

$$E = Pt = I^2 R t = (1.50 \times 10^{-2} \text{ A})^2 \cdot 100\Omega \cdot 0.100 \text{ s} = 2.25 \times 10^{-3} \text{ J}$$

Force for each coil is  $\vec{F} = I\vec{l} \times \vec{B}$     Force for N coil is  $\vec{F} = N I \vec{l} \times \vec{B}$

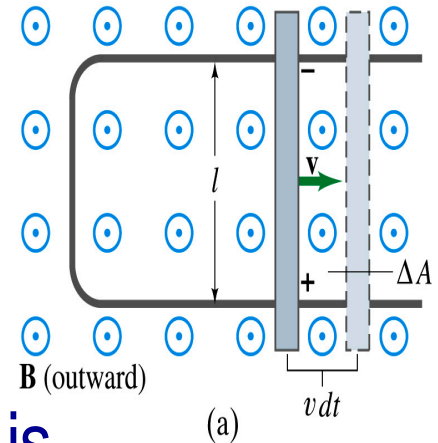
$$|F| = N I l B = 100 \cdot (1.50 \times 10^{-2} \text{ A}) \cdot (4 \times 5 \times 10^{-2}) \cdot 0.600 \text{ T} = 0.045 \text{ N}$$

# EMF Induced on a Moving Conductor

- Another way of inducing emf is using a U shaped conductor with a movable rod resting on it.
- As the rod moves at a speed  $v$ , it travels  $vdt$  in time  $dt$ , changing the area of the loop by  $dA = \ell v dt$ .
- Using Faraday's law, the induced emf for this loop is

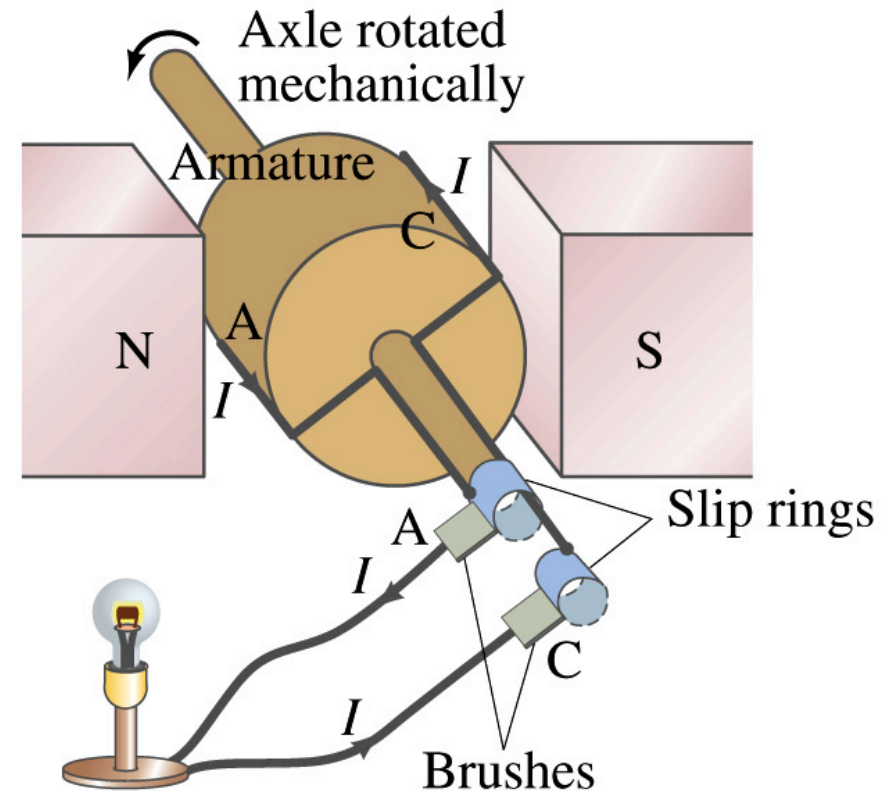
$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

- This equation is valid as long as  $B$ ,  $\ell$  and  $v$  are perpendicular to each other. What do we do if not?
  - Use the scalar product of vector quantities
- An emf induced on a conductor moving in a magnetic field is called a **motional emf**



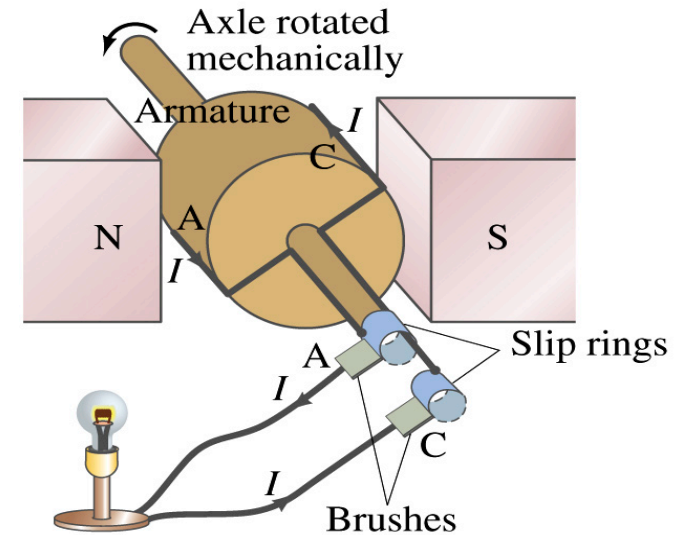
# Electric Generators

- What does a generator do?
  - Transforms mechanical energy into the electrical energy
  - What does this look like?
    - An inverse of an electric motor which transforms electrical energy to mechanical energy
  - An electric generator is also called a dynamo
- Whose law does the generator based on?
  - Faraday's law of induction



# How does an Electric Generator work?

- An electric generator consists of
  - Many coils of wires wound on an armature that can rotate by mechanical means in a magnetic field
- An emf is induced in the rotating coil
- Electric current is the output of a generator
- Which direction does the output current flow when the armature rotates counterclockwise?
  - The conventional current flows outward on wire A toward the brush
  - After half the revolution the wire A will be where the wire C is and the current flow on A is reversed
- Thus the current produced is alternating its direction



# How does an Electric Generator work?

- Let's assume the loop is rotating in a uniform B field w/ a constant angular velocity  $\omega$ . The induced emf is

- $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} [BA \cos \theta]$$

- What is the variable that changes above?

- The angle  $\theta$ . What is  $d\theta/dt$ ?

- The angular speed  $\omega$ .

- So  $\theta = \theta_0 + \omega t$

- If we choose  $\theta_0 = 0$ , we obtain

- $$\mathcal{E} = -BA \frac{d}{dt} [\cos \omega t] = BA\omega \sin \omega t$$

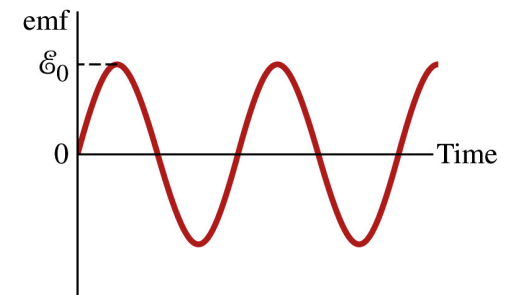
- If the coil contains N loops: 
$$\mathcal{E} = -N \frac{d\Phi_B}{dt} = NBA\omega \sin \omega t = \mathcal{E}_0 \sin \omega t$$

- What is the shape of the output?

- Sinusoidal w/ amplitude  $\mathcal{E}_0 = NBA\omega$

- USA frequency is 60Hz. Europe is at 50Hz

- Most the U.S. power is generated at steam plants





## Example 29 – 9

**An AC generator.** The armature of a 60-Hz AC generator rotates in a 0.15-T magnetic field. If the area of the coil is  $2.0 \times 10^{-2} \text{m}^2$ , how many loops must the coil contain if the peak output is to be  $\mathcal{E}_0 = 170 \text{V}$ ?

The maximum emf of a generator is  $\mathcal{E}_0 = NBA\omega$

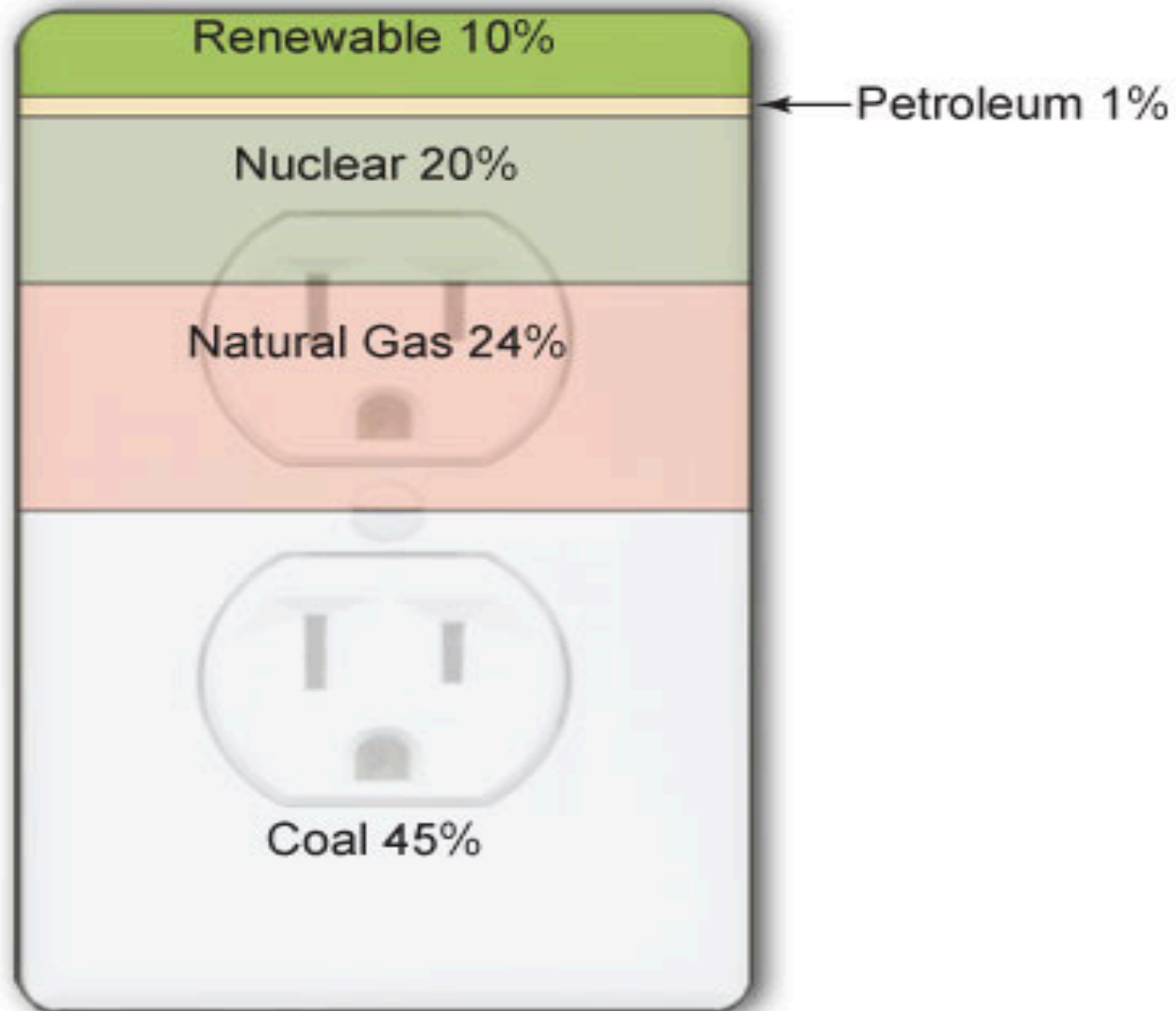
**Solving for N**  $N = \frac{\mathcal{E}_0}{BA\omega}$

Since  $\omega = 2\pi f$  We obtain

$$N = \frac{\mathcal{E}_0}{2\pi BAf} = \frac{170 \text{V}}{2\pi \cdot (0.15 \text{T}) \cdot (2.0 \times 10^{-2} \text{m}^2) \cdot (60 \text{s}^{-1})} = 150 \text{turns}$$



# Sources of U.S. Electricity Generation, 2010

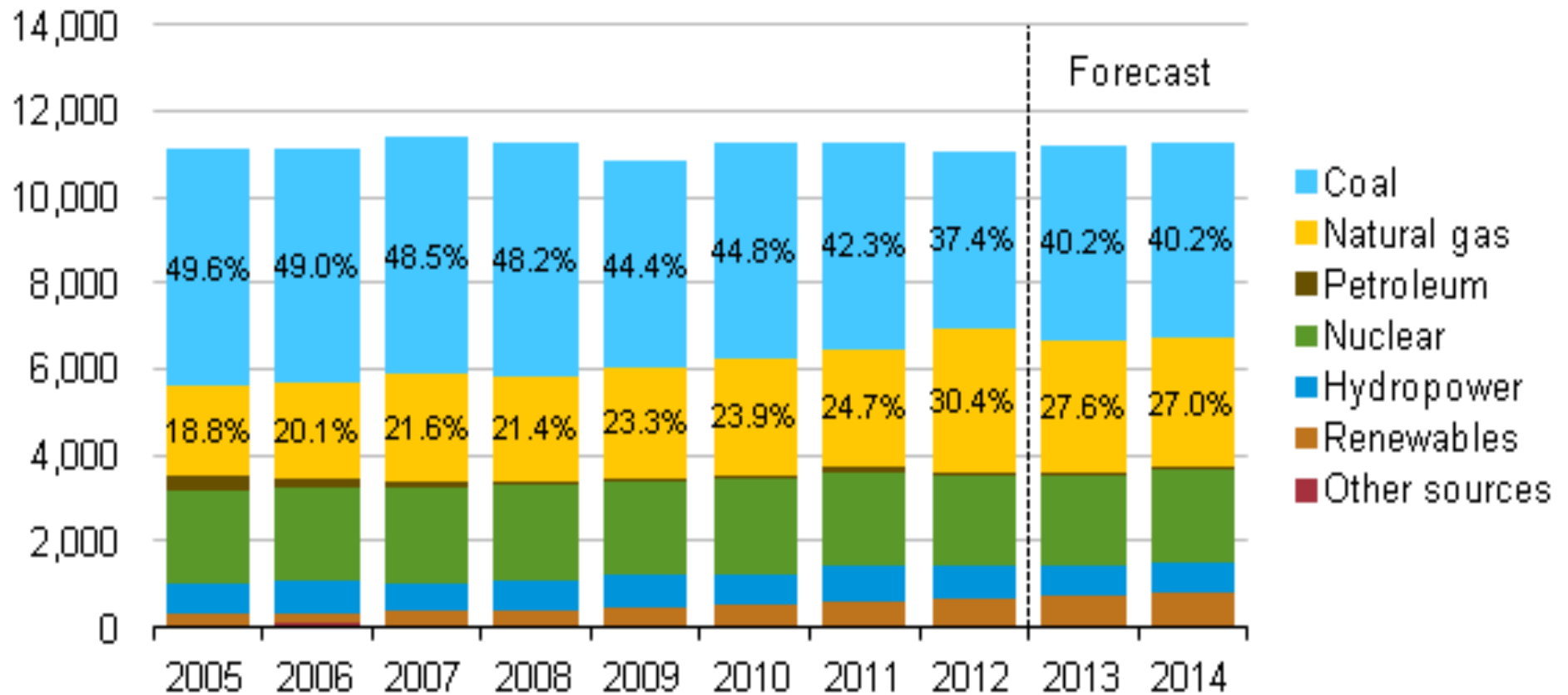


Source: U.S. Energy Information Administration, *Monthly Energy Review* (June 2011). Percentages based on Table 7.2a, preliminary 2010 data.

# US Electricity Consumption by Source

## U.S. Electricity Generation by Fuel, All Sectors

thousand megawatt hours per day



Note: Labels show percentage share of total generation provided by coal and natural gas.

US Energy Information Administration <http://www.eia.gov/electricity/>

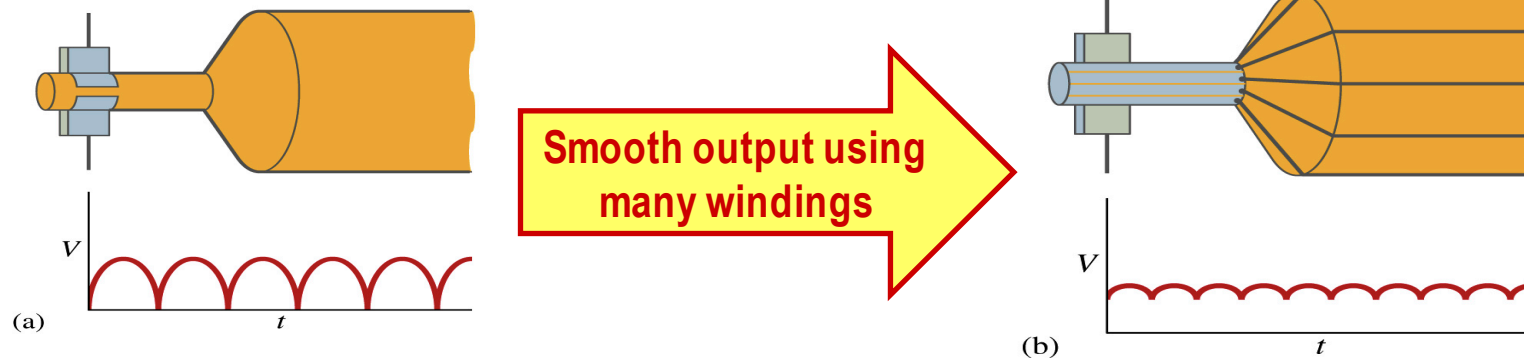


# The World Energy Consumption

- In 2008, total worldwide energy consumption was 474 EJ ( $474 \times 10^{18} \text{ J} = 132 \text{ PWh}$ ).
  - Equivalent to an average energy consumption rate of 15 terawatts ( $1.504 \times 10^{13} \text{ W}$ )
  - US uses 26.6 PWh (as of 2008)
- The potential for renewable energy
  - solar energy 1600 EJ (444,000 TWh)
  - wind power 600 EJ (167,000 TWh)
  - geothermal energy 500 EJ (139,000 TWh),
  - biomass 250 EJ (70,000 TWh)
  - hydropower 50 EJ (14,000 TWh) an
  - ocean energy 1 EJ (280 TWh)

# A DC Generator

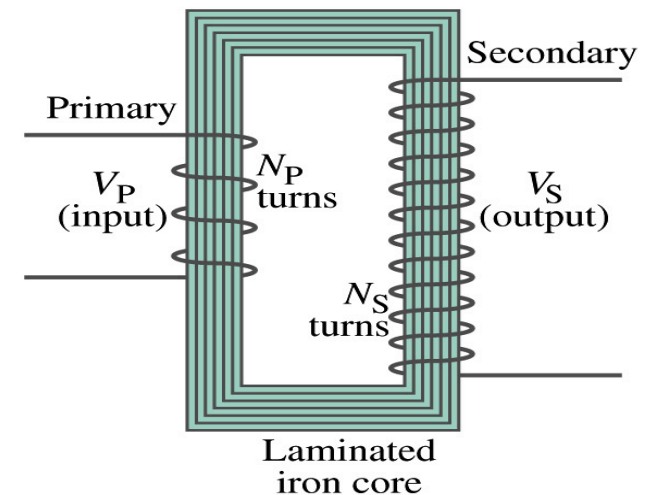
- A DC generator is almost the same as an AC generator except the slip rings are replaced by split-ring commutators



- Output can be smoothed out by placing a capacitor on the output
  - More commonly done using many armature windings

# Transformer

- What is a transformer?
  - A device for increasing or decreasing an AC voltage
  - A few examples?
    - TV sets to provide High Voltage to picture tubes, portable electronic device converters, transformers on the pole, etc
- A transformer consists of two coils of wires known as the primary and the secondary
  - The two coils can be interwoven or linked by a laminated soft iron core to reduce losses due to Eddy current
- Transformers are designed so that all magnetic flux produced by the primary coil pass through the secondary



Tuesday, July 5, 2016



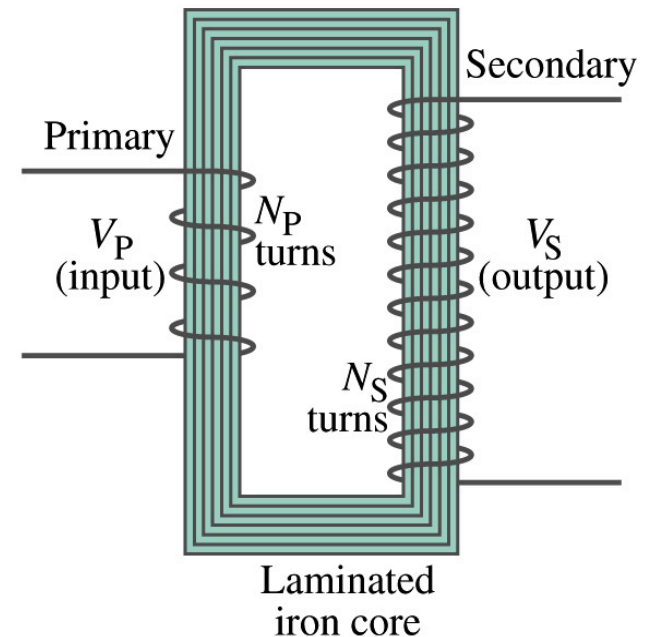
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# How does a transformer work?

- When an AC voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary wire
- So how would we make the voltage different?
  - By varying the number of loops in each coil
  - From Faraday's law, the induced emf in the secondary is
  - $V_S = N_S \frac{d\Phi_B}{dt}$
  - The input primary voltage is
  - $V_P = N_P \frac{d\Phi_B}{dt}$
  - Since  $d\Phi_B/dt$  is the same, we obtain

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Transformer  
Equation



# Transformer Equation

- The transformer equation does not work for DC current
  - Since there is no change of magnetic flux!!
- If  $N_S > N_P$ , the output voltage is greater than the input so it is called a step-up transformer while  $N_S < N_P$  is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?
  - Wrong! Wrong! Wrong! Energy is always conserved!
  - A well designed transformer can be more than 99% efficient
  - The power output is the same as the input:
  - $V_P I_P = V_S I_S$

$$\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

The output current for step-up transformer will be lower than the input, while it is larger for step-down x-former than the input.

# Example for A Transformer

**Portable radio transformer.** A transformer for home use of a portable radio reduces 120-V AC to 9.0V AC. The secondary contains 30 turns, and the radio draws 400mA. Calculate (a) the number of turns in the primary (b) the current in the primary and (c) the power transformed.

(a) What kind of a transformer is this? A step-down x-former

Since  $\frac{V_P}{V_S} = \frac{N_P}{N_S}$  We obtain  $N_P = N_S \frac{V_P}{V_S} = 30 \frac{120V}{9V} = 400 \text{ turns}$

(b) Also from the transformer equation  $\frac{I_S}{I_P} = \frac{V_P}{V_S}$  We obtain  $I_P = I_S \frac{V_S}{V_P} = 0.4A \frac{9V}{120V} = 0.03A$

(c) Thus the power transformed is

$$P = I_S V_S = (0.4A) \cdot (9V) = 3.6W$$

How about the input power? The same assuming 100% efficiency.



# Example 29 – 13: Power Transmission

**Transmission lines.** An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of  $0.4\Omega$ . Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use  $P=V^2/R$  since we do not know the voltage along the transmission line. We, however, can use  $P=I^2R$ .

(a) If 120kW is sent at 240V, the total current is  $I = \frac{P}{V} = \frac{120 \times 10^3}{240} = 500A$ .

Thus the power loss due to transmission line is

$$P = I^2 R = (500A)^2 \cdot (0.4\Omega) = 100kW$$

(b) If 120kW is sent at 24,000V, the total current is  $I = \frac{P}{V} = \frac{120 \times 10^3}{24 \times 10^3} = 5.0A$ .

Thus the power loss due to transmission line is

$$P = I^2 R = (5A)^2 \cdot (0.4\Omega) = 10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

# Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

# Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference  $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field

- $\mathcal{E} = \oint \vec{E} \cdot d\vec{l}$

- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux  $\Phi_B$  is changing.

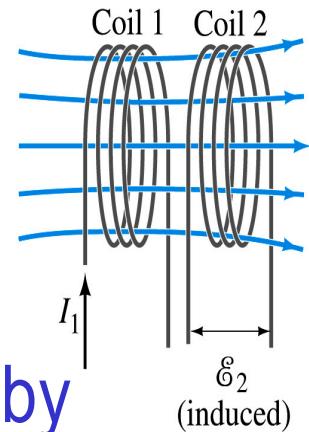
# Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit → Mutual inductance
  - Or induce an emf in itself → Self inductance



# Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf,  $\varepsilon_2$ , in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current  $I_1$  in coil 1
- If  $\Phi_{21}$  is the magnetic flux in each loop of coil2 created by coil1 and  $N_2$  is the number of closely packed loops in coil2, then  $N_2\Phi_{21}$  is the total flux passing through coil2.
- If the two coils are fixed in space,  $N_2\Phi_{21}$  is proportional to the current  $I_1$  in coil 1,  $N_2\Phi_{21} = M_{21} I_1$ .
- The proportionality constant for this is called the Mutual Inductance and defined as  $M_{21} = N_2\Phi_{21}/I_1$ .
- The emf induced in coil2 due to the changing current in coil1 is



$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d(N_2\Phi_{21})}{dt} = -M_{21} \frac{dI_1}{dt}$$

# Mutual Inductance

- The mutual induction of coil2 with respect to coil1,  $M_{21}$ ,
  - is a constant and does not depend on  $I_1$ .
  - depends only on “geometric” factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present
 

What? Does this make sense?

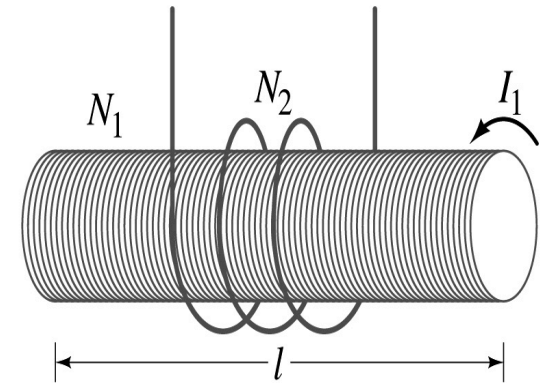
    - The farther apart the two coils are the less flux can pass through coil, 2, so  $M_{21}$  will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $\epsilon_1 = -M_{12} \frac{dI_2}{dt}$ 
  - $M_{12}$  is the mutual inductance of coil1 with respect to coil2 and  $M_{12} = M_{21}$
  - We can put  $M = M_{12} = M_{21}$  and obtain
  - SI unit for mutual inductance is henry (H)

$$\epsilon_1 = -M \frac{dI_2}{dt} \text{ and } \epsilon_2 = -M \frac{dI_1}{dt}$$

$$1H = 1V \cdot s / A = 1\Omega \cdot s$$

# Example 30 – 1

**Solenoid and coil.** A long thin solenoid of length  $l$  and cross-sectional area  $A$  contains  $N_1$  closely packed turns of wire. Wrapped around it is an insulated coil of  $N_2$  turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid?  $B = \frac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual

inductance of coil 2 is  $M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$

# Self Inductance

- The concept of inductance applies to a single isolated coil of  $N$  turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impede its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current



# Self Inductance

- Since the magnetic flux  $\Phi_B$  passing through  $N$  turn coil is proportional to current  $I$  in the coil,  $N\Phi_B = LI$
- We define self-inductance,  $\mathcal{L}$ :

$$L = \frac{N\Phi_B}{I}$$


Self Inductance
- The induced emf in a coil of self-inductance  $\mathcal{L}$  is
  - $\mathcal{E} = -N \frac{d\Phi_B}{dt} = -L \frac{dI}{dt}$
  - What is the unit for self-inductance?  $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of  $\mathcal{L}$  depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

# So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance



# Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance,  $\mathcal{L}$ , is called an inductor and is express with the symbol 
  - Precision resistors are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a “non-inductive winding”
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term reactance or impedance

# Example 30 – 3

**Solenoid inductance.** (a) Determine the formula for the self inductance  $\mathcal{L}$  of a tightly wrapped solenoid ( a long coil) containing  $N$  turns of wire in its length  $l$  and whose cross-sectional area is  $A$ . (b) Calculate the value of  $\mathcal{L}$  if  $N=100$ ,  $l=5.0\text{cm}$ ,  $A=0.30\text{cm}^2$  and the solenoid is air filled. (c) calculate  $\mathcal{L}$  if the solenoid has an iron core with  $\mu=4000\mu_0$ .

What is the magnetic field inside a solenoid?  $B = \mu_0 nI = \mu_0 NI / l$

The flux is, therefore,  $\Phi_B = BA = \mu_0 NIA / l$

Using the formula for self inductance:  $L = \frac{N\Phi_B}{I} = \frac{N \cdot \mu_0 NIA / l}{I} = \frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) 100^2 (0.30 \times 10^{-4} \text{ m}^2)}{5.0 \times 10^{-2} \text{ m}} = 7.5 \mu\text{H}$$

(c) The magnetic field with an iron core solenoid is  $B = \mu NI / l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000 (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) 100^2 (0.30 \times 10^{-4} \text{ m}^2)}{5.0 \times 10^{-2} \text{ m}} = 0.030 \text{ H} = 30 \text{ mH}$$

# Energy Stored in the Magnetic Field

- When an inductor of inductance  $\mathcal{L}$  is carrying current  $I$  which is changing at a rate  $dI/dt$ , energy is supplied to the inductor at a rate
  - $P = I\mathcal{E} = I\mathcal{L} \frac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to  $I$ ?
  - The work,  $dW$ , done in time  $dt$  is  $dW = Pdt = \mathcal{L}IdI$
  - Thus the total work needed to bring the current from 0 to  $I$  in an inductor is
$$W = \int dW = \int_0^I \mathcal{L}IdI = \mathcal{L} \left[ \frac{1}{2} I^2 \right]_0^I = \frac{1}{2} \mathcal{L}I^2$$

# Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current  $I$

- $$U = \frac{1}{2} LI^2$$

Energy Stored in a magnetic field inside an inductor

- This is compared to the energy stored in a capacitor,  $C$ , when the potential difference across it is  $V$ :  $U = \frac{1}{2} CV^2$
- Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
- The energy in an inductor can be considered to be stored in its magnetic field

# Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?

- Inductance of an ideal solenoid without a fringe effect

$$L = \mu_0 N^2 A / l$$

- The magnetic field in a solenoid is  $B = \mu_0 NI / l$

- Thus the energy stored in an inductor is

$$U = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left( \frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} \underbrace{Al}$$

$$U = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

E

- Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

What is this?

Volume V

$$u = \frac{1}{2} \frac{B^2}{\mu_0}$$

E density

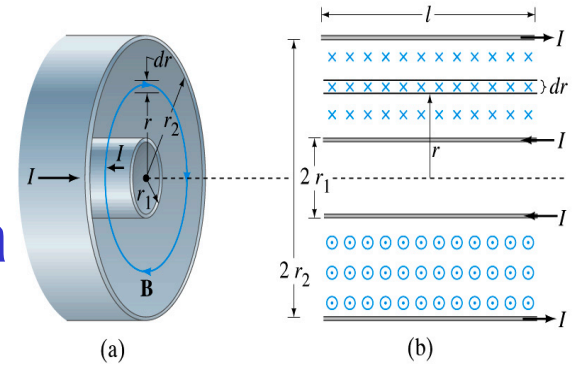
- This formula is valid in any region of space
- If a ferromagnetic material is present,  $\mu_0$  becomes  $\mu$ .

What volume does  $Al$  represent?

The volume inside a solenoid!!

# Example 30 – 5

**Energy stored in a coaxial cable.** (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii  $r_1$  and  $r_2$  and which carry a current  $I$ ? (b) Where is the energy density highest?



(a) The total flux through  $l$  of the cable is  $\Phi_B = \int B l dr = \frac{\mu_0 I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is  $\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}$

Thus the energy stored per unit length is

$$\frac{U}{l} = \frac{1}{2} \frac{L I^2}{l} = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}$$

(b) Since the magnetic field is  $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is  $u = \frac{1}{2} \frac{B^2}{\mu_0}$

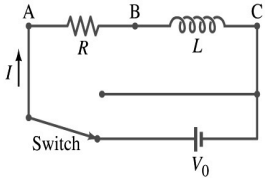
The energy density is highest where  $B$  is highest. Since  $B$  is highest close to  $r=r_1$ , near the surface of the inner conductor.



# LR Circuits

- What happens when an emf is applied to an inductor?

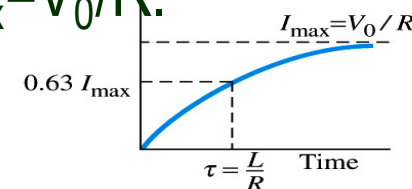
- An inductor has some resistance, however negligible



- So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**

- What happens at the instance the switch is thrown to apply emf to the circuit?

- The current starts to flow, gradually increasing from 0
    - This change is opposed by the induced emf in the inductor → the emf at point B is higher than point C
    - However there is a voltage drop at the resistance which reduces the voltage across inductance
    - Thus the current increases less rapidly
    - The overall behavior of the current is a gradual increase, reaching to the maximum current  $I_{\max} = V_0 / R$ .

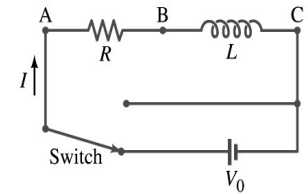


Tuesday, July 5, 2016



PHYS 1444-001, Surr  
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# LR Circuits



- This can be shown w/ Kirchhoff loop rules

- The emfs in the circuit are the battery voltage  $V_0$  and the emf  $\mathcal{E} = -L(dI/dt)$  in the inductor opposing the current increase

- The sum of the potential changes through the circuit is

$$V_0 + \mathcal{E} - IR = V_0 - L dI/dt - IR = 0$$

- Where  $I$  is the current at any instance

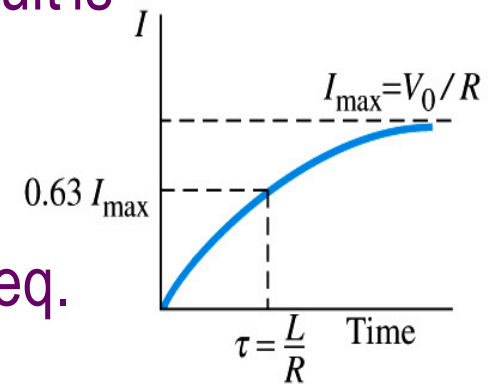
- By rearranging the terms, we obtain a differential eq.

- $L dI/dt + IR = V_0$

- We can integrate just as in RC circuit

- So the solution is  $-\frac{1}{R} \ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$

- Where  $\tau = L/R$



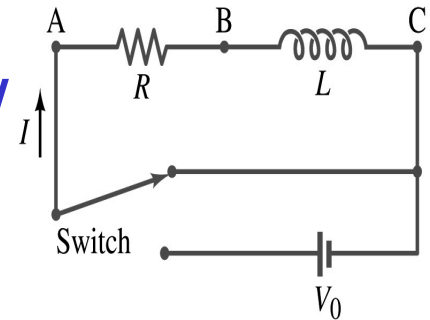
$$\int_{I=0}^I \frac{dI}{V_0 - IR} = \int_{t=0}^t \frac{dt}{L}$$

$$I = V_0 (1 - e^{-t/\tau}) / R = I_{\max} (1 - e^{-t/\tau})$$

- This is the time constant  $\tau$  of the LR circuit and is the time required for the current  $I$  to reach 0.63 of the maximum

# Discharge of LR Circuits

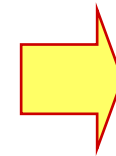
- If the switch is flipped away from the battery



- The differential equation becomes

- $L \frac{dI}{dt} + IR = 0$

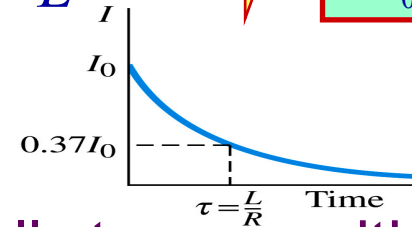
- So the integration is  $\int_{I_0}^I \frac{dI}{IR} = \int_{t=0}^t \frac{dt}{L}$



$$\ln \frac{I}{I_0} = -\frac{R}{L} t$$

- Which results in the solution

- $I = I_0 e^{-\frac{R}{L} t} = I_0 e^{-t/\tau}$



- The current decays exponentially to zero with the time constant  $\tau = L/R$

- So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.

- The current in LR circuit behaves almost the same as that in RC circuit but the time constant is inversely proportional to R in LR circuit unlike the RC circuit

# AC Circuit w/ Resistance only

- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \omega t = V_0 \sin \omega t$$

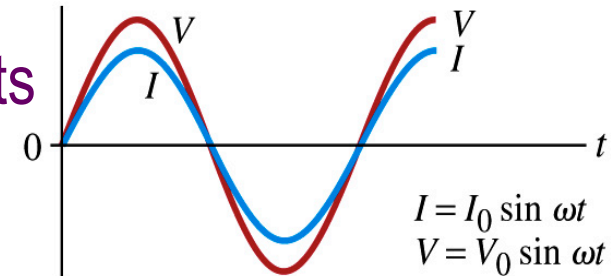
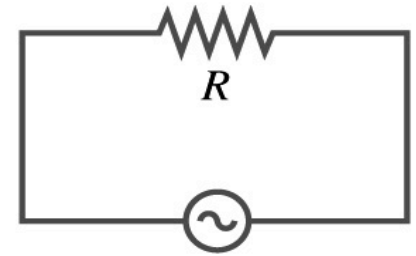
– where  $V_0 = I_0 R$

- What does this mean?

- Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
- Current and voltage are “in phase”

- Energy is lost via the transformation into heat at an average rate

$$\bar{P} = \bar{I} \bar{V} = I_{rms}^2 R = V_{rms}^2 / R$$



# AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

$$V - L \frac{dI}{dt} = 0$$

- Thus

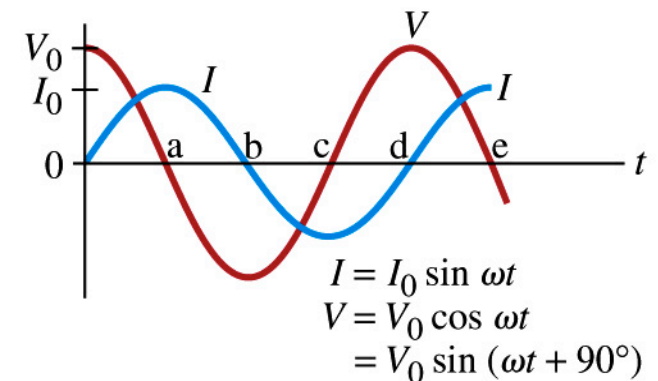
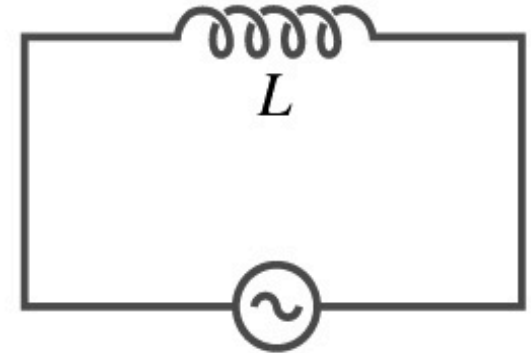
$$V = L \frac{dI}{dt} = L \frac{d(I_0 \sin \omega t)}{dt} = \omega L I_0 \cos \omega t$$

- Using the identity  $\cos \theta = \sin(\theta + 90^\circ)$
- $V = \omega L I_0 \sin(\omega t + 90^\circ) = V_0 \sin(\omega t + 90^\circ)$ 
  - where  $V_0 = \omega L I_0$
- What does this mean?

- Current and voltage are “out of phase by  $\pi/2$  or  $90^\circ$ ”. In other words the current reaches its peak  $\frac{1}{4}$  cycle after the voltage

- What happens to the energy?

- No energy is dissipated
- The average power is 0 at all times
- The energy is stored temporarily in the magnetic field
- Then released back to the source



# AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
  - Inductor Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they the same?
  - They both impede the flow of charge
  - For a resistance  $R$ , the peak voltage and current are related to  $V_0 = I_0 R$
  - Similarly, for an inductor we may write  $V_0 = I_0 X_L$ 
    - Where  $X_L$  is the inductive reactance of the inductor  $X_L = \omega L$  0 when  $\omega=0$ .
    - What do you think is the unit of the reactance?  $\Omega$
    - The relationship  $V_0 = I_0 X_L$  is not valid at a particular instance. Why not?
      - Since  $V_0$  and  $I_0$  do not occur at the same time

# Example 30 – 9

**Reactance of a coil.** A coil has a resistance  $R=1.00\Omega$  and an inductance of  $0.300\text{H}$ . Determine the current in the coil if (a)  $120\text{ V DC}$  is applied to it; (b)  $120\text{ V AC (rms)}$  at  $60.0\text{Hz}$  is applied.

Is there a reactance for DC? Nope. Why not? Since  $\omega=0$ ,  $X_L = \omega L = 0$

So for DC power, the current is from Kirchhoff's rule  $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120\text{V}}{1.00\Omega} = 120\text{A}$$

For an AC power with  $f=60\text{Hz}$ , the reactance is

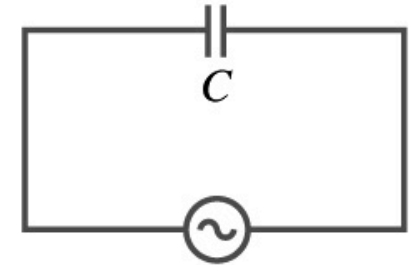
$$X_L = \omega L = 2\pi fL = 2\pi \cdot (60.0\text{s}^{-1}) \cdot 0.300\text{H} = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120\text{V}}{113\Omega} = 1.06\text{A}$$

# AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a DC power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since the capacitor prevents the flow of the DC current
- What do you think will happen if it is connected to an AC power source?
  - The current flows continuously. Why?
  - When the AC power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction





# AC Circuit w/ Capacitance only

- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- The current at any instance is  $I = \frac{dQ}{dt} = I_0 \sin \omega t$

- The charge Q on the plate at any instance is

$$Q = \int_{Q=0}^Q dQ = \int_{t=0}^t I_0 \sin \omega t dt = -\frac{I_0}{\omega} \cos \omega t$$

- Thus the voltage across the capacitor is

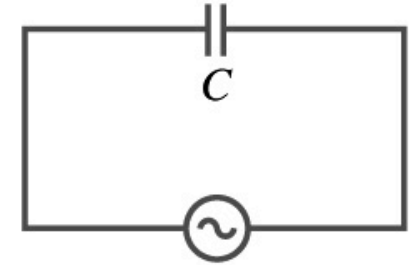
$$V = \frac{Q}{C} = -I_0 \frac{1}{\omega C} \cos \omega t$$

- Using the identity  $\cos \theta = -\sin(\theta - 90^\circ)$

$$V = I_0 \frac{1}{\omega C} \sin(\omega t - 90^\circ) = V_0 \sin(\omega t - 90^\circ)$$

- Where

$$V_0 = \frac{I_0}{\omega C}$$

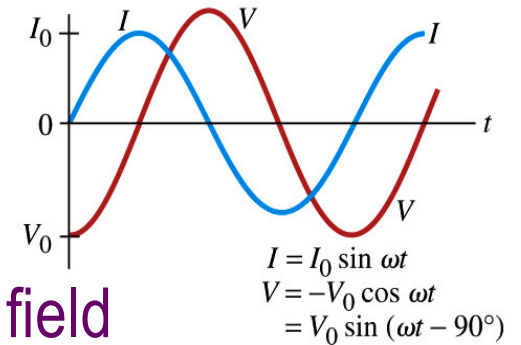


# AC Circuit w/ Capacitance only

- So the voltage is  $V = V_0 \sin(\omega t - 90^\circ)$
- What does this mean?
  - Current and voltage are “out of phase by  $\pi/2$  or  $90^\circ$ ” but in this case, the voltage reaches its peak  $\frac{1}{4}$  cycle after the current

- What happens to the energy?

- No energy is dissipated
- The average power is 0 at all times
- The energy is stored temporarily in the electric field
- Then released back to the source



- Applied voltage and the current in the capacitor can be written as  $V_0 = I_0 X_C$

- Where the capacitive reactance  $X_C$  is defined as
- Again, this relationship is only valid for rms quantities

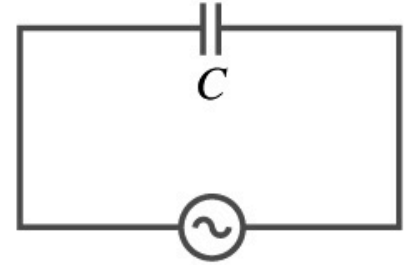
$$X_C = \frac{1}{\omega C}$$

Infinite  
when  
 $\omega=0$ .

$$V_{rms} = I_{rms} X_C$$

# Example 30 – 10

**Capacitor reactance.** What are the peak and rms current in the circuit in the figure if  $C=1.0\mu\text{F}$  and  $V_{\text{rms}}=120\text{V}$ ? Calculate for (a)  $f=60\text{Hz}$ , and then for (b)  $f=6.0\times 10^5\text{Hz}$ .



The peak voltage is  $V_0 = \sqrt{2}V_{\text{rms}} = 120\text{V} \cdot \sqrt{2} = 170\text{V}$

The capacitance reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot (60\text{s}^{-1}) \cdot 1.0 \times 10^{-6}\text{F}} = 2.7\text{k}\Omega$$

Thus the peak current is

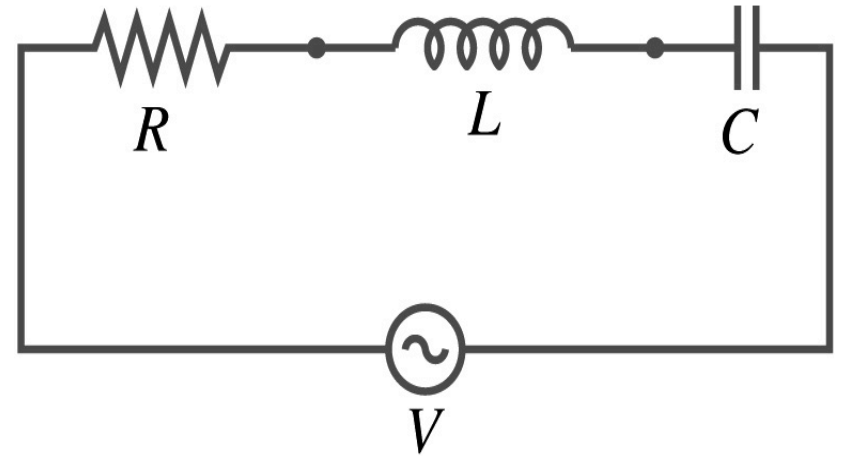
$$I_0 = \frac{V_0}{X_C} = \frac{170\text{V}}{2.7\text{k}\Omega} = 63\text{mA}$$

The rms current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{120\text{V}}{2.7\text{k}\Omega} = 44\text{mA}$$

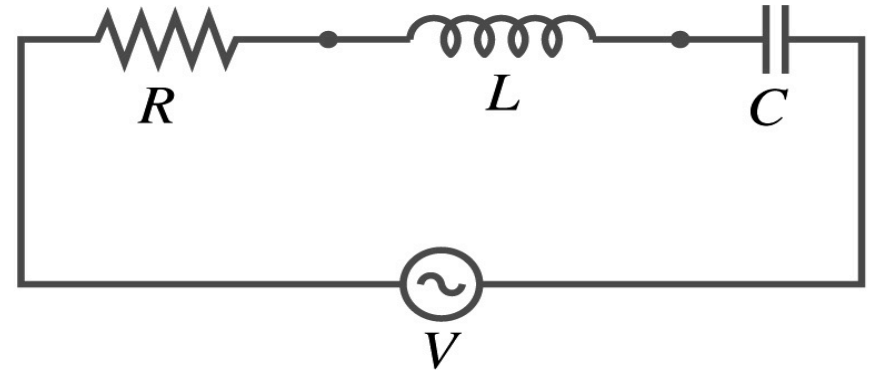
# AC Circuit w/ LRC

- The voltage across each element is
  - $V_R$  is in phase with the current
  - $V_L$  leads the current by  $90^\circ$
  - $V_C$  lags the current by  $90^\circ$
- From Kirchhoff's loop rule
- $V = V_R + V_L + V_C$ 
  - However since they do not reach the peak voltage at the same time, the peak voltage of the source  $V_0$  will not equal  $V_{R0} + V_{L0} + V_{C0}$
  - The rms voltage also will not be the simple sum of the three
- Let's try to find the total impedance, peak current  $I_0$  and the phase difference between  $I_0$  and  $V_0$ .



# AC Circuit w/ LRC

- The current at any instance is the same at all point in the circuit
  - The currents in each elements are in phase
  - Why?
    - Since the elements are in series
  - How about the voltage?
    - They are not in phase.



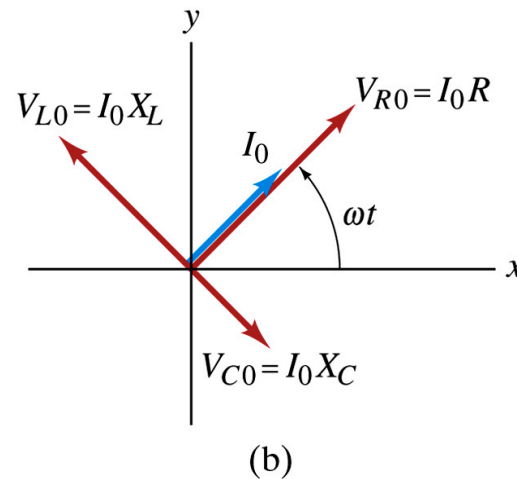
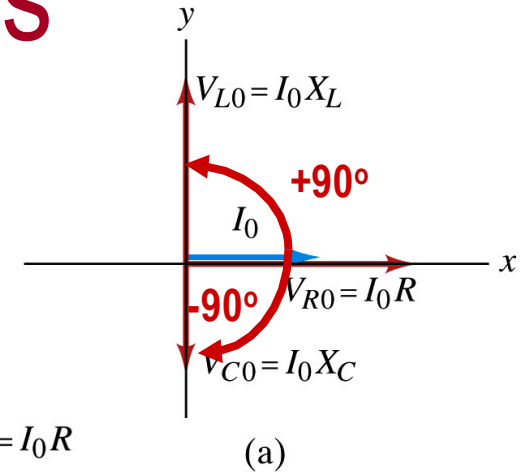
- The current at any given time is

$$I = I_0 \sin \omega t$$

- The analysis of LRC circuit is done using the “phasor” diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
  - The lengths of the arrows represent the magnitudes of the peak voltages across each element;  $V_{R0} = I_0 R$ ,  $V_{L0} = I_0 X_L$  and  $V_{C0} = I_0 X_C$
  - The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency  $\omega$  to take into account the time dependence.
    - The projection of each arrow on y axis represents voltage across each element at any given time

# Phasor Diagrams

- At  $t=0$ ,  $I=0$ .
  - Thus  $V_{R0}=0$ ,  $V_{L0}=I_0X_L$ ,  $V_{C0}=I_0X_C$
- At  $t=t$ ,  $I = I_0 \sin \omega t$

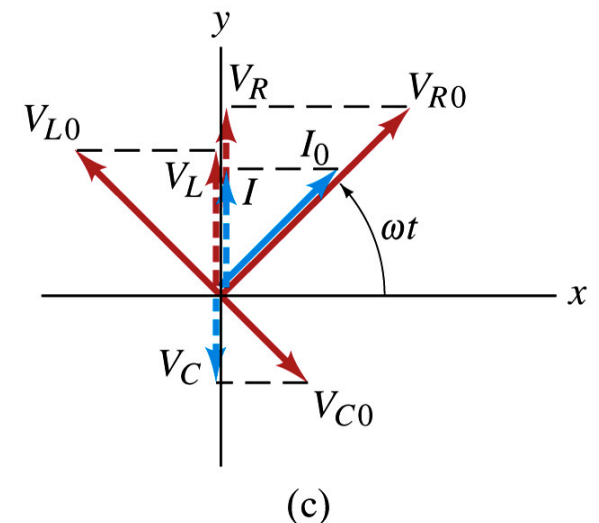


- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \omega t$$

$$V_L = V_{L0} \sin(\omega t + 90^\circ)$$

$$V_C = V_{C0} \sin(\omega t - 90^\circ)$$



Tuesday, July 5, 2016

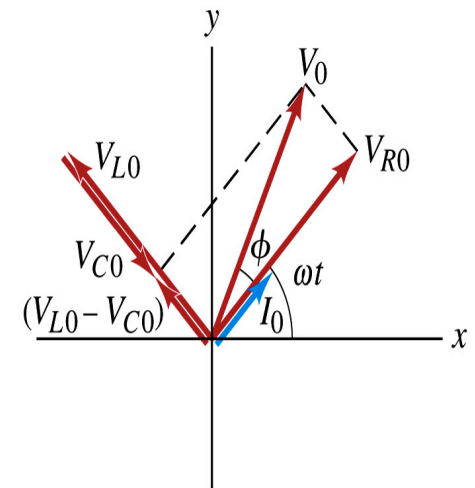


PHYS 1444-001, Summer 2016  
Dr. Jaehoon Yu

# AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.

- The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
- So we can use the sum of all vectors as the representation of the peak source voltage  $V_0$ .



- $V_0$  forms an angle  $\phi$  to  $V_{R0}$  and rotates together with the other vectors as a function of time,  $V = V_0 \sin(\omega t + \phi)$
- We determine the total impedance  $Z$  of the circuit defined by the relationship  $V_{rms} = I_{rms} Z$  or  $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is  $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

# AC Circuit w/ LRC

- The phase angle  $\phi$  is

$$\tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0 (X_L - X_C)}{I_0 R} = \frac{(X_L - X_C)}{R}$$

- or

$$\cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

- What is the power dissipated in the circuit?

- Which element dissipates the power?
- Only the resistor

- The average power is  $\bar{P} = I_{rms}^2 R$

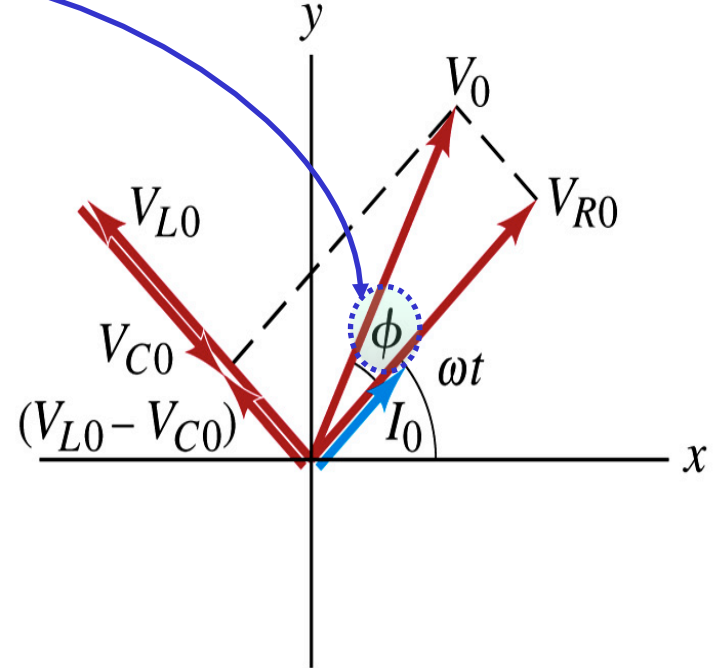
- Since  $R = Z \cos \phi$

- We obtain  $\bar{P} = I_{rms}^2 Z \cos \phi = I_{rms} V_{rms} \cos \phi$

- The factor  $\cos \phi$  is referred as the power factor of the circuit

- For a pure resistor,  $\cos \phi = 1$  and  $\bar{P} = I_{rms} V_{rms}$

- For a capacitor or inductor alone  $\phi = -90^\circ$  or  $+90^\circ$ , so  $\cos \phi = 0$  and  $\bar{P} = 0$ .





# Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19<sup>th</sup> century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

# Ampere's Law

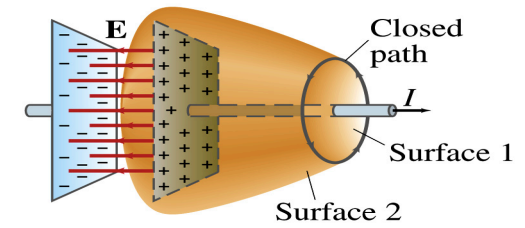
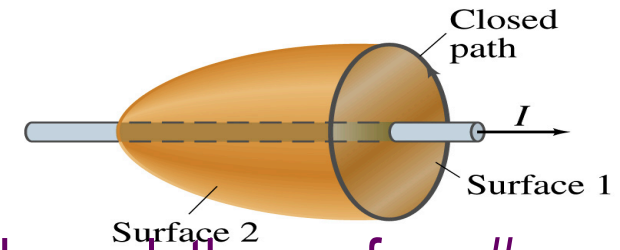
- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field
- Then can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
  - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism

# Expanding Ampere's Law

- Let's consider a wire carrying current  $I$ 
  - The current that is enclosed in the loop passes through the surface # 1 in the figure
  - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
    - Still  $I$ .
  - So the Ampere's law still works
- We could then consider a capacitor being charged up or being discharged.
  - The current  $I$  enclosed in the loop passes through the surface #1
  - However the surface #2 that shares the same closed loop do not have any current passing through it.
    - There is magnetic field present since there is current  $\rightarrow$  In other words there is a changing electric field in between the plates
    - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field



$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{encl}$$

# Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
  - The charge  $Q$  on the capacitor with capacitance  $C$  is  $Q=CV$ 
    - Where  $V$  is the potential difference between the plates
  - Since  $V=Ed$ 
    - Where  $E$  is the uniform field between the plates, and  $d$  is the separation of the plates
  - And for parallel plate capacitor  $C=\epsilon_0 A/d$
  - We obtain

$$Q = CV = \left( \epsilon_0 \frac{A}{d} \right) Ed = \epsilon_0 AE$$

# Modifying Ampere's Law

- If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt}$$

- Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d(AE)}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

- Where  $\Phi_E = EA$  is the electric flux through the surface between the plates

- So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Extra term  
by Maxwell

- This equation represents the general form of Ampere's law
  - This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

# Example 31 – 1

**Charging capacitor.** A 30-pF air-gap capacitor has circular plates of area  $A=100\text{cm}^2$ . It is charged by a 70-V battery through a  $2.0\text{-}\Omega$  resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume  $\mathbf{E}$  is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is:  $Q = CV_0(1 - e^{-t/RC})$

For the initial current ( $t=0$ ), we differentiate the charge with respect to time.

$$I_0 = \left. \frac{dQ}{dt} \right|_{t=0} = \left. \frac{CV_0}{RC} e^{-t/RC} \right|_{t=0} = \frac{V_0}{R} = \frac{70\text{V}}{2.0\Omega} = 35\text{A}$$

The electric field is  $E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$

Change of the electric field is  $\frac{dE}{dt} = \frac{dQ/dt}{A\epsilon_0} = \frac{35\text{A}}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (1.0 \times 10^{-2} \text{ m}^2)} = 4.0 \times 10^{14} \text{ V/m} \cdot \text{s}$

# Example 31 – 1

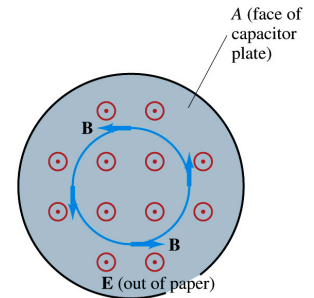
(c) Determine the magnetic field induced between the plates. Assume  $\mathbf{E}$  is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to  $\mathbf{E}$  and is circular due to symmetry

Whose law can we use to determine  $B$ ?

Extended Ampere's Law w/  $I_{\text{encl}}=0$ !

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



We choose a circular path of radius  $r$ , centered at the center of the plane, following the  $B$ .

For  $r < r_{\text{plate}}$ , the electric flux is  $\Phi_E = EA = E\pi r^2$  since  $\mathbf{E}$  is uniform throughout the plate

So from Ampere's law, we obtain  $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r^2)}{dt} = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}$

**Solving for B**

$$B = \mu_0 \epsilon_0 \frac{r}{2} \frac{dE}{dt}$$

For  $r < r_{\text{plate}}$

Since we assume  $E=0$  for  $r > r_{\text{plate}}$ , the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{\text{plate}}^2$$

So from Ampere's law, we obtain  $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r_{\text{plate}}^2)}{dt} = \mu_0 \epsilon_0 \pi r_{\text{plate}}^2 \frac{dE}{dt}$

**Solving for B**

$$B = \frac{\mu_0 \epsilon_0 r_{\text{plate}}^2}{2r} \frac{dE}{dt}$$

For  $r > r_{\text{plate}}$

Tuesday, July 5, 2016

01, Summer 2016

Dr. Jaehoon Yu

# Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as the displacement current,  $I_D$
  - While the other term is called as the conduction current,  $I$
- Ampere's law then can be written as

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I + I_D)$$

- Where

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

- While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself



# Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field  $B$ , the magnetic flux  $\Phi_B$  through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Where the integration is over the area of either an open or a closed surface

- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge  $Q$  enclosed by the surface divided by  $\epsilon_0$ .

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss' Law  
for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for  
magnetism

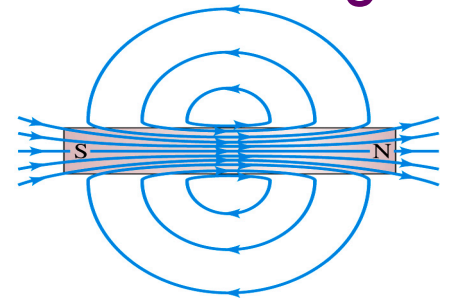
- Why is result of the integral zero?
  - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

# Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0$$

- There are as many magnetic flux lines that enter the enclosed volume as leave it
- If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
  - Electricity do have the source and the sink
- Magnetic field lines must be continuous
- Even for bar magnets, the field lines exist both inside and outside of the magnet



# Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

## Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

## Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

## Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

## Ampère's Law

A magnetic field is produced by an electric current or by a changing electric field

# Maxwell's Amazing Leap of Faith

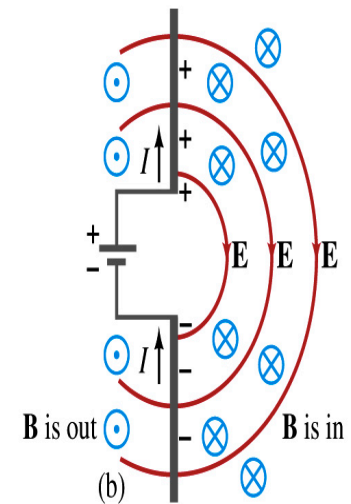
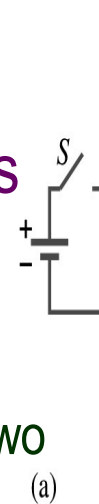
- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes.
    - This changing magnetic field then in turn produces the electric field that changes.
    - This process continues.
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

# Production of EM Waves

- Consider two conducting rods that will serve as an antenna are connected to a DC power source

- What do you think will happen when the switch is closed?

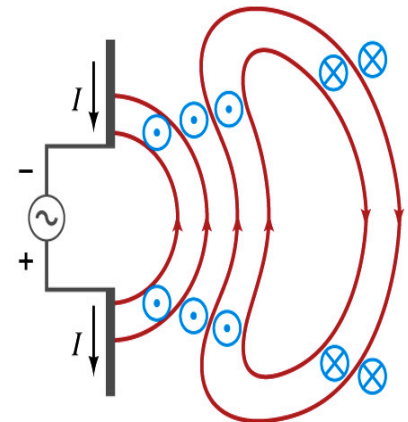
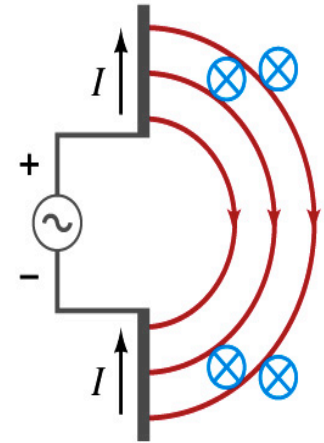
- The rod connected to the positive terminal is charged positive and the other negatively
- Then the electric field will be generated between the two rods
- Since there is current that flows through, the rods generates a magnetic field around them



- How far would the electric and magnetic fields extend?
  - In static case, the field extends indefinitely
  - When the switch is closed, the fields are formed nearby the rods quickly but
  - The stored energy in the fields won't propagate w/ infinite speed

# Production of EM Waves

- What happens if the antenna is connected to an ac power source?
  - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - The new field lines with the opposite direction forms
    - While the original field lines still propagates away from the rod reaching out far
      - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
  - The fields far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other



# Properties of Radiation Fields

- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the direction is 0
- The magnitude of  $E$  and  $B$  in the radiation field decrease with distance as  $1/r$
- The energy carried by the EM wave is proportional to the square of the amplitude,  $E^2$  or  $B^2$ 
  - So the intensity of wave decreases as  $1/r^2$

# Properties of Radiation Fields

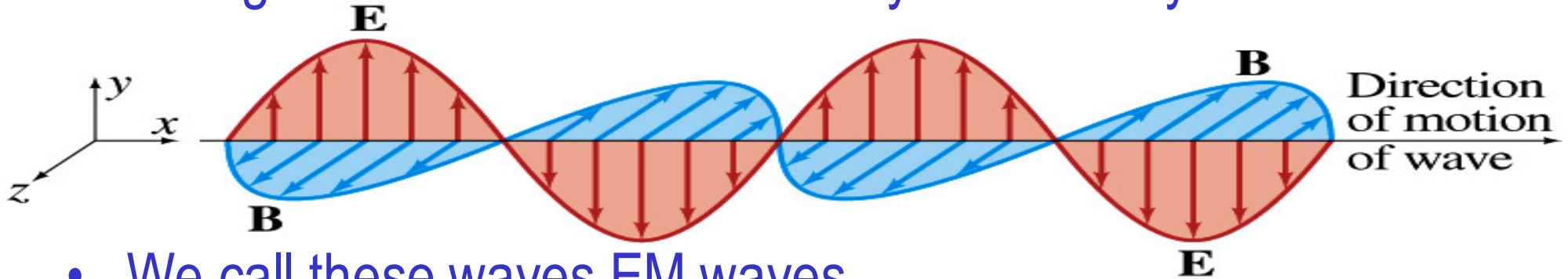
- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to max in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
  - Called plane waves





# EM Waves

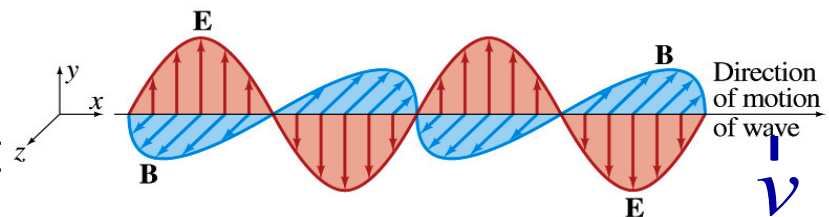
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
  - Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally by Heinrich Hertz through the discovery of radio waves

# EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there is no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - Plane waves
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity,  $\mathbf{v} = v\mathbf{i}$ , and that **E** is parallel to y axis and **B** is parallel to z axis



Tuesday, July 5, 2016



PHYS 1444-001, Summer 2016  
Dr. Jaehoon Yu

# Maxwell's Equations w/ $Q=I=0$

- In this region of free space,  $Q=0$  and  $I=0$ , thus the four Maxwell's equations become

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$	$Q_{encl}=0$	$\oint \vec{E} \cdot d\vec{A} = 0$
$\oint \vec{B} \cdot d\vec{A} = 0$	$\text{No Changes}$	$\oint \vec{B} \cdot d\vec{A} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\text{No Changes}$	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$I_{encl}=0$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

# EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength  $\lambda$  and frequency  $f$ , such traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

– Where

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad \text{Thus} \quad f\lambda = \frac{\omega}{k} = v$$

– What is  $v$ ?

- It is the speed of the traveling wave

– What are  $E_0$  and  $B_0$ ?

- The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

# From Faraday's Law

- Let's apply Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- to the rectangular loop of height  $\Delta y$  and width  $dx$
- $\vec{E} \cdot d\vec{l}$  along the top and bottom of the loop is 0. Why?
  - Since  $\vec{E}$  is perpendicular to  $d\vec{l}$
  - So the result of the integral through the loop counterclockwise becomes

$$\begin{aligned} \oint \vec{E} \cdot d\vec{l} &= \vec{E} \cdot d\vec{x} + (\vec{E} + d\vec{E}) \cdot \Delta\vec{y} + \vec{E} \cdot d\vec{x}' + \vec{E} \cdot \Delta\vec{y}' = \\ &= 0 + (E + dE)\Delta y - 0 - E\Delta y = dE\Delta y \end{aligned}$$

- For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt} dx \Delta y$$

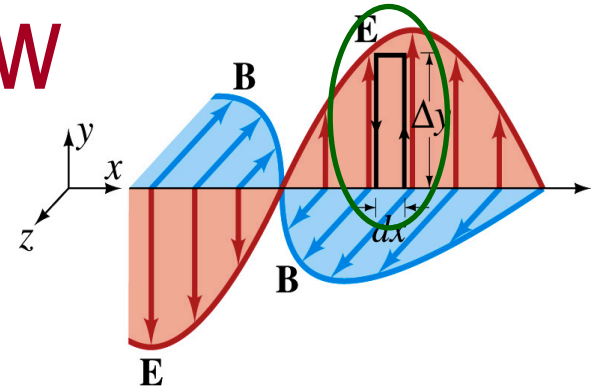
Thus

$$dE\Delta y = -\frac{dB}{dt} dx \Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since  $E$  and  $B$  depend on  $x$  and  $t$

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



# From Modified Ampère's Law

- Let's apply Maxwell's 4<sup>th</sup> equation

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

- to the rectangular loop of length  $\Delta z$  and width  $dx$

- $\vec{B} \cdot d\vec{l}$  along the x-axis of the loop is 0

- Since  $\vec{B}$  is perpendicular to  $d\vec{l}$
- So the result of the integral through the loop counterclockwise

becomes

$$\oint \vec{B} \cdot d\vec{l} = B\Delta z - (B + dB)\Delta z = -dB\Delta z$$

- For the right-hand side of the equation is

$$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{dE}{dt} dx \Delta z$$

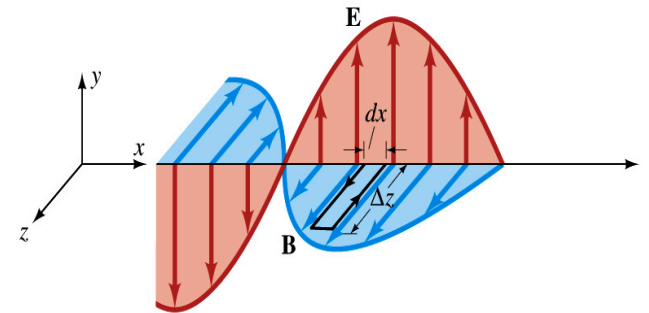
Thus

$$-dB\Delta z = \mu_0 \epsilon_0 \frac{dE}{dt} dx \Delta z$$

$$-\frac{dB}{dx} = -\mu_0 \epsilon_0 \frac{dE}{dt}$$

Since E and B  
depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$




# Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law  $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x} = \frac{\partial}{\partial x} (E_0 \sin(kx - \omega t)) = kE_0 \cos(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t} (B_0 \sin(kx - \omega t)) = -\omega B_0 \cos(kx - \omega t)$$

Since  $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$   **We obtain**  $kE_0 \cos(kx - \omega t) = \omega B_0 \cos(kx - \omega t)$

 **Thus**  $\frac{E_0}{B_0} = \frac{\omega}{k} = v$

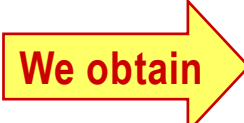
- Since E and B are in phase, we can write  $E/B = v$ 
  - This is valid at any point and time in space. What is v?
    - The velocity of the wave


# Speed of EM Waves

- Let's now use the relationship from Ampere's law  $\frac{\partial B}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x} (B_0 \sin(kx - \omega t)) = kB_0 \cos(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} (E_0 \sin(kx - \omega t)) = -\omega E_0 \cos(kx - \omega t)$$

Since  $\frac{\partial B}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$   **We obtain**  $kB_0 \cos(kx - \omega t) = \epsilon_0 \mu_0 \omega E_0 \cos(kx - \omega t)$

 **Thus**  $\frac{B_0}{E_0} = \frac{\epsilon_0 \mu_0 \omega}{k} = \epsilon_0 \mu_0 v$

– However, from the previous page we obtain  $E_0/B_0 = v = \frac{1}{\epsilon_0 \mu_0 v}$

– Thus  $v^2 = \frac{1}{\epsilon_0 \mu_0}$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})}} = 3.00 \times 10^8 \text{ m/s}$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

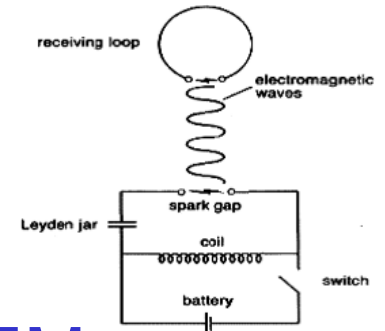


# Speed of Light w/o Sinusoidal Wave Forms

- Taking the time derivative on the relationship from Ampere's laws, we obtain  $\frac{\partial^2 B}{\partial x \partial t} = -\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$
- By the same token, we take position derivative on the relationship from Faraday's law  $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$
- From these, we obtain  $\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2}$  **and**  $\frac{\partial^2 B}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}$
- Since the equation for traveling wave is  $\frac{\partial^2 x}{\partial t^2} = v^2 \frac{\partial^2 x}{\partial x^2}$
- By correspondence, we obtain  $v^2 = \frac{1}{\epsilon_0 \mu_0}$
- A natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling w/ speed  $v = 1/\sqrt{\epsilon_0 \mu_0}$ 
  - Maxwell predicted the existence of EM waves based on this

# Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but ...
  - They did not know what kind of waves they are.
    - Most importantly what is it that oscillates in light?
- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  - Charge was rushed back and forth in a short period of time, generating waves with frequency about  $10^9\text{Hz}$  (these are called radio waves)
  - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  - These waves were later shown to travel at the speed of light and behave exactly like the light just not visible

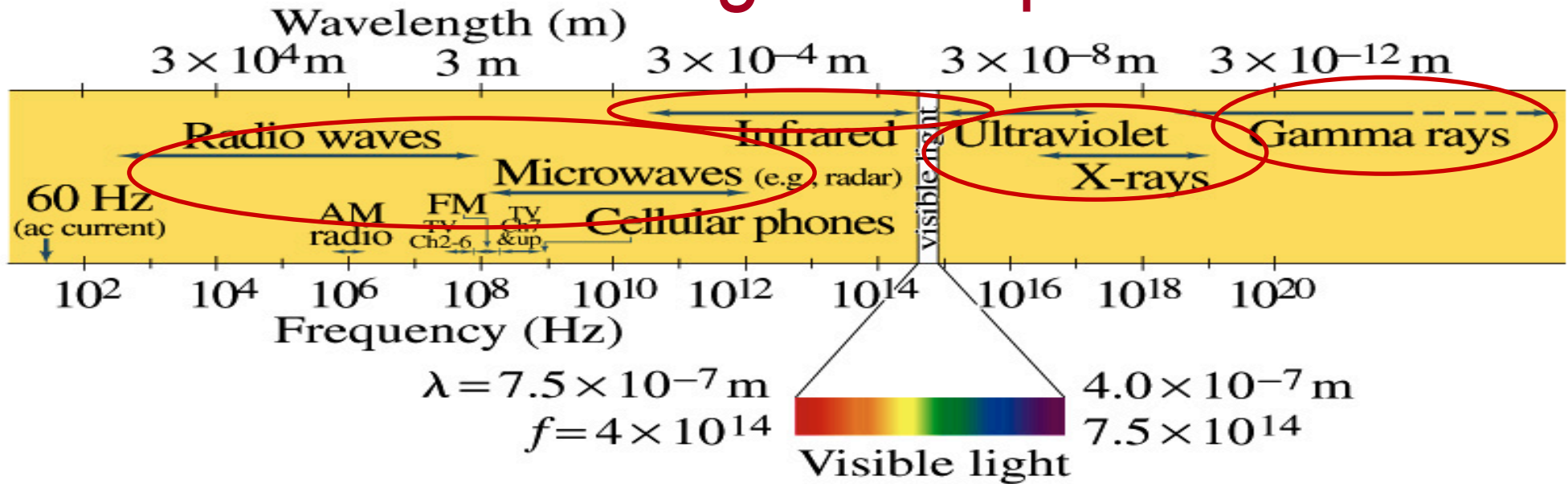


# Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19<sup>th</sup> century
  - The visible light wave length were found to be between  $4.0 \times 10^{-7} \text{m}$  (400nm) and  $7.5 \times 10^{-7} \text{m}$  (750nm)
  - The frequency of visible light is  $f\lambda = c$ 
    - Where  $f$  and  $\lambda$  are the frequency and the wavelength of the wave
      - What is the range of visible light frequency?
      - $4.0 \times 10^{14} \text{Hz}$  to  $7.5 \times 10^{14} \text{Hz}$
    - $c$  is  $3 \times 10^8 \text{m/s}$ , the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum



# Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
  - The Sun emits visible lights, IR and UV
    - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

# Example 31 – 3

**Wavelength of EM waves.** Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency  $4.74 \times 10^{14} \text{ Hz}$ .

What is the relationship between speed of light, frequency and the wavelength?  $c = f \lambda$

Thus, we obtain  $\lambda = \frac{c}{f}$

For  $f=60 \text{ Hz}$   $\lambda = \frac{3 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5 \times 10^6 \text{ m}$

For  $f=93.3 \text{ MHz}$   $\lambda = \frac{3 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{ s}^{-1}} = 3.22 \text{ m}$

For  $f=4.74 \times 10^{14} \text{ Hz}$   $\lambda = \frac{3 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{ s}^{-1}} = 6.33 \times 10^{-7} \text{ m}$

# EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
  - Can it not just travel through the empty space?
  - Nope. It sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
  - When two wires are separated via air, the EM wave travel through the air at the speed of light,  $c$ .
  - However, through medium w/ permittivity  $\epsilon$  and permeability  $\mu$ , the speed of the EM wave is given  $v = 1/\sqrt{\epsilon\mu} < c$ 
    - Is this faster than  $c$ ? **Nope! It is slower.**

# Example 31 – 5

**Phone call time lag.** You make a telephone call from New York to London. Estimate the time the electrical signal to travel to reach London (a) carried on a 5000km telephone cable under the Atlantic Ocean and (b) sent via satellite 36,000km above the ocean. Would this cause a noticeable delay in either case?

Time delay via the cable:  $t = \frac{d}{c} = \frac{5 \times 10^6}{3 \times 10^8} = 0.017s$

Delay via satellite  $t = \frac{2d_s}{c} = \frac{2 \times 3.6 \times 10^7}{3.0 \times 10^8} = 0.24s$

So in case of satellite, the delay is likely noticeable!!

# Energy in EM Waves

- Since  $B=E/c$  and  $c = 1/\sqrt{\epsilon_0\mu_0}$ , we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2 \quad \boxed{u = \epsilon_0 E^2}$$

- Note that the energy density associated with B field is the same as that associated with E
- So each field contributes half to the total energy

- By rewriting in B field only, we obtain

$$u = \frac{1}{2}\epsilon_0 \frac{B^2}{\epsilon_0\mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0}$$

$$\boxed{u = \frac{B^2}{\mu_0}}$$

- We can also rewrite to contain both E and B

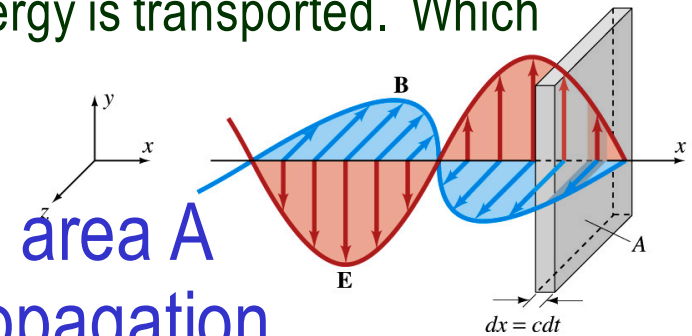
$$u = \epsilon_0 E^2 = \epsilon_0 E c B = \frac{\epsilon_0 E B}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} E B$$

$$\boxed{u = \sqrt{\frac{\epsilon_0}{\mu_0}} E B}$$



# Energy Transport

- What is the energy the wave transport per unit time per unit area?
  - This is given by the vector **S**, the Poynting vector
    - The unit of **S** is W/m<sup>2</sup>.
    - The direction of **S** is the direction in which the energy is transported. Which direction is this?
      - The direction the wave is moving
- Let's consider a wave passing through an area **A** perpendicular to the x-axis, the axis of propagation
  - How much does the wave move in time  $dt$ ?
    - $dx = cdt$
  - The energy that passes through **A** in time  $dt$  is the energy that occupies the volume  $dV$ ,  $dV = A dx = A c dt$
  - Since the energy density is  $u = \epsilon_0 E^2$ , the total energy,  $dU$ , contained in the volume  $V$  is  $dU = u dV = \epsilon_0 E^2 A c dt$



# Energy Transport

- Thus, the energy crossing the area  $A$  per time  $dt$  is

$$S = \frac{1}{A} \frac{dU}{dt} = \epsilon_0 c E^2$$

- Since  $E=cB$  and  $c = 1/\sqrt{\epsilon_0 \mu_0}$ , we can also rewrite

$$S = \epsilon_0 c E^2 = \frac{c B^2}{\mu_0} = \frac{EB}{\mu_0}$$

- Since the direction of  $S$  is along  $\mathbf{v}$ , perpendicular to  $\mathbf{E}$  and  $\mathbf{B}$ , the Poynting vector  $\mathbf{S}$  can be written

$$\mathbf{S} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B})$$

What is the unit?  $\text{W/m}^2$

- This gives the energy transported per unit area per unit time at any instant

# Average Energy Transport

- The average energy transport in an extended period of time is useful since sometimes we do not detect the rapid variation with respect to time because the frequency is so high.
- If E and B are sinusoidal,  $\overline{E^2} = E_0^2 / 2$
- Thus we can write the magnitude of the average Poynting vector as

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0}$$

- This time averaged value of S is the intensity, defined as the average power transferred across unit area.  $E_0$  and  $B_0$  are maximum values.
- We can also write 
$$\bar{S} = \frac{E_{rms} B_{rms}}{\mu_0}$$
  - Where  $E_{rms}$  and  $B_{rms}$  are the rms values ( $E_{rms} = \sqrt{\overline{E^2}}$ ,  $B_{rms} = \sqrt{\overline{B^2}}$  )

# Example 31 – 6

**E and B from the Sun.** Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about  $1350 \text{ W/m}^2$ . Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2\mu_0}$$

For  $E_0$ , 
$$E_0 = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2 \cdot 1350 \text{ W/m}^2}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (3.00 \times 10^8 \text{ m/s})}} = 1.01 \times 10^3 \text{ V/m}$$

For  $B_0$  
$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 3.37 \times 10^{-6} \text{ T}$$