# PHYS 1441 – Section 001
# Lecture #6

*Tuesday, June 12, 2018*

*Dr. Jaehoon Yu*

- Chapter 22
  - Gauss' Law
  - Electric Flux
  - Gauss' Law with Multiple Charges
  - What is Gauss' Law Good For?
- Chapter 23
  - Electric Potential due to Point Charge
  - Shape of the Electric Potential

# Announcements

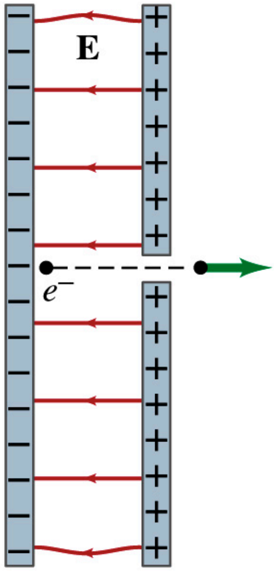
- Quiz #2
  - At the beginning of the class this Thursday, June 14
  - Covers up to what we've learned tomorrow, Wed. June 13
  - You can bring your calculator but it must not have any relevant formula pre-input
    - Cell phones or any types of computers cannot replaced a calculator!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of any problems !
  - No additional formulae or values of constants will be provided!
- Reading assignments
  - CH22.4

# Reminder: SP#2 – Angels & Demons

- Compute the total possible energy released from an annihilation of x-grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class tomorrow Wed. June. 13

# Reminder: Special Project #3

- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction $f$ of the speed of light $c$. Express E in terms of **M, Q, D,** $f$, **c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. ( 8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)

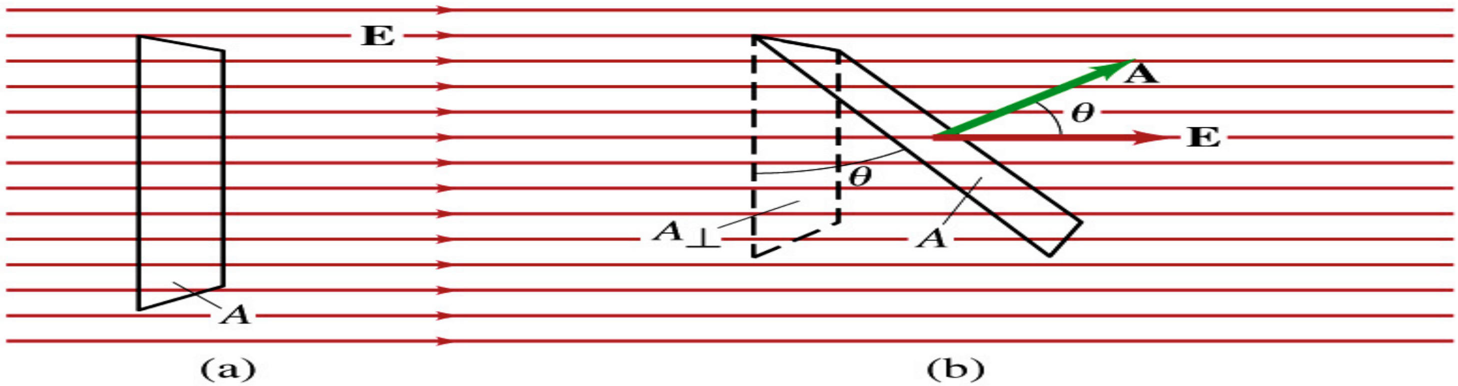



- Due beginning of the class Monday, June 18

# Gauss' Law

- Gauss' law states the relationship between the electric charge and the electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

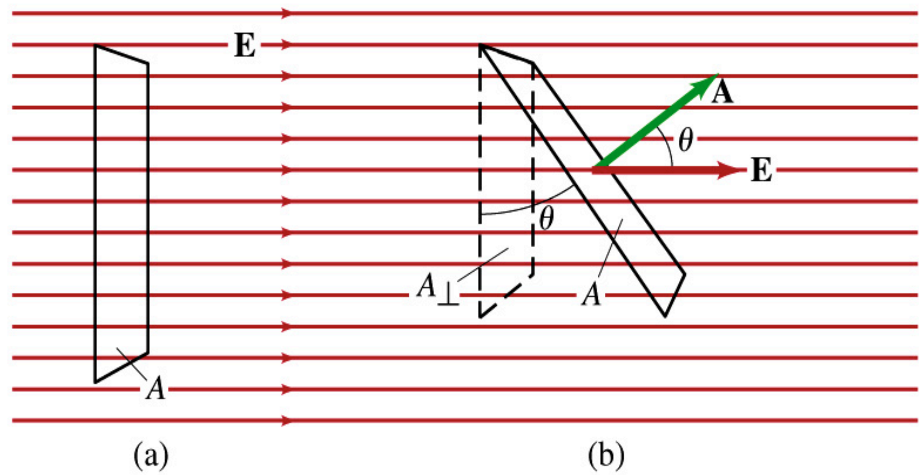
# Electric Flux




- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux $\Phi_E$ is defined as
  - $\Phi_E$=EA, if the field is perpendicular to the surface
  - $\Phi_E$=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

# Example 22 – 1

- **Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux if the angle is 30 degrees?




The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot(0.1\times 0.2m^2) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees, we obtain

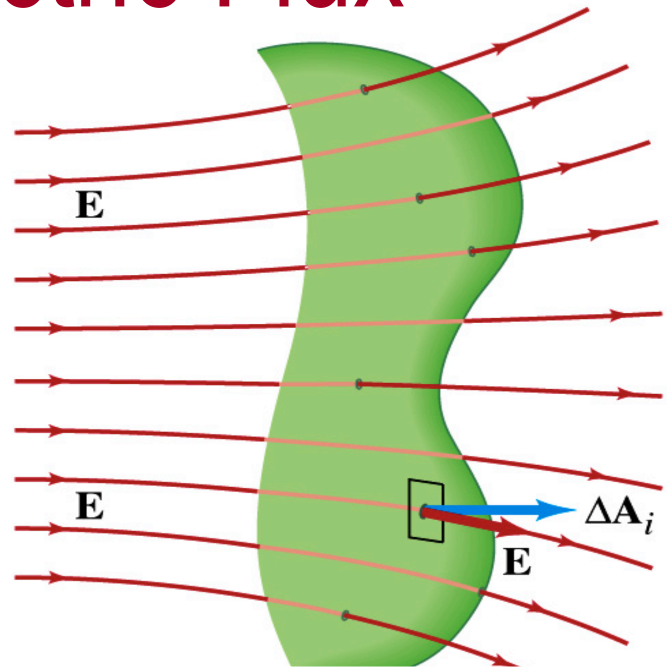
$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot(0.1\times 0.2m^2)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

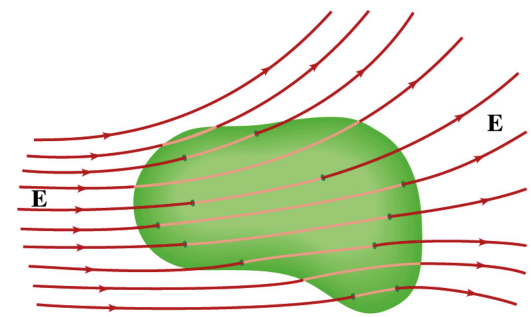
# Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta \mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is approximately

$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

- In the limit where $\Delta \mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

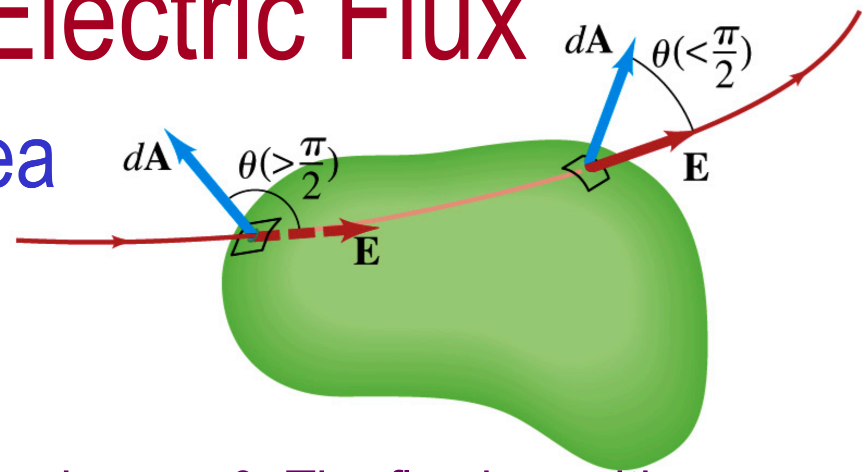


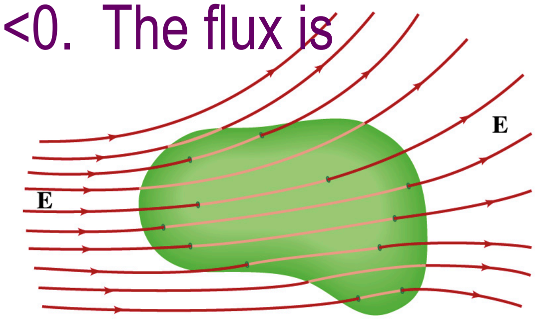

$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

# Generalization of the Electric Flux

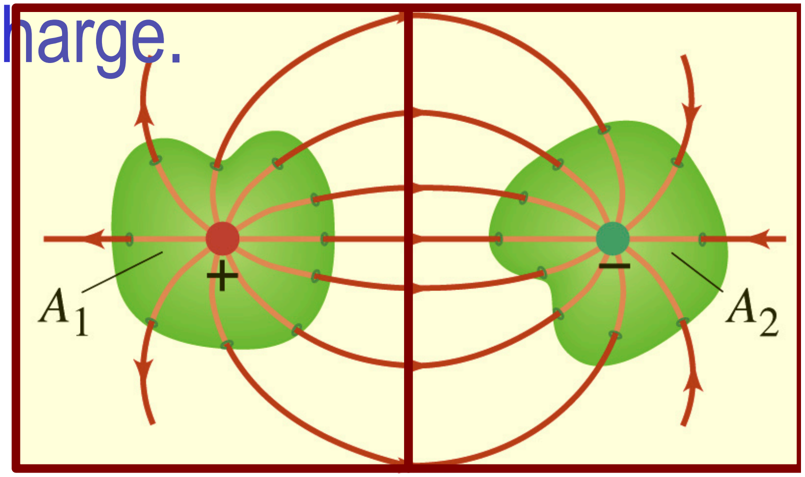



- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $|\theta|<\pi/2$ and $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $|\theta|>\pi/2$ and $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.

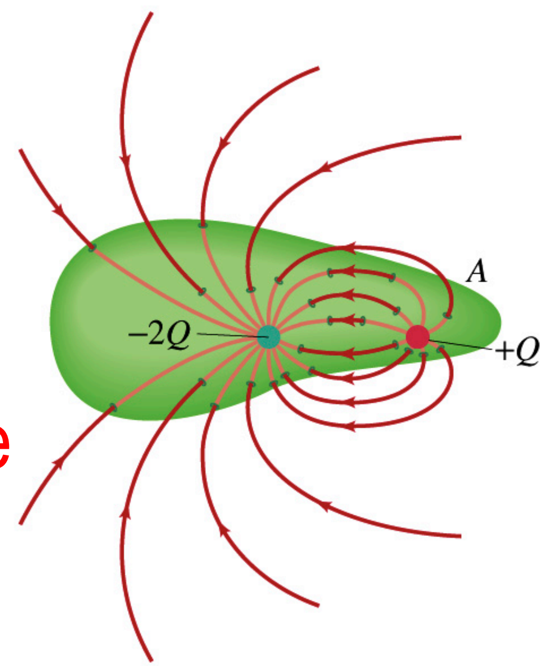



- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E}\cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

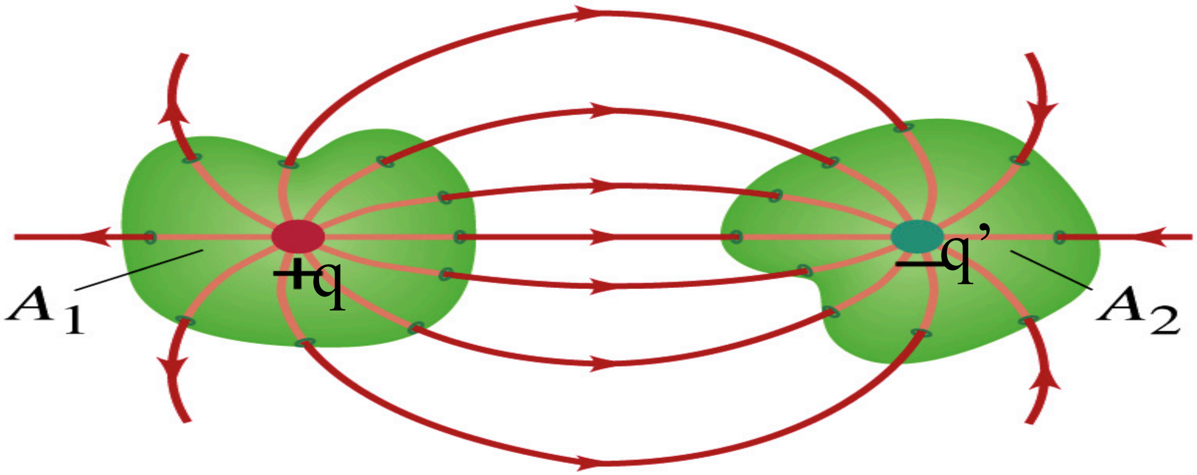
# Generalization of the Electric Flux




- The field line starts or ends only on a charge.
- Sign of the net flux on the surface $A_1$?
  - Net outward flux (positive flux)
- How about $A_2$?
  - Net inward flux (negative flux)
- What is the flux in the bottom figure?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

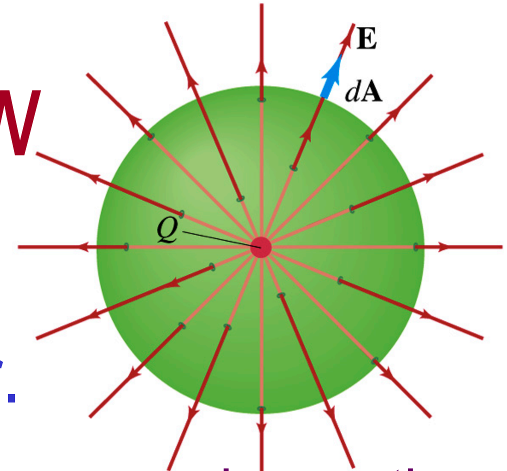


# Gauss' Law




- Let's consider the case in the above figure.
- What are the results of the closed integral of the Gaussian surfaces $A_1$ and $A_2$?
  - For $A_1$ $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For $A_2$ $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

# Coulomb's Law from Gauss' Law




- Let's consider a charge Q enclosed inside our imaginary Gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Have the same magnitude (uniform) at any point on the surface
    - Points radially outward parallel to the surface vector d**A**.
- The Gaussian integral can be written as

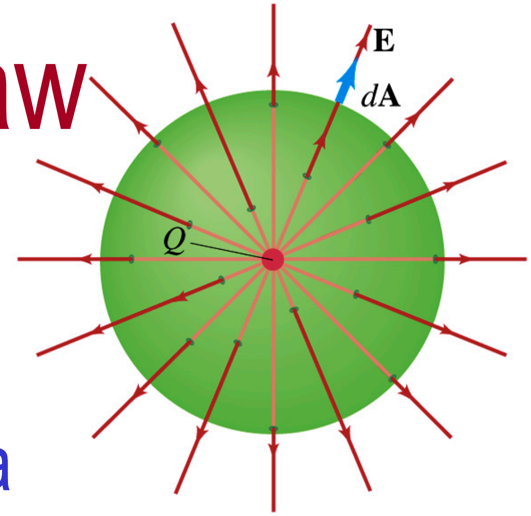
$$\oint \vec{E}\cdot d\vec{A} = \oint E\,dA = E\oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

**Solve for E**

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

**Electric Field of Coulomb's Law**

# Gauss' Law from Coulomb's Law



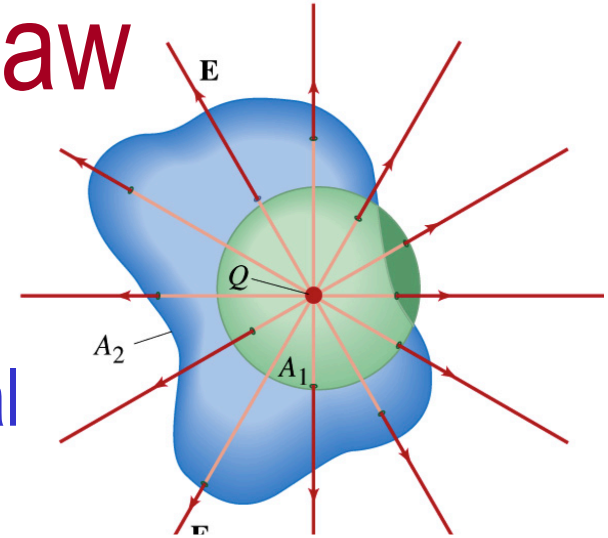

- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface of radius r is $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E} \cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

**Gauss' Law**

# Gauss' Law from Coulomb's Law Irregular Surface




- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface $A_1$ and a randomly shaped surface $A_2$.
- What is the difference in the total number of field lines due to the charge Q, passing through the two surfaces?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $$\oint_{A_1} \vec{E}\cdot d\vec{A} = \oint_{A_2} \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$
  - What does this mean?
    - The flux due to the given enclosed charge is the same independent of the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.

# Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, $Q_i$ inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

**What is $\vec{E}_i$?**

**The electric field produced by $Q_i$ alone!**

- Since electric fields can be added vectorially, following the superposition principle, the total field **E** is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ and any external fields. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left( \vec{E}_{ext} + \sum \vec{E}_i \right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

**What is $Q_{encl}$?**

**The total enclosed charge!**

- The value of the flux depends only on the charge enclosed in the surface!! ➔ Gauss' law.

# So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for **static electric charge**.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, forces or obtain charges

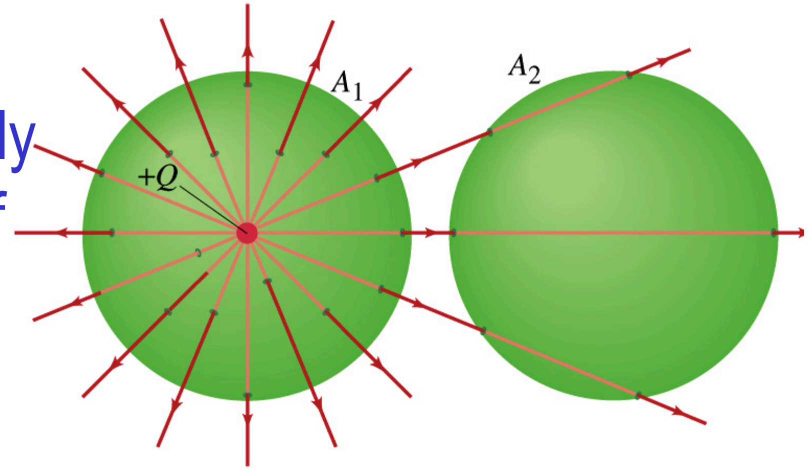
Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge inside that surface!!!

# Solving problems with Gauss' Law

- Identify the symmetry of the charge distributions
- Draw an appropriate Gaussian surface, making sure it passes through the point you want to know the electric field
- Use the symmetry of charge distribution to determine the direction of E at the point of the Gaussian surface
- Evaluate the flux
- Calculate the enclosed charge by the Gaussian surface
  - Ignore all the charges outside the Gaussian surface
- Equate the flux to the enclosed charge and solve for E

# Example 22 – 2

**Flux from Gauss' Law**: Consider two Gaussian surfaces, $A_1$ and $A_2$, shown in the figure. The only charge present is the charge +Q at the center of surface $A_1$. What is the net flux through each surface $A_1$ and $A_2$?




- The surface $A_1$ encloses the charge +Q, so from Gauss' law we obtain the total net flux

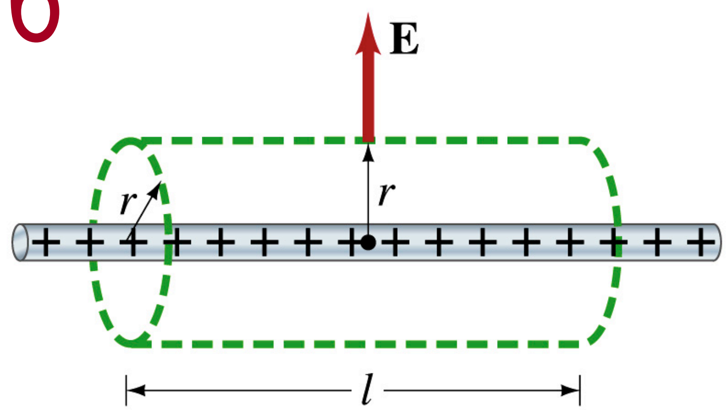
$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- The surface $A_2$ the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

# Example 22 – 6

**Long uniform line of charge**: A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field a point near but outside the wire, far from the ends.




- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of radial vector **r**.
- Due to cylindrical symmetry, the field is the same on the Gaussian surface of a cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector **E** is perpendicular to the surface vector d**A**.
- From Gauss' law

$$\oint \vec{E}\cdot d\vec{A} = E\oint dA = E(2\pi rl) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

# A Brain Teaser of Electric Flux

- What would change the electric flux through a circle lying in the xz plane where the electric field is (10N/C)**j**?
  1. Changing the magnitude of the electric field
  2. Changing the surface area of the circle
  3. Tipping the circle so that it is lying in the xy plane
  4. All of the above
  5. None of the above

# Gauss' Law Summary

- The precise relationship between flux and the enclosed charge is given by Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$
  - $\varepsilon_0$ is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
  - Freedom to choose!!
    - The surface integral is performed over the value of **E** on a closed surface of our choice in any given situation.
  - Test of existence of electrical charge!!
    - The charge $Q_{encl}$ is the net charge enclosed by the arbitrary closed surface of our choice.
  - Universality of the law!
    - It does NOT matter where or how much charge is distributed inside the surface. Gauss' law still applies!
  - The charge outside the surface does not contribute to $Q_{encl}$. Why?
    - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface.