PHYS 1441 – Section 001
Lecture #7

Wednesday, June 13, 2018

Dr. Jaehoon Yu

- Chapter 23
  - Electric Potential Energy
  - Electric Potential due to Point Charges
  - Shape of the Electric Potential
  - V due to Charge Distributions
  - Equi-potential Lines and Surfaces
  - Electric Potential Due to Electric Dipole

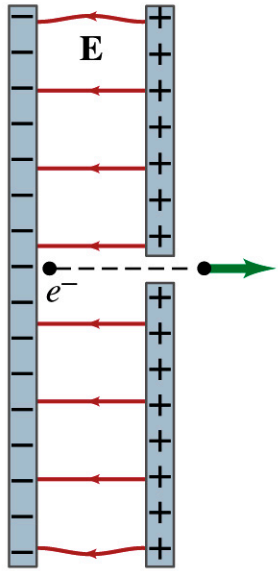
Today's homework is homework #4, due 11pm, Monday, June 18!!

Announcements

- Quiz #2
  - At the beginning of the class tomorrow, June 14
  - Covers up to what we've learned today
  - You can bring your calculator but it must not have any relevant formula pre-input
    - Cell phones or any types of computers cannot replaced a calculator!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, or solutions of any problems !
  - No additional formulae or values of constants will be provided!
- Bring out special project #2 during the mid-class break

Reminder: Special Project #3

- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction f of the speed of light c. Express E in terms of **M, Q, D,** f, **c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. (8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)



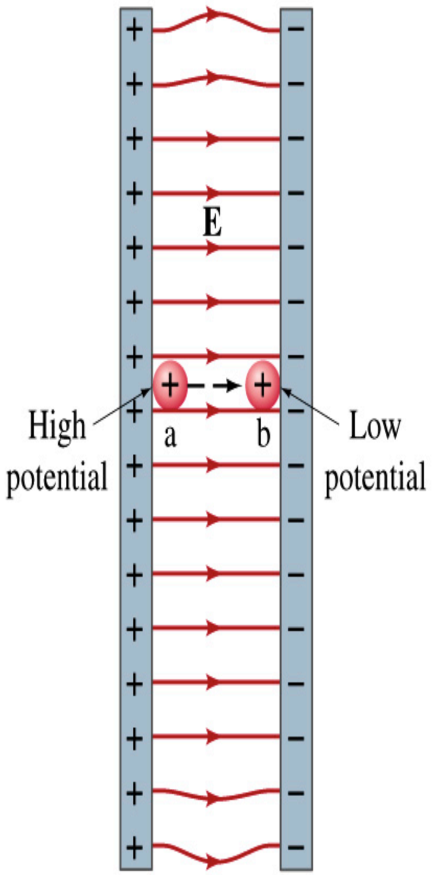
- Due beginning of the class Monday, June 18

Electric Potential Energy

- Concept of energy is very useful solving mechanical problems
- Conservation of energy makes solving complex problems easier.
- When can the potential energy be defined?
  - Only for a conservative force.
  - The work done by a conservative force is independent of the path. What does it only depend on??
    - The difference between the initial and final positions
  - Can you give me an example of a conservative force?
    - Gravitational force
- Is the electrostatic force between two charges a conservative force?
  - Yes. Why?
  - The dependence of the force to the distance is identical to that of the gravitational force.
    - The only thing matters is the direct linear distance between the objects not the path.

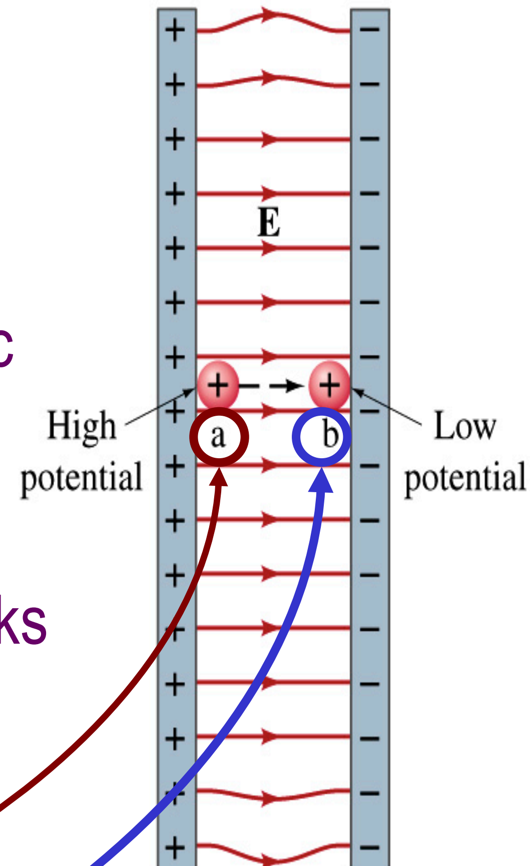
Electric Potential Energy

- How would you define the change in electric potential energy $U_b - U_a$?
  - The potential to work gained by the charge as it moves from point *a* to point *b*.
  - The negative work done on the charge by the electric force to move it from *a* to *b*.
- Let's consider an electric field between two parallel plates w/ equal but opposite charges
  - The field between the plates is uniform since the gap is small and the plates are infinitely long…
- What happens when we place a small charge, +q, on a point at the positive plate and let go?
  - The electric force will accelerate the charge toward the negative plate.
  - What kind of energy does the charged particle gain?
    - Kinetic energy



Electric Potential Energy

- What does this mean in terms of energies?
  - The electric force is a conservative force.
  - Thus, the mechanical energy (K+U) is conserved under this force.
  - The positively charged object has only the electric potential energy (no KE) at the positive plate.
  - The electric potential energy decreases and
  - Turns into kinetic energy as the electric force works on the charged object, and the charged object gains speed.



- Point of the greatest potential energy for
  - Positively charged object
  - Negatively charged object

PE= U 0
KE= 0 K
ME=U K
U+K

Electric Potential

- How is the electric field defined?
  - Electric force per unit charge: F/q
- We can define electric potential (potential) as
  - The electric potential energy per unit charge
  - This is like the voltage of a battery…
- Electric potential is written with the symbol V
  - If a positive test charge q has the potential energy U_a at a point *a*, the electric potential of the charge at that point is

$$V_a = \frac{U_a}{q}$$

Electric Potential

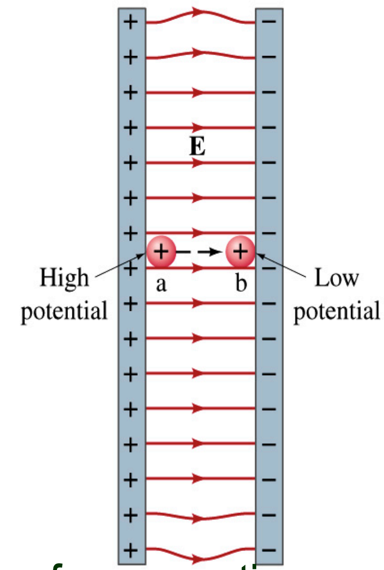
- Since only the difference in potential energy is meaningful, only the potential difference between two points is measurable
- What happens when the electric force does a “positive work”?
  - The charge gains kinetic energy
  - Electric potential energy of the charge decreases
- Thus the difference in potential energy is the same as the negative of the work, W_{ba}, done on the charge by the electric field to move the charge from point a to b.
- The potential difference V_{ba} is

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = \frac{-W_{ba}}{q}$$

  - Electric potential is independent of the test charge!! Unit?

A Few Things about the Electric Potential

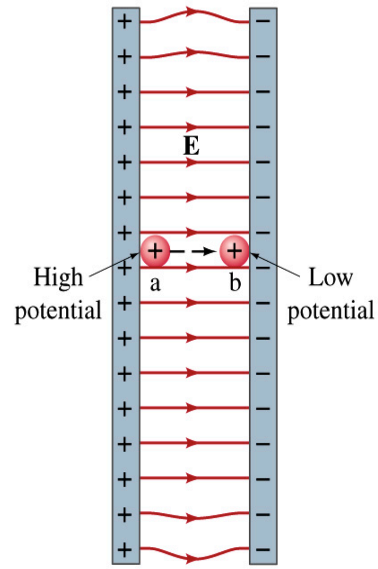
- What does the electric potential depend on?
  - Other charges that creates the field
  - What about the test charge?
    - No, the electric potential is independent of the test charge
    - Test charge gains potential energy by existing in the potential created by other charges
- Which plate is at a higher potential?
  - Positive plate. Why?
    - Since positive charge has the greatest potential energy on it.
  - What happens to the positive charge if it is let go?
    - It moves from higher potential to lower potential
  - How about the negative charge?
    - Its potential energy is higher on the negative plate. Thus, it moves from negative plate to positive. Potential difference is the same.
- The unit of the electric potential is Volt (V).
- From the definition, 1V = 1J/C.



Zero point of electric potential can be chosen arbitrarily.

Often the ground, a conductor connected to Earth, is zero.

Example 23 – 1



A negative charge: Suppose a negative charge, such as an electron, is placed at point *b* in the figure. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?

- An electron placed at point *b* will move toward the positive plate since it was released at its highest **potential energy** point.
- It will gain kinetic energy as it moves toward left, decreasing its potential energy.
- The electron, however, moves from the point *b* at a lower potential to point *a* at a higher **potential**. $\Delta V = V_a - V_b > 0$.
- This is because the **potential is generated by the charges on the plates** not by the electron.

Electric Potential and Potential Energy

- What is the definition of the electric potential?
  - The potential energy difference per unit charge

$$V_{ba} = \frac{U_b - U_a}{q}$$

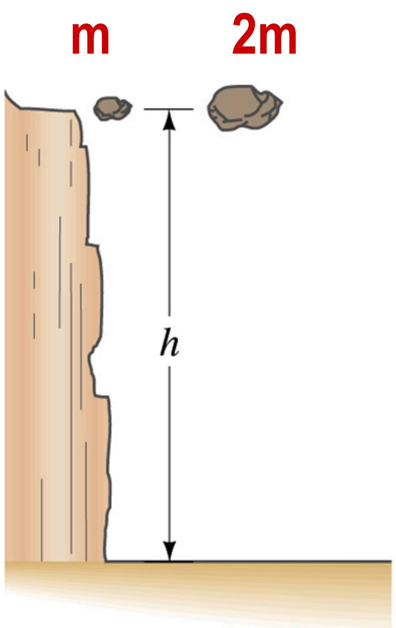
- OK, then, how would you express the potential energy that a charge q would obtain when it is moved between point *a* and *b* with the potential difference V_{ba}?

$$U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

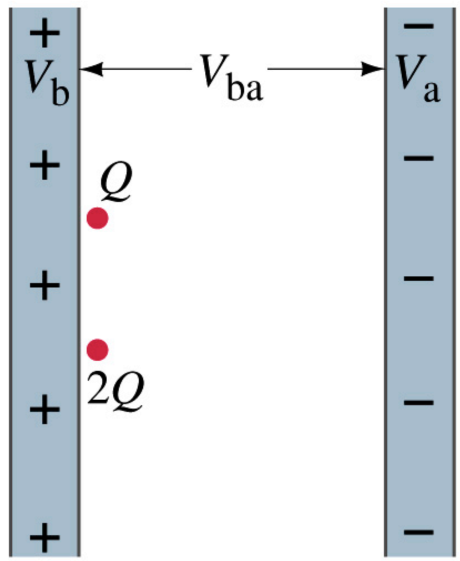
  - In other words, if an object with charge q moves through a potential difference V_{ba}, its potential energy changes by qV_{ba}.
- So based on this, how differently would you describe the electric potential in words?
  - A measure of how much energy an electric charge can acquire in a given situation
  - A measure of how much work a given charge can do.

Comparisons of Potential Energies

- Let's compare gravitational and electric potential energies



(a)



(b)

- What are the potential energies of the rocks?
  - mgh and 2mgh
- Which rock has a bigger potential energy?
  - The rock with a larger mass
- Why?
  - It's got a bigger mass.

- What are the potential energies of the charges?
  - QV_{ba} and $2QV_{ba}$
- Which object has a bigger potential energy?
  - The object with a larger charge.
- Why?
  - It's got a bigger charge.

The potential is the same but the heavier rock or larger charge can do a greater work.

Electric Potential and Potential Energy

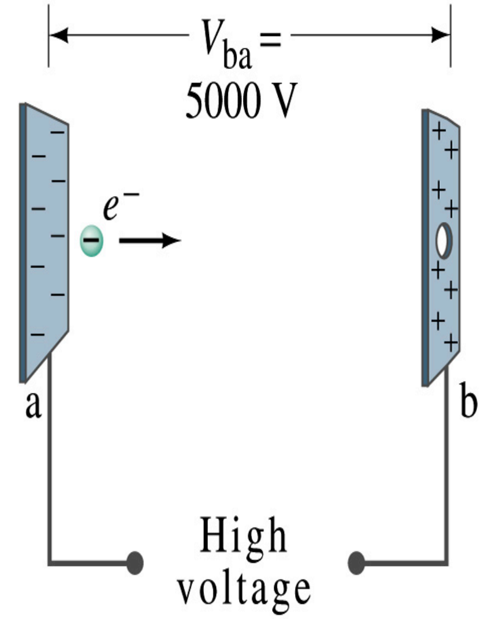
- The electric potential difference gives potential energy or the possibility to perform work based on the charge of the object.
- So what is happening in a battery or a generator?
  - They maintain a potential difference.
  - The actual amount of energy used or transformed depends on how much charge flows.
  - How much is the potential difference maintained by a car's battery?
    - 12Volts
  - If for a given period, 5C charge flows through the headlight lamp, what is the total energy transformed?
    - E_{tot}=5C*12V=60 Umm… What is the unit? **Joules**
  - If it is left on twice as long? E_{tot}=10C*12V=120J.

Some Typical Voltages

Sources	Approximate Voltage
Thundercloud to ground	10^8 V
High-Voltage Power Lines	10^6 V
Power supply for TV tube	10^4 V
Automobile ignition	10^4 V
Household outlet	10^2 V
Automobile battery	12 V
Flashlight battery	1.5 V
Resting potential across nerve membrane	10^{-1} V
Potential changes on skin (EKG and EEG)	10^{-4} V

In a typical lightening strike, 15C of electrons are released in 500μs. What is the total kinetic energy of these electrons when they strike ground? What is the power released during this strike? What do you think will happen to a tree hit by this lightening?

Example 23 – 2



Electrons in TV tube: Suppose an electron in the picture tube of a television set is accelerated from rest through a potential difference V_{ba}=+5000V. (a) What is the change in potential energy of the electron? (b) What is the speed of the electron ($m=9.1\times10^{-31}kg$) as a result of this acceleration? (c) Repeat for a proton ($m=1.67\times10^{-27}kg$) that accelerates through a potential difference of V_{ba}=-5000V.

- (a) What is the charge of an electron?
  - $e=-1.6\times10^{-19}C$

- So what is the change of its potential energy?

$$\Delta U = qV_{ba} = eV_{ba} = \left(-1.6\times10^{-19}C\right)\left(+5000V\right) = -8.0\times10^{-16}J$$

Example 23 – 2

- (b) Speed of the electron?
  - The entire potential energy of the electron turns to its kinetic energy. Thus the equation is

$$\Delta K = \frac{1}{2} m_e v_e^2 - 0 = W = -\Delta U = -eV_{ba} =$$
$$= -\left(-1.6\times10^{-19} C\right) 5000V = 8.0\times10^{-16} J$$

$$v_e = \sqrt{\frac{2\times eV_{ba}}{m_e}} = \sqrt{\frac{2\times 8.0\times10^{-16}}{9.1\times10^{-31}}} = 4.2\times10^{7} m/s$$

- (C) Speed of a proton?

$$\Delta K = \frac{1}{2} m_p v_p^2 - 0 = W = -\Delta U = -\left\{\left(-e\right)\left(-V_{ba}\right)\right\} = -eV_{ba} = 8.0\times10^{-16} J$$

$$v_p = \sqrt{\frac{2\times eV_{ba}}{m_p}} = \sqrt{\frac{2\times 8.0\times10^{-16}}{1.67\times10^{-27}}} = 9.8\times10^{5} m/s$$

Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, how are these two quantities related?

Electric Potential and Electric Field

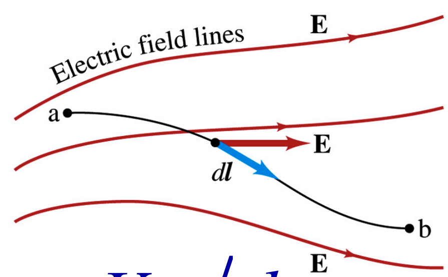
- **Potential energy** change is expressed in terms of a conservative force (point ***a*** at a higher potential)

$$U_b - U_a = -\vec{F}\cdot\vec{D} = -W_C$$

- For the electrical case, we are more interested in the **potential** difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q}\cdot\vec{D} = -\vec{E}\cdot\vec{D} = -ED\cos\theta$$

  – This formula can be used to determine V_{ba} when the electric field is given.



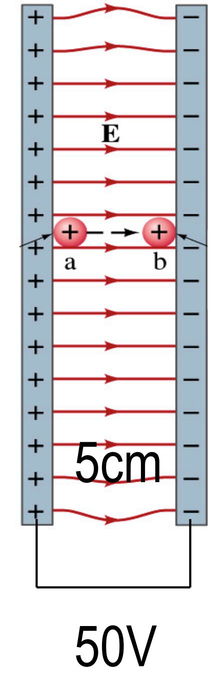
- When the field is uniform

$$V_b - V_a = -\vec{E}\cdot\vec{D} = -ED\cos\theta = -Ed \quad \text{so} \quad E = -V_{ba}/d$$

What does "-" sign mean? The direction of E is along that of decreasing potential.

Unit of the electric field in terms of potential? V/m Can you derive this from N/C?

Example



Uniform electric field obtained from voltage: Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.

What is the relationship between electric field and the potential for a uniform field?

$$V = Ed$$

Solving for E

$$E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5\times10^{-2}m} = 1000V/m$$

Which direction is the field? Direction of decreasing potential!

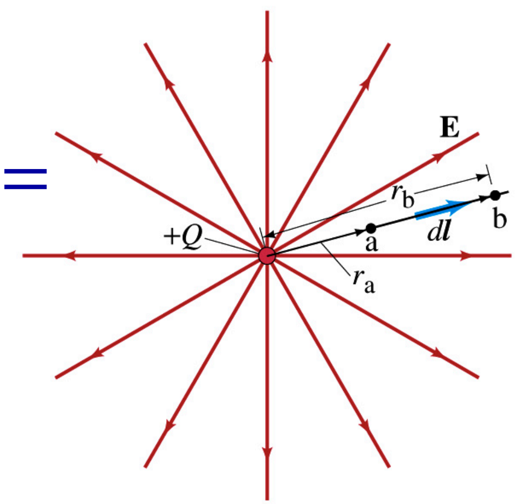
Electric Potential due to Point Charges

- What is the electric field by a single point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in radial direction away from the charge Q is

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\hat{r}}{r^2}\cdot\hat{r}\,dr =$$

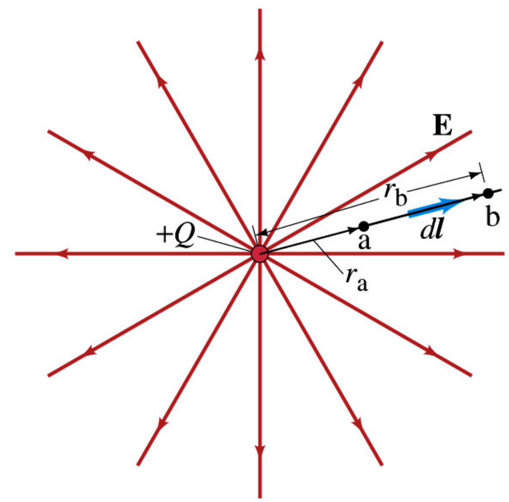


$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of **the potential difference by a single point charge between r and infinity**

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?
  - Electric potential
    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the source charges to obtain the total potential from multiple charges, since potential is a scalar quantity

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

  - Electric field
    - Electric force per unit charge
    - Inversely proportional to the **square** of the distance
    - Need vector sums to obtain the total field from multiple source charges

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

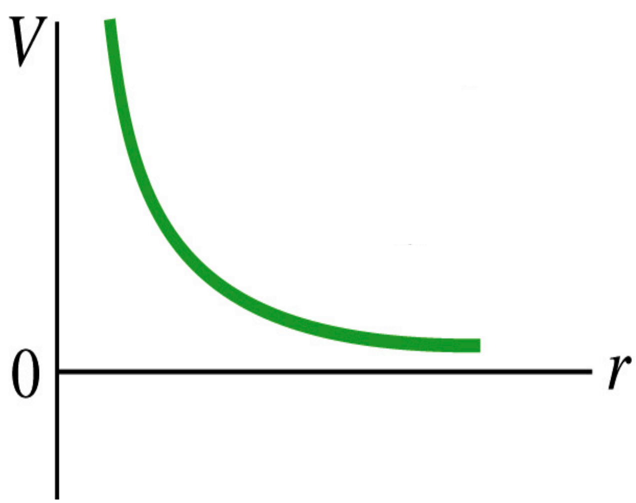
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at the large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at a large distance.

Shape of the Electric Potential

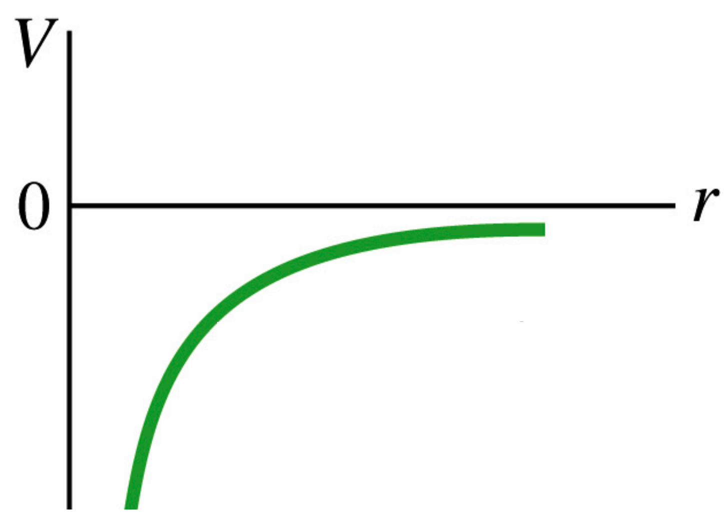
- So, how does the electric potential look like as a function of distance?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive Charge



Negative Charge



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 23 – 6

Work to bring two positive charges close together: What is minimum work required by an external force to bring the charge q=3.00μC from a great distance away (r=∞) to a point 0.500m from a charge Q=20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(3.00\times10^{-6}C\right)\left(20.00\times10^{-6}C\right)}{0.500m} = -1.08J$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge 3.00μC from infinity to 0.500m to the charge 20.0μC.