PHYS 1441 – Section 001
Lecture #8

Thursday, June 14, 2018

Dr. Jaehoon Yu

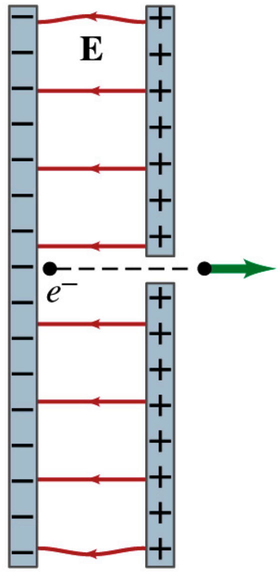
- Chapter 23
  - V due to Charge Distributions
  - Equi-potential Lines and Surfaces
  - Electric Potential Due to Electric Dipole
- Chapter 24 Capacitance etc..
  - Capacitors
  - Capacitors in Series or Parallel
  - Electric Energy Storage

Announcements

- Reading Assignment
  - CH 23.9
- Mid-term exam
  - In class Wednesday, June 20
  - Comprehensive exam which covers CH21.1 through what we learn Tuesday, June 19 plus appendices A and B the math refresher
  - BYOF
- Term 1 results
  - Class Average: 57.7/96
    - Equivalent to 60.1/100
  - Top score: 91/96

Reminder: Special Project #3

- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction f of the speed of light c. Express E in terms of **M, Q, D,** f**, c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. (8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)



- Due beginning of the class Monday, June 18

Electric Potential by Charge Distributions

- Let's consider a case of n individual point charges in a given space and V=0 at r=∞.
- Then the potential V_{ia} due to the charge Q_i at point a, at a distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

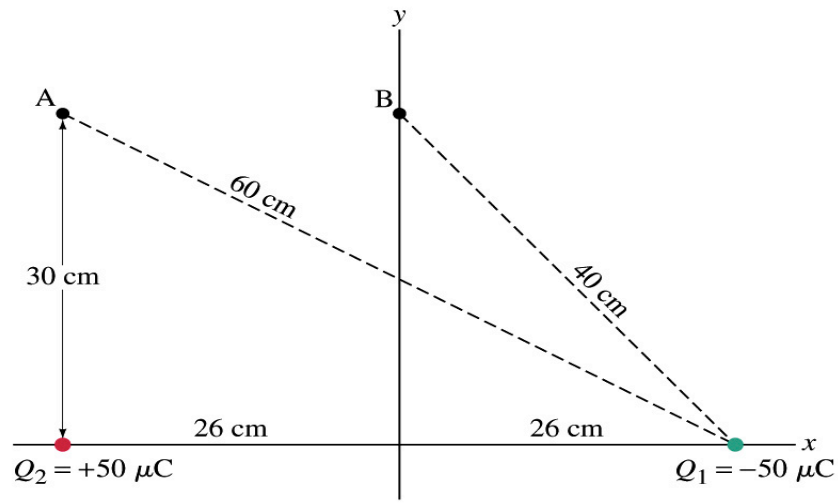
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Example

- **Potential due to two charges**: Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



- Potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.

(a) potential at A is $V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{iA}} =$

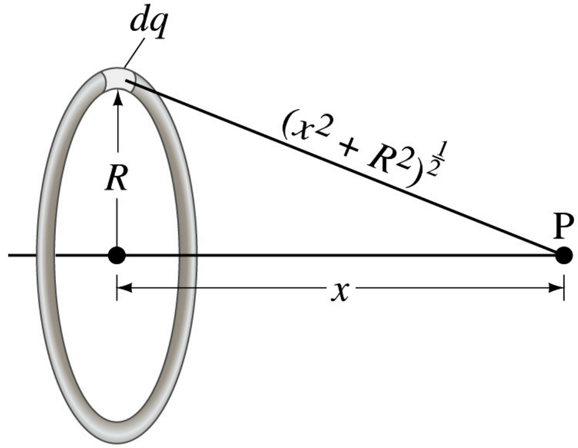
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_{2A}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^9\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^5 V$$

(b) How about potential at B?

Example 23 – 8

- **Potential due to a ring of charge**: A thin circular ring of radius R carries a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center.



- Each point on the ring is at the same distance from the point P. What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0 r}\int dq =$$

What's this?

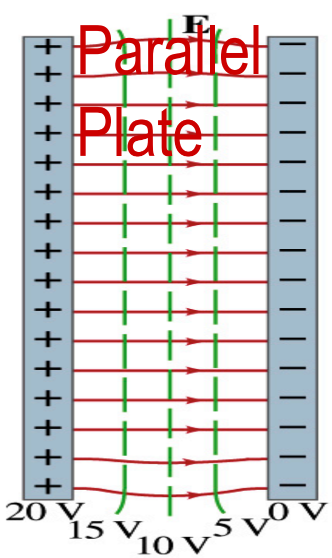
$$\frac{1}{4\pi\varepsilon_0\sqrt{x^2+R^2}}\int dq = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

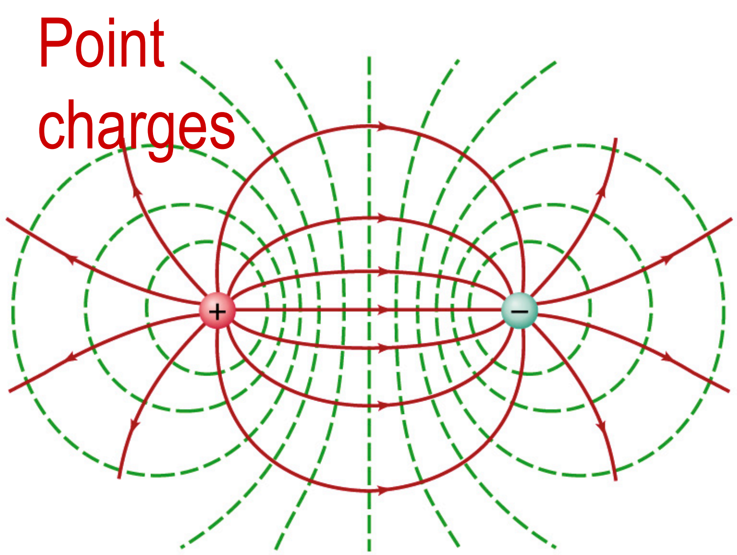
Equi-potential Surfaces

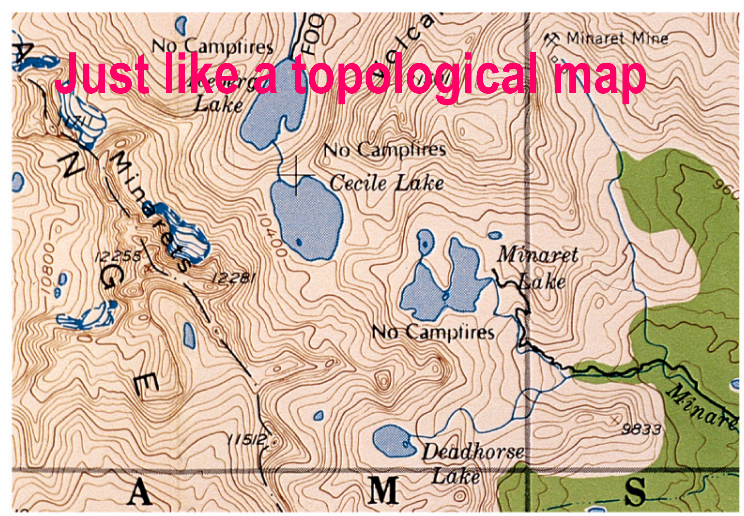
- Electric potential can be graphically shown using the equipotential lines in 2-D or the equipotential surfaces in 3-D
- Any two points on the equipotential surfaces (lines) are at the same potential
- What does this mean in terms of the potential difference?
  - The potential difference between any two points on an equipotential surface is 0.
- How about the potential energy difference?
  - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  - No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- Since there can be no electric field within a conductor in a static case, the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.

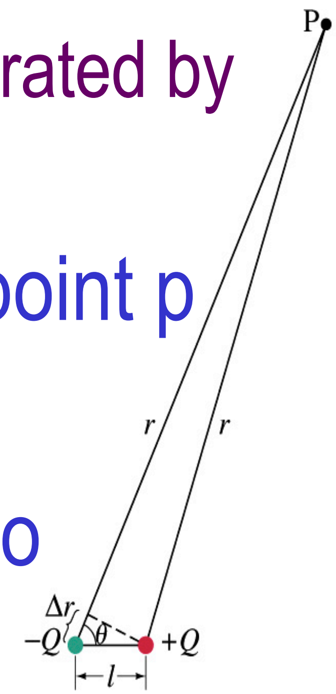






Electric Potential due to Electric Dipoles

- What is an electric dipole?
  - Two equal point charge Q of opposite signs separated by a distance ℓ and behaves like one entity: $P=Q\ell$
- For the electric potential due to a dipole at a point p
  - We take V=0 at r=∞
- The simple sum of the potential at p by the two charges is



$$V = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{(-Q)}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

- Since $\Delta r=\ell\cos\theta$ and if $r \gg \ell$, $r \gg \Delta r$, thus $r \sim r+\Delta r$ and

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}$$

V by a dipole at a distance r from the dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r}$$

E Determined from V

- Potential difference between two points under the electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$
- So in a differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

  - What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between the two points separated by a distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.
- Thus we can write the field component E_ℓ as

$$E_l = -\frac{dV}{dl}$$

Physical Meaning?

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction.!!

E Determined from V

- The quantity dV/dℓ is called the **gradient of V** in a particular direction
  - If no direction is specified, the term gradient refers to the direction on which **V changes most rapidly** and this would be the direction of the field vector **E** at that point.
  - So if **E** and dℓ are parallel to each other, $E = -\frac{dV}{dl}$
- If E is written as a function x, y and z, the ℓ refers to x, y and z $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$
- $\frac{\partial V}{\partial x}$ is the "partial derivative" of V with respect to x, while y and z are held constant
- In vector form, $\vec{E} = -gradV = -\vec{\nabla}V = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)V$

$\vec{\nabla} = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called ***del*** or the ***gradient operator*** and is a ***vector operator***.

Electrostatic Potential Energy

- Consider a case in which a point charge q is moved between points *a* and *b* where the electrostatic potential due to other charges in the system is V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  – Let's choose V=0 at r=∞
  – If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is under no electric force

Electrostatic Potential Energy; Two charges

- If a second point charge Q_2 is brought close to Q_1 at a distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r= ∞ is

$$U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$$

  - This is the work that needs to be done by an external force to bring Q_2 from infinity to the distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain location without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$
  - Work need to bring Q_3 to certain distances to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2 Q_3}{r_{23}}$$

- So the total electrostatic potential energy of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 Q_2}{r_{12}} + \frac{Q_1 Q_3}{r_{13}} + \frac{Q_2 Q_3}{r_{23}}\right) \quad \left[V = 0 \text{ at } r = \infty\right]$$

  - What about a four charge system or N charge system?

Electrostatic Potential Energy: electron Volt

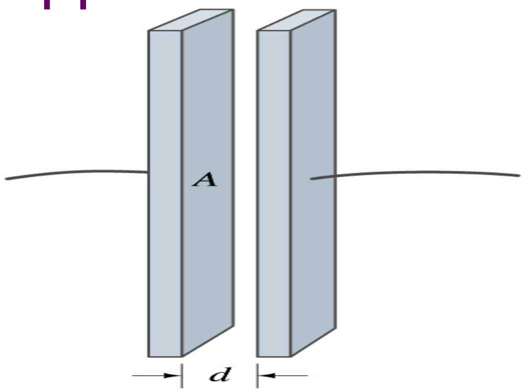
- What is the unit of electrostatic potential energy?
  - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot1V = 1.6\times10^{-19}J$
- eV however is **NOT a standard SI unit**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?

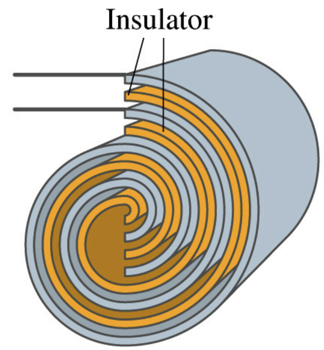
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge
  - But does not let them flow through
- What does it consist of?
  - Usually consists of two conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charge will neutralize…
- Can you give some examples?
  - Camera flash, UPS, Surge protectors, binary circuits, memory, etc…
- How is the capacitor different than the battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charges but very little energy.

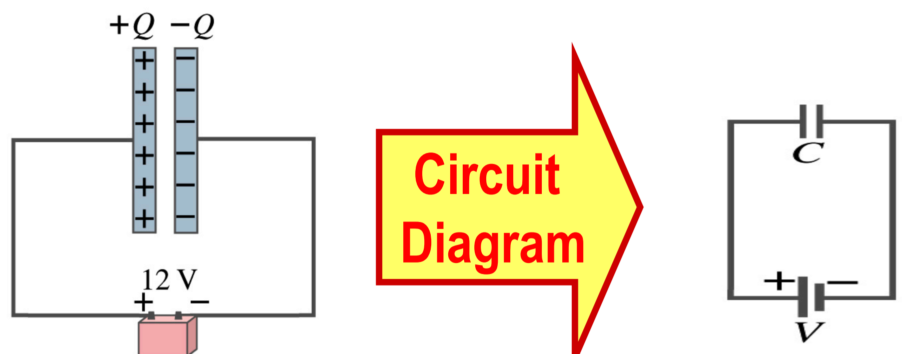
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



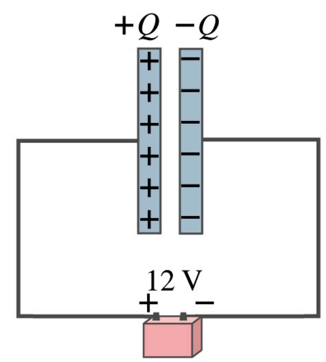


- How would you draw symbols for a capacitor and a battery in a circuit diagram?
  - Capacitor -||-
  - Battery (+) -|ı- (-)



Capacitors

- What do you think will happen if a battery is connected (or the voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative in equal amount.



- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential. And?
  - So the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored on each capacitor plate is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

$$Q = CV_{ba}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called the capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use µF or pF.