PHYS 1441 – Section 001
Lecture #14

Wednesday, June 27, 2018

Dr. Jaehoon Yu

- Chapter 27: Magnetism and Magnetic Field
  - Magnetic Forces on Electric Current
  - Magnetic Forces on a Moving Charge
  - Charged Particle Path in a Magnetic Field
  - Cyclotron Frequency
  - Torque on a Current Loop
  - Magnetic Dipole Moment
- Chapter 28:Sources of Magnetic Field
  - Sources of Magnetic Field
  - Magnetic Field Due to Straight Wire

Today's homework is #8, due 11pm, Monday, July 2!!

Announcements

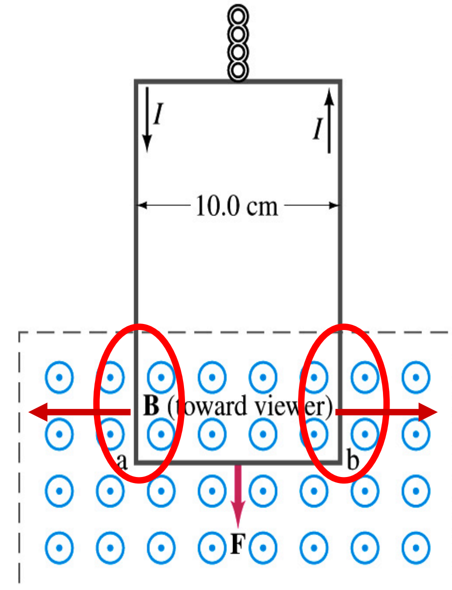
- Term exam 2 tomorrow, Thursday, June 28 in class
  - Non-comprehensive exam covers CH25.6 – what we finish today
  - BYOF
  - Will have a 30min class after the exam
- Class feedback survey at http://uta.mce.cc/
  - Bring your devices tomorrow for the survey after the exam
- Reading Assignments
  - CH28.6 – 10
- Quiz 3 results
  - Class average: 37.8/55
    - Equivalent to: 68.7/100
    - Previous quizzes: 35/100 and 49/100! Marked improvement!
  - Top score: 55/55

Magnetic Forces on Electric Current

- Magnetic field strength B can be defined using the proportionality relationship w/ the constant 1: $F = IlB\sin\theta$
- if θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$
- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of the wire carrying the current I when the wire is perpendicular to **B**
- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - l is the vector whose magnitude is the length of the wire in B and its direction is along the wire in the direction of the conventional current
  - This formula works only if **B** is uniform.
- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

Example 27 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire ab (length l=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of F=3.48x10^{-2}N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is $\vec{F} = I\vec{l} \times \vec{B}$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

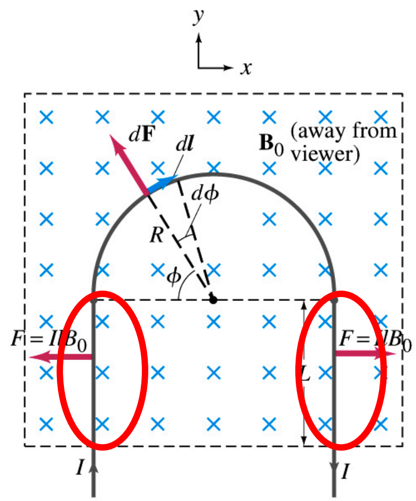
Solving for B $B = \frac{F}{Il} = \frac{3.48 \times 10^{-2}\,N}{0.245A \cdot 0.10m} = 1.42T$

Something is not right! What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.

Example 27 – 3

Magnetic force on a semi-circular wire. A rigid wire, carrying the current I, consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field $\mathbf{B_0}$. The straight portions each has length ℓ within the field. Determine the net force on the wire due to the magnetic field $\mathbf{B_0}$.



As in the previous example, the forces on the straight sections of the wire is equal and in opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle? $d\vec{F} = Id\vec{l} \times \vec{B}$

We divide the semicircle into infinitesimal straight sections. $dl = Rd\phi$

What is the net x component of the force exerting on the circular section? 0 **Why?**

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = d\left(F\sin\phi\right) = IRB_0 d\phi$

Integrating over φ=0 - π

$$F = \int_0^\pi d\left(F\sin\phi\right) = IB_0 R\int_0^\pi \sin\phi d\phi = -IB_0 R\left[\cos\phi\right]_0^\pi = 2RIB_0$$

Which direction?

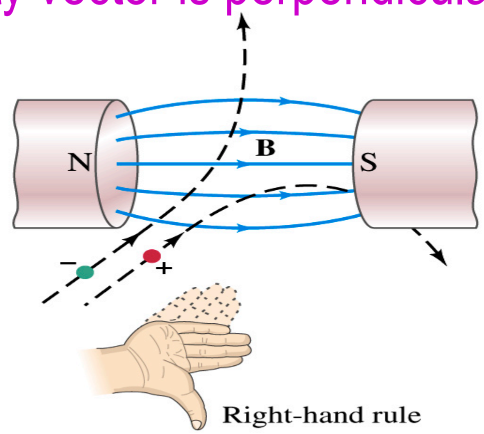
Vertically upward direction. The wire will be pulled deeper into the field.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since the wire carrying a current (moving charge) experiences force in a magnetic field, a freely moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by a given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for a charge q to travel a distance L in the magnetic field **B**
    - Then, the length vector ℓ becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\boxed{\vec{F} = q\vec{v} \times \vec{B}}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity v in a field B is
    - $F = qvB\sin\theta$
    - What is θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the angle between the field and the velocity vector is perpendicular.
    - $F_{max} = qvB \rightarrow B = \frac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 27 – 5

Magnetic force on a proton. A proton having a speed of $5x10^6$m/s in a magnetic field feels a force of $F=8.0x10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6\times10^{-19}C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

Since the particle feels force toward West, the field should be pointing to? North

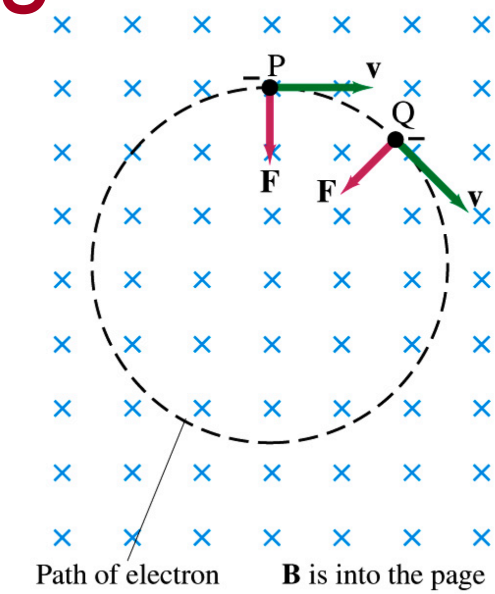
Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0\times10^{-14}N}{1.6\times10^{-19}C\cdot5.0\times10^{6}m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field

- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward



  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity (the speed) is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^7 m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\dfrac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

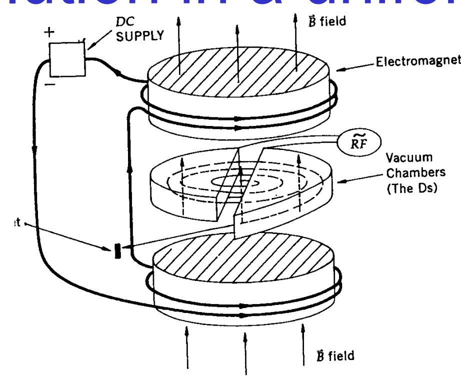
Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\dfrac{v^2}{r}$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ a constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



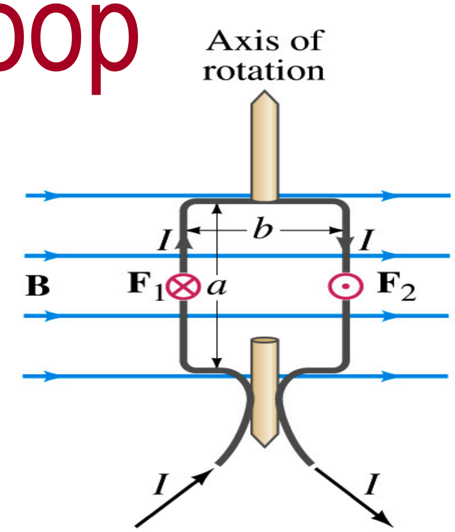
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

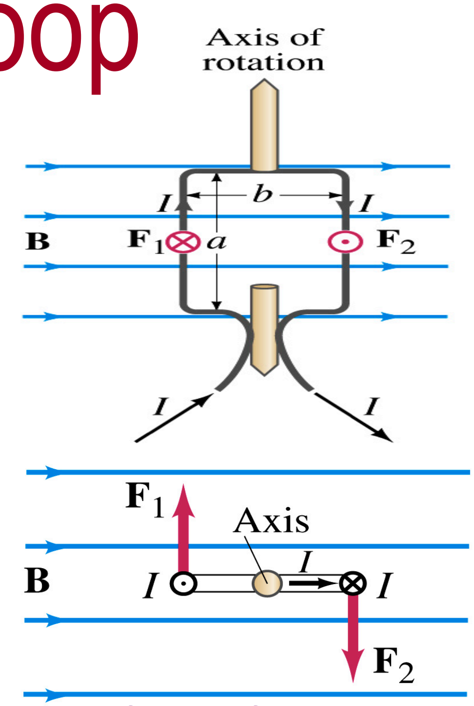
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with an electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length *a*?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

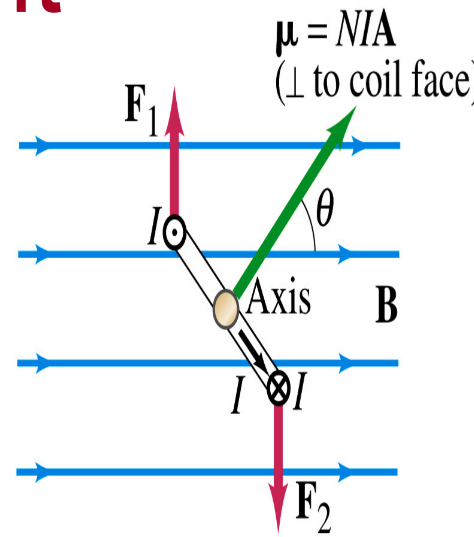
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A=ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity N$I\mathcal{A}$ is called the **magnetic dipole moment of the coil**
  - It is considered a vector $\vec{\mu} = NI\overrightarrow{A}$
    - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the same direction of the current
  - Using the definition of magnetic moment, the torque can be rewritten in vector form $\vec{\tau} = NI\overrightarrow{A} \times \vec{B} = \vec{\mu} \times \vec{B}$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at θ=π/2, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from $F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$

Solving for v

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^6\,m/s$$

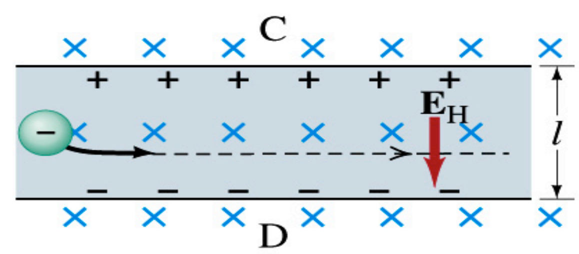
Since the electric current is the charge that passes through the given point per unit time, we can obtain the current $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

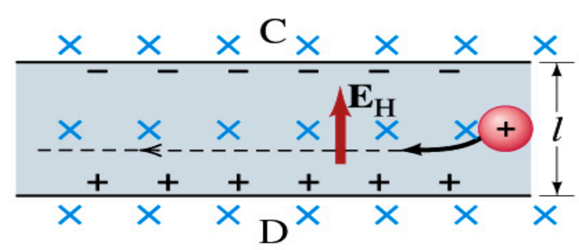
$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$

The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force



(a)

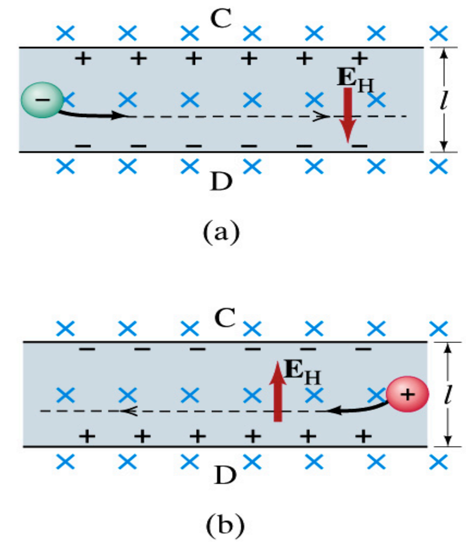


(b)

- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of charge is called the Hall field, $\mathbf{E}_H$, and it points to the direction opposite to the magnetic force

The Hall Effect

- In an equilibrium, the force due to Hall field is balanced by the magnetic force ev_dB, so we obtain
- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d Bl$
  - Where l is the width of the conductor



- What do we use the Hall effect for?
  - The current of negative charge moving to right is equivalent to the positive charge moving to the left
  - The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
  - Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the B-field strength
    - Hall probe