

PHYS 1441 – Section 001

Lecture #16

Monday, July 2, 2018

Dr. Animesh Chatterjee

- Chapter 28: Sources of Magnetic Field
 - Magnetic Field Due to Straight Wire
 - Magnetic Materials
 - Hysteresis
- Chapter 29: EM Induction & Faraday's Law
 - Induced EMF and EM Induction
 - Faraday's Law of Induction
 - Lenz's Law
 - Generation of Electricity

Today's homework is #9, due 11pm, Thursday, July 5!!



PHYS 1441 – Section 001

Lecture #17

Tuesday, July 3, 2018

Dr. Animesh Chatterjee

- Chapter 29: EM Induction & Faraday's Law
 - Transformer
 - Mutual and Self Inductance
 - Energy Stored in Magnetic Field
 - Alternating Current and AC Circuits
- Chapter 30: Inductance
 - Mutual and Self Inductance
 - Energy Stored in Magnetic Field

Today's homework is #10, due 11pm, Saturday, July 7!!

Thursday, July 5, 2018



PHYS 1444-001, Summer 2018
Dr. Jaehoon Yu

PHYS 1441 – Section 001

Lecture #18

Thursday, July 5, 2018

Dr. Animesh Chatterjee

- Chapter 30: Inductance
 - Energy Stored in Magnetic Field
 - Alternating Current and AC Circuits
 - AC Circuit W/ LRC
- Chapter 31: Maxwell's Equations
 - Expansion of Ampere's Law
 - Gauss' Law for Magnetism
 - Production of EM Waves
 - Light as EM Waves



Announcement

- Quiz #4
 - Beginning of the class Thursday, July 5
 - Covers CH 28.1 – what we finish Tuesday, July 3
 - BYOF
- Final Exam
 - In class 10:30 – 12:30, Monday, July 9
 - Comprehensive Exam: Covers CH21.1 – what we finish Thursday, July 5 + Math refresher Appendices
 - BYOF
- Reading Assignments:
 - Ch29.5, CH29.8, CH30.10 and CH30.11
- Be sure to bring your planetarium extra credit sheet
Monday, July 9

Thursday, July 5, 2018

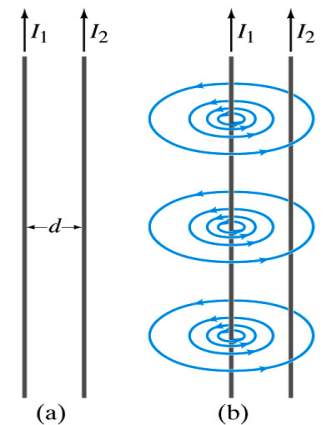


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Force Between Two Parallel Wires

- We have learned that a wire carrying the electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
 - They will exert force onto each other. Repel or attract?
 - Depending on the direction of the currents
- This was first pointed out by Ampère.
- Let's consider two long parallel conductors separated by a distance d , carrying currents I_1 and I_2 .
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is

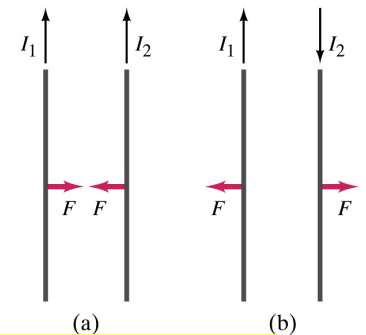
$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$



Force Between Two Parallel Wires

- The force F by a magnetic field B_1 on a wire of length l , carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
 - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi} \frac{I_1}{d}$
 - This force is only due to the magnetic field generated by the wire carrying the current I_1
 - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in opposite direction
- So the force per unit length is $\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$
- How about the direction of the force?

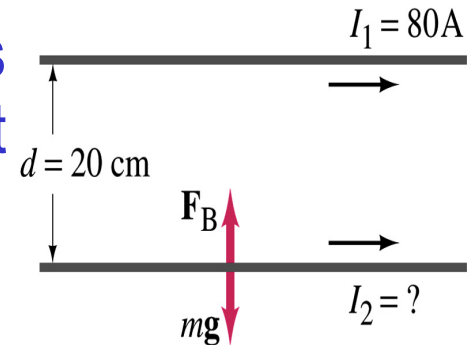
$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current $I_1=80\text{A}$ DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l} \frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi (9.8 \text{ m/s}^2) \cdot (0.12 \times 10^{-3} \text{ kg}) \cdot (0.20 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \cdot (80 \text{ A})} = 15 \text{ A}$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \frac{1\text{A} \cdot 1\text{A}}{1\text{m}} = 2 \times 10^{-7} \text{ N/m}$$

- So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2 \times 10^{-7} \text{ N/m}$.
- Coulomb is then defined as exactly $1\text{C} = 1\text{A} \cdot \text{s}$.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.

Ampère's Law

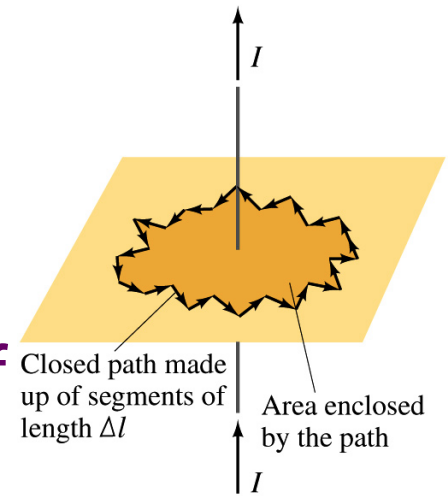
- What is the relationship between the magnetic field strength and the current?

$$B = \frac{\mu_0 I}{2\pi r}$$

 - Does this work in all cases?
 - Nope!
 - OK, then when?
 - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shapes of the wire?
 - French scientist André Marie Ampère proposed such a relationship soon after Oersted's discovery

Ampère's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
 - Let's split this path in small segments each of Δl long.
 - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path



- $$\sum B_{\parallel} \Delta l = \mu_0 I_{encl}$$

- In the limit $\Delta l \rightarrow 0$, this relation becomes

- $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

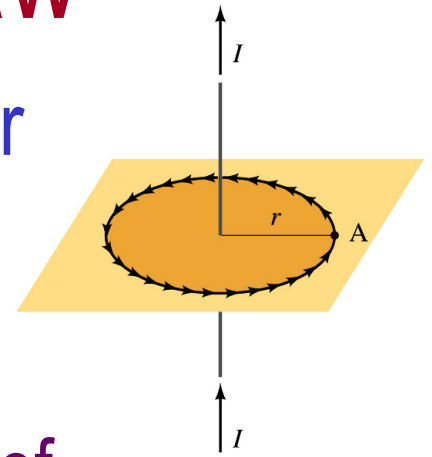
Ampère's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampère's Law

- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
 - This is a verification of Ampere's Law
 - We can apply Ampere's law to a circular path of radius r .



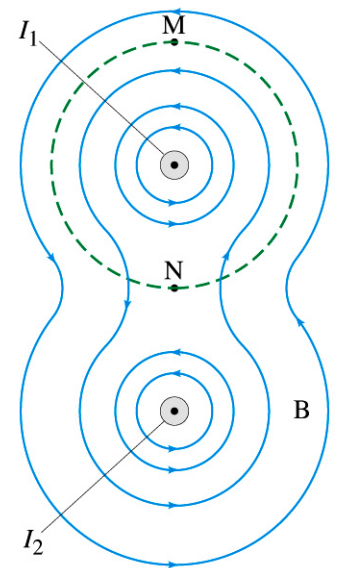
$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

Solving for B $\Rightarrow B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi} \frac{I}{r}$

- We just verified that Ampere's law works in a simple case
- Experiments verified that it works for other cases too
- The importance of this formula, however, is that it provides means to relate magnetic field to current

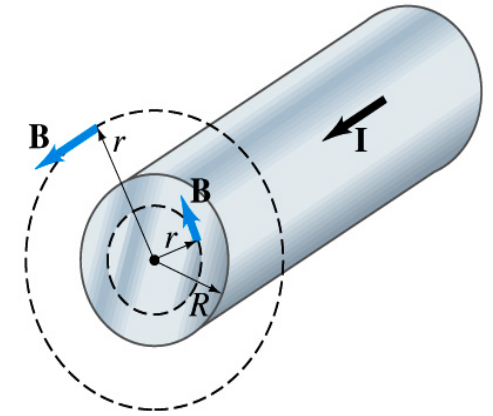
Verification of Ampère's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl} .
- B is the field at each point in space along the chosen path due to all sources
 - Including the current I enclosed by the path but also due to any other sources
 - How do you obtain B in the figure at any point?
 - Vector sum of the field by the two currents
 - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
 - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor ($r > R$) and (b) points inside the conductor ($r < R$). Assume that r , the radial distance from the axis, is much less than the length of the wire. (c) If $R = 2.0\text{mm}$ and $I = 60\text{A}$, what is B at $r = 1.0\text{mm}$, $r = 2.0\text{mm}$ and $r = 3.0\text{mm}$?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire ($r > R$). What is I_{encl} ? $I_{\text{encl}} = I$

So using Ampere's law

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

$B = \frac{\mu_0}{2\pi} \frac{I}{r}$

Example 28 – 6 cont'd

For $r < R$, the current inside the closed path is less than I .
How much is it?

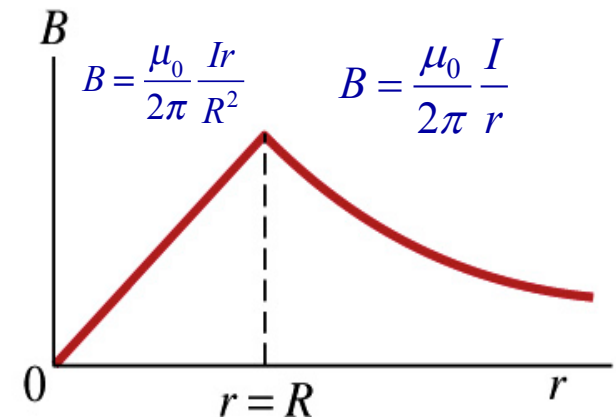
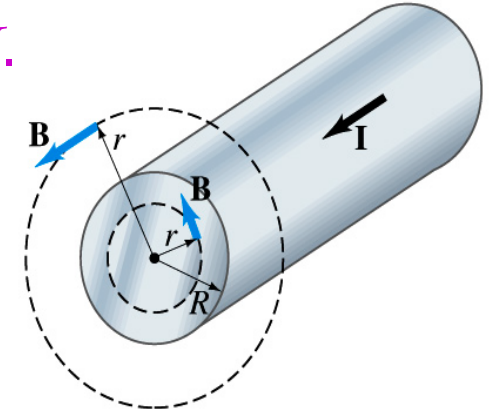
$$I_{encl} = I \frac{\pi r^2}{\pi R^2} = I \left(\frac{r}{R} \right)^2$$

So using Ampere's law

$$\mu_0 I \left(\frac{r}{R} \right)^2 = \oint \vec{B} \cdot d\vec{l} = 2\pi r B \quad \text{Solving for B} \quad B = \frac{\mu_0}{2\pi} \frac{I}{r} \left(\frac{r}{R} \right)^2 = \frac{\mu_0}{2\pi} \frac{Ir}{R^2}$$

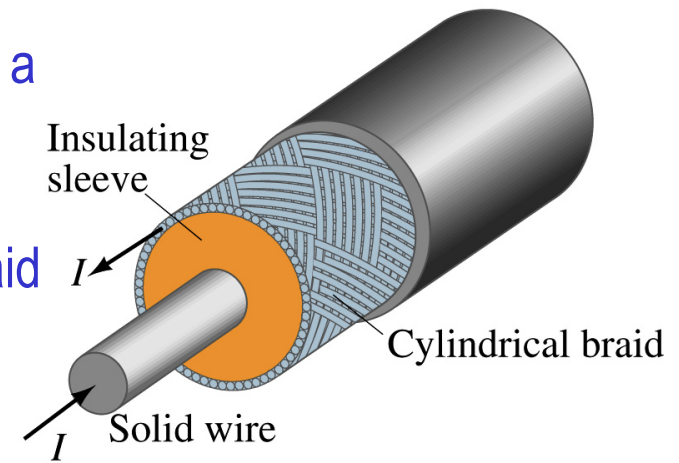
What does this mean?

The field is 0 at $r=0$ and increases linearly as a function of the distance from the center of the wire up to $r=R$ then decreases as $1/r$ beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

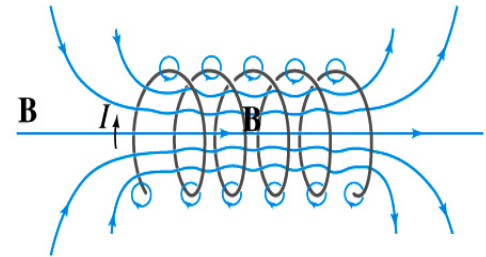
(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field

- What is a solenoid?

- A long coil of wire consisting of many loops
- If the space between loops are wide

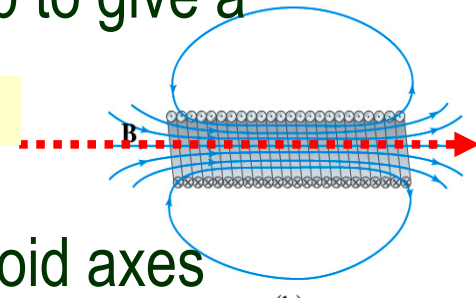


- The field near the wires are nearly circular
- Between any two wires, the fields due to each loop cancel
- Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform

- For a long, densely packed loops

- The field is nearly uniform and parallel to the solenoid axes within the entire cross section
- The field outside the solenoid is very small compared to the field inside, except the ends

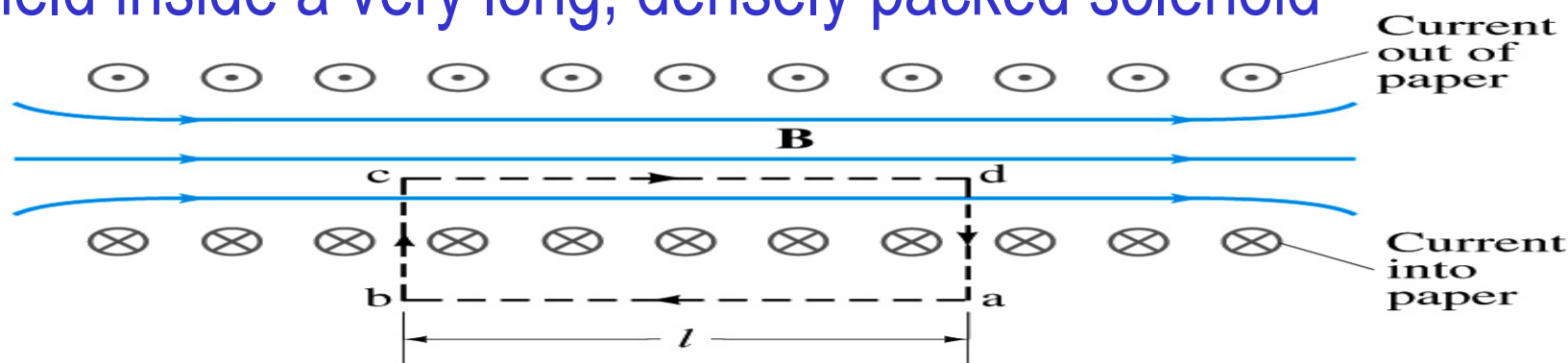
Solenoid Axis



- The same number of field lines spread out to an open space

Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



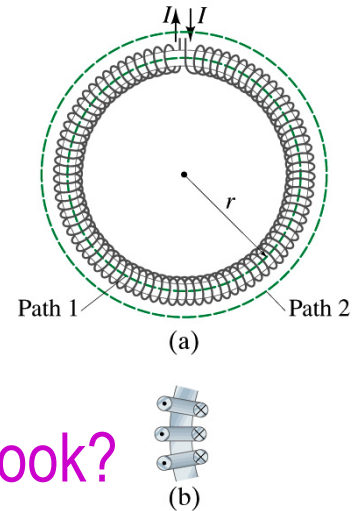
- Let's choose the path $ab cd$, far away from the ends
 - We can consider four segments of the loop for integral
 - $$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$
 - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
 - Now the field B is perpendicular to the bc and da segments. So these integrals become 0, also.

Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
 - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 \mathcal{N}I$
- If we let $n = \mathcal{N}/l$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $B = \mu_0 nI$
 - B depends on the number of loops per unit length, n , and the current I
 - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 10

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.



(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid. If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI \quad \xrightarrow{\text{Solving for } B} \quad B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The field in this case is $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

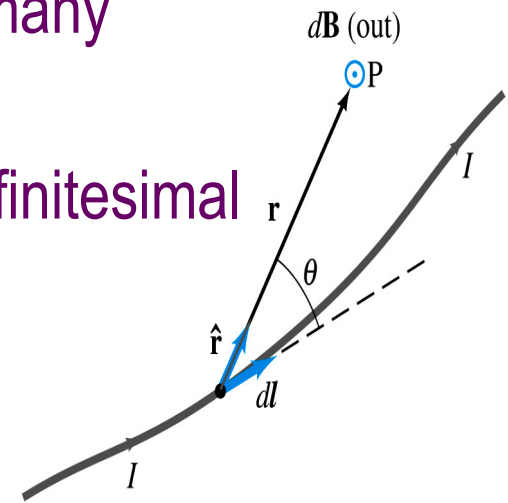
Biot-Savart Law

- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method of using infinitesimal current segments for B field
 - Jean Baptiste Biot and Felix Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
 - The infinitesimal magnetic field $d\mathbf{B}$ caused by the infinitesimal length $d\mathbf{l}$ that carries current I is

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

Biot-Savart Law

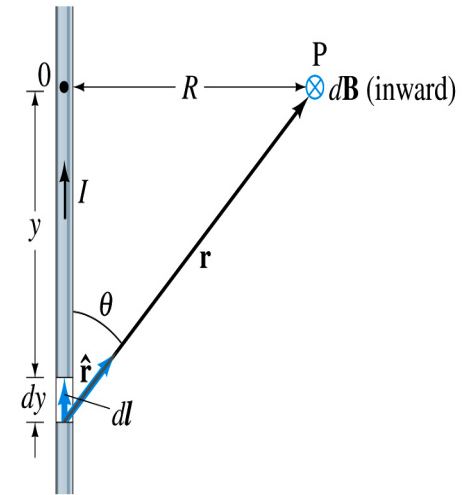
- $\mathbf{r}$ is the displacement vector from the element $d\mathbf{l}$ to the point P
- Biot-Savart law is the magnetic equivalent to Coulomb's law



B field in Biot-Savart law is only that by the current, nothing else.

Example 28 – 11

B due to current I in a straight wire. For the field near a long straight wire carrying a current I , show that the Biot-Savart law gives the same result as the simple long straight wire, $B = \mu_0 I / 2\pi R$.



What is the direction of the field **B** at point P? Going into the page.

All **dB** at point P has the same direction based on right-hand rule.

The magnitude of **B** using Biot-Savart law is

$$B = \oint dB = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{|d\vec{l} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{+\infty} \frac{dy \sin \theta}{r^2}$$

Where $dy = dl$ and $r^2 = R^2 + y^2$ and since $y = -R \cot \theta$ we obtain

$$dy = +R \csc^2 \theta d\theta = \frac{R d\theta}{\sin^2 \theta} = \frac{R d\theta}{(R/r)^2} = \frac{r^2 d\theta}{R}$$

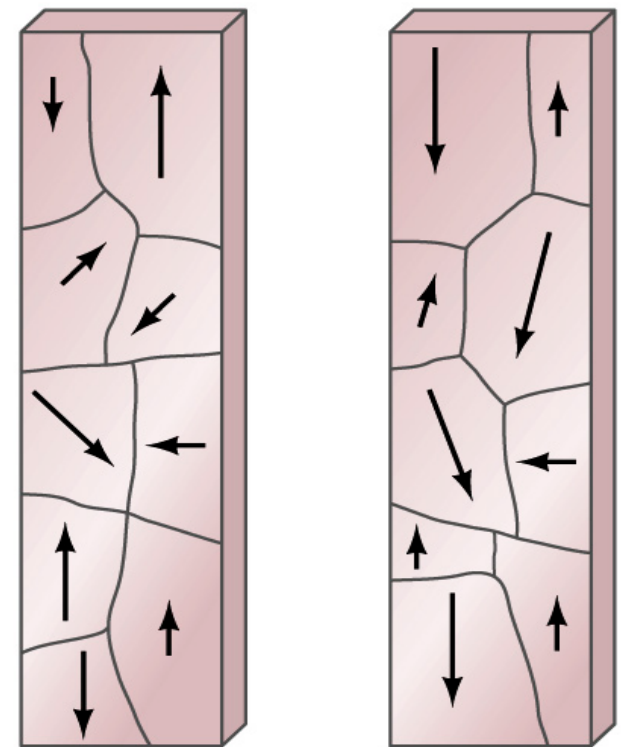
Integral becomes

$$B = \frac{\mu_0 I}{4\pi} \int_{y=-\infty}^{+\infty} \frac{dy \sin \theta}{r^2} = \frac{\mu_0 I}{4\pi} \frac{1}{R} \int_{\theta=0}^{\pi} \sin \theta d\theta = -\frac{\mu_0 I}{4\pi} \frac{1}{R} \cos \theta \Big|_0^{\pi} = \frac{\mu_0 I}{2\pi} \frac{1}{R}$$

Thursd The same as the simple, long straight wire!! It works!!

Magnetic Materials - Ferromagnetism

- Iron is a material that can turn into a strong magnet
 - This kind of material is called **ferromagnetic** material
- In microscopic sense, ferromagnetic materials consist of many tiny regions called **domains**
 - Domains are like little magnets usually smaller than 1mm in length or width
- What do you think the alignment of domains are like when they are not magnetized?
 - Randomly arranged
- What if they are magnetized?
 - The size of the domains aligned with the external magnetic field direction grows while those of the domains not aligned reduce
 - This gives magnetization to the material
- How do we demagnetize a bar magnet?
 - Hit the magnet hard or heat it over the Curie temperature



(b)

B in Magnetic Materials

- What is the magnetic field inside a solenoid?
- $B_0 = \mu_0 nI$
 - Magnetic field in a long solenoid is directly proportional to the current.
 - This is valid only if air is inside the coil
- What do you think will happen to B if we have something other than the air inside the solenoid?
 - It will be increased dramatically, when the current flows
 - Especially if a ferromagnetic material such as an iron is put inside, the field could increase by several orders of magnitude
- Why?
 - Since the domains in the iron aligns permanently by the external field.
 - The resulting magnetic field is the sum of that due to current and due to the iron

B in Magnetic Materials

- It is sometimes convenient to write the total field as the sum of two terms
- $\vec{B} = \vec{B}_0 + \vec{B}_M$
 - $\mathbf{B}_0$ is the field due only to the current in the wire, namely the external field
 - The field that would be present without a ferromagnetic material
 - $\mathbf{B}_M$ is the additional field due to the ferromagnetic material itself; often $\mathbf{B}_M \gg \mathbf{B}_0$
- The total field in this case can be written by replacing μ_0 with another proportionality constant μ , the magnetic permeability of the material $B = \mu nI$
 - μ is a property of a magnetic material
 - μ is not a constant but varies with the external field

Hysteresis

- What is a toroid?

- A solenoid bent into a shape

- Toroid can be used for magnetic field measurement

- Why?

- Since it does not leak magnetic field outside of itself, it fully contains all the magnetic field created within it.

- Consider an un-magnetized iron core toroid, without any current flowing in the wire

- What do you think will happen if the current slowly increases?

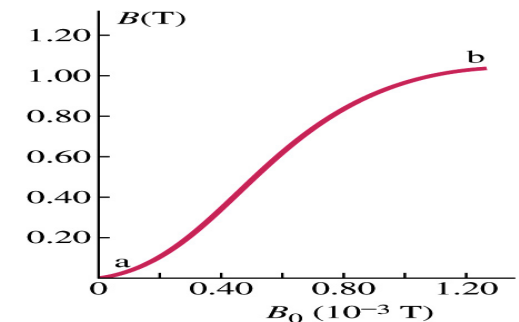
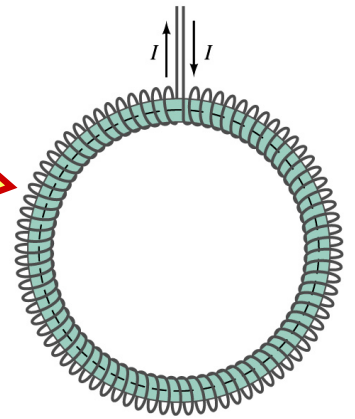
- B_0 increases linearly with the current.

- And B increases also but follows the curved line shown in the graph

- As B_0 increases, the domains become more aligned until nearly all are aligned (point b on the graph)

- The iron is said to be approaching saturation

- Point b is typically at 70% of the max



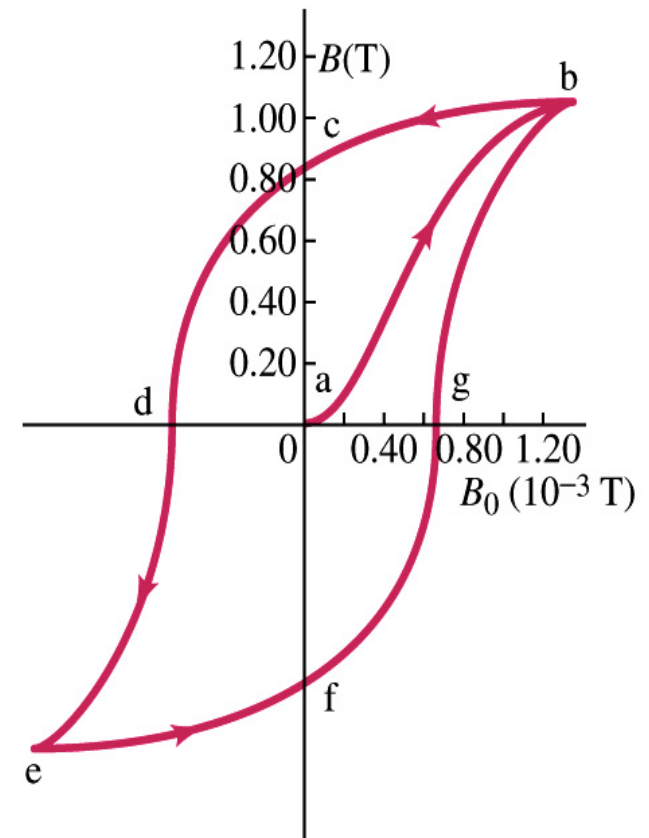
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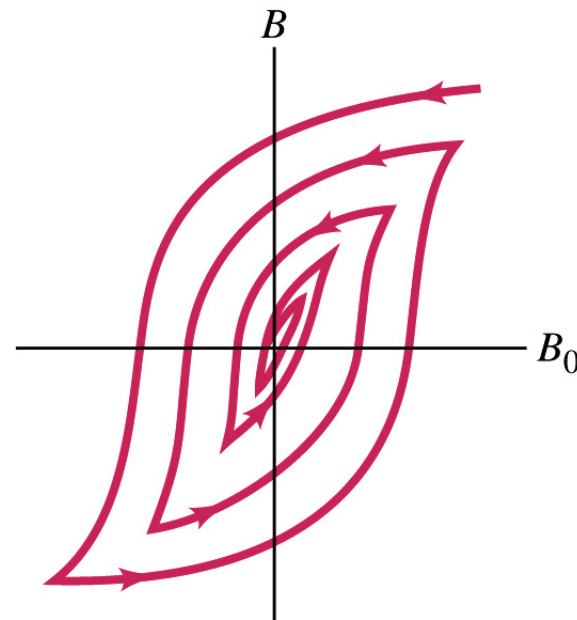
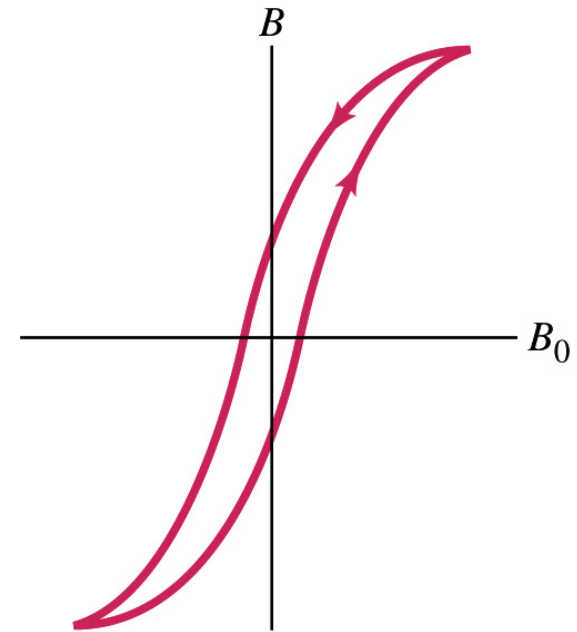
Hysteresis

- What do you think will happen to B if the external field B_0 is reduced to 0 by decreasing the current in the coil?
 - ~~Of course it goes to 0!!~~
 - Wrong! Wrong! Wrong! They do not go to 0. Why not?
 - The domains do not completely return to random alignment state
- Now if the current direction is reversed, the external magnetic field direction is reversed, causing the total field B pass 0, and the direction reverses to the opposite side
 - If the current is reversed again, the total field B will increase but never goes through the origin
- This kind of curve whose path does not retrace themselves and does not go through the origin is called the **Hysteresis**.



Magnetically Soft Material

- In a hysteresis cycle, much energy is transformed to thermal energy. Why?
 - Due to the microscopic friction between domains as they change directions to align with the external field
- The energy dissipated in the hysteresis cycle is proportional to the area of the hysteresis loop
- Ferromagnetic material with large hysteresis area is called magnetically hard while the small ones are called soft
 - Which one do you think are preferred in electromagnets or transformers?
 - Soft. Why?
 - Since the energy loss is small and much easier to switch off the field
- Then how do we demagnetize a ferromagnetic material?
 - Keep repeating the Hysteresis loop, reducing the range of B_0 .



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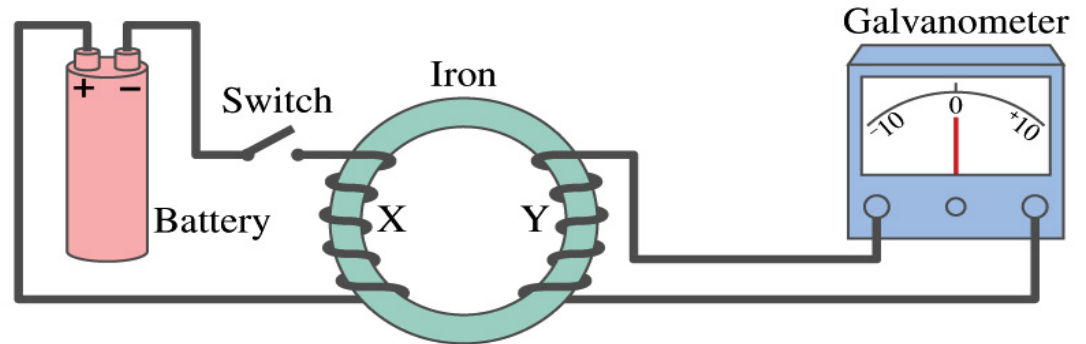
Induced EMF

- It has been discovered by Oersted and company in early 19th century that
 - Magnetic field can be produced by the electric current
 - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
 - Yes, you are absolutely right! You would wonder if the magnetic field can create the electric current.
 - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
 - Though, Faraday was given the credit since he published his work before Henry did
 - He also did a lot of detailed studies on magnetic induction



Electromagnetic Induction

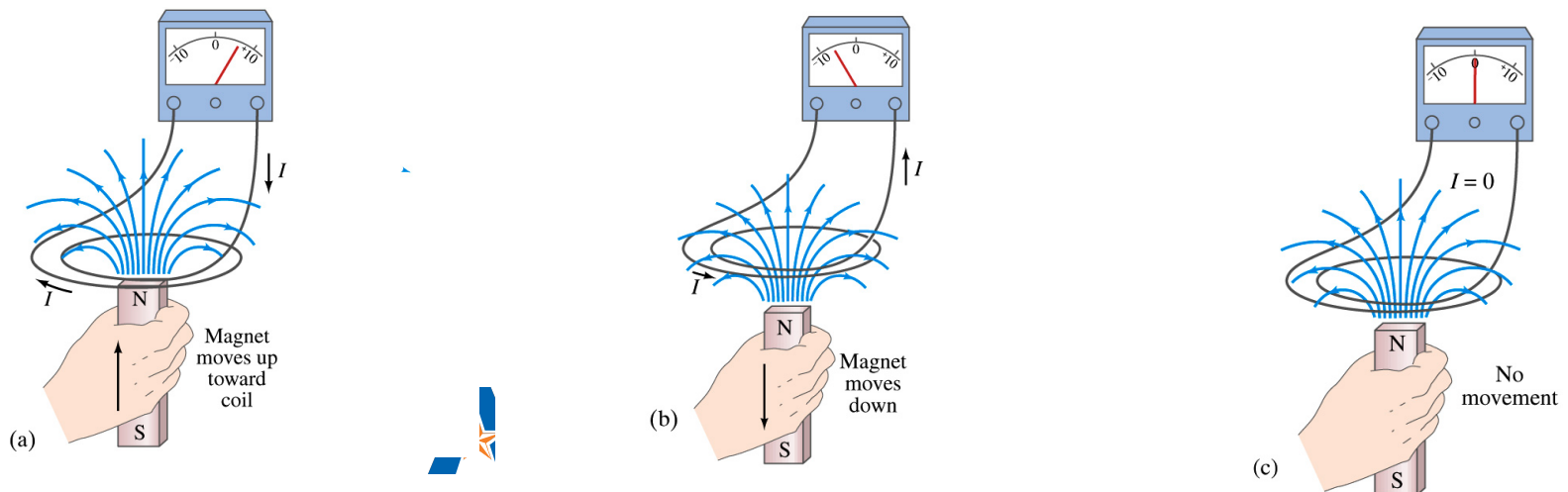
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
 - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

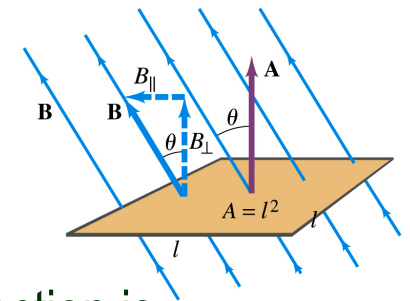
Electromagnetic Induction

- Further studies on electromagnetic induction taught
 - If a magnet is moved quickly into a coil of wire, a current is induced in the wire.
 - If a magnet is removed from the coil, a current is induced in the wire in the opposite direction
 - By the same token, the current can also be induced if the magnet stays put but the coil moves toward or away from the magnet
 - Current is also induced if the coil rotates.
- In other words, it does not matter whether the magnet or the coil moves. It is the relative motion that counts.



Magnetic Flux

- So what do you think is the induced emf proportional to?
 - The rate of changes of the magnetic field?
 - the higher the changes the higher the induction
 - Not really, it rather depends on the rate of change of the **magnetic flux**, Φ_B .
 - Magnetic flux is defined as (just like the electric flux)
 - $\Phi_B = B_{\perp} A = BA \cos \theta = \vec{B} \cdot \vec{A}$
 - θ is the angle between $\mathbf{B}$ and the area vector $\mathbf{A}$ whose direction is perpendicular to the face of the loop based on the right-hand rule
 - What kind of quantity is the magnetic flux?
 - Scalar. Unit?
 - $T \cdot m^2$ or weber
- If the area of the loop is not simple or B is not uniform, the magnetic flux can be written as



$$1Wb = 1T \cdot m^2$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

Faraday's Law of Induction

- In terms of magnetic flux, we can formulate Faraday's findings
 - The emf induced in a circuit is equal to the rate of change of magnetic flux through the circuit

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Faraday's Law of Induction

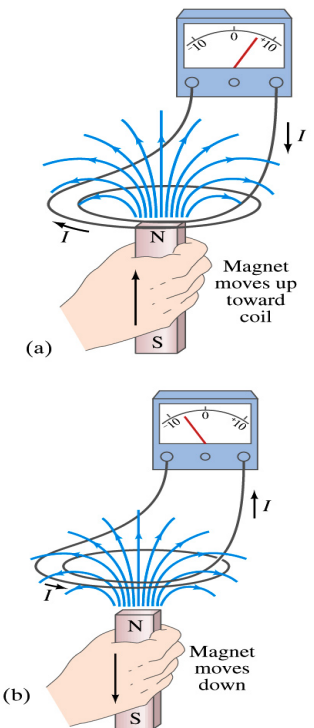
- If the circuit contains N closely wrapped loops, the total induced emf is the sum of emf induced in each loop

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- Why negative?
 - Has got a lot to do with the direction of induced emf...

Lenz's Law

- It is experimentally found that
 - An induced emf gives rise to a current whose magnetic field opposes the original change in flux → This is known as **Lenz's Law**
 - In other words, an induced emf is always in a direction that opposes the original change in flux that caused it.
 - We can use Lenz's law to explain the following cases in the figures
 - When the magnet is moving into the coil
 - Since the external flux increases, the field inside the coil takes the opposite direction to minimize the change and causes the current to flow clockwise
 - When the magnet is moving out
 - Since the external flux decreases, the field inside the coil takes the opposite direction to compensate the loss, causing the current to flow counter-clockwise- Which law is Lenz's law result of?
 - Energy conservation. Why?



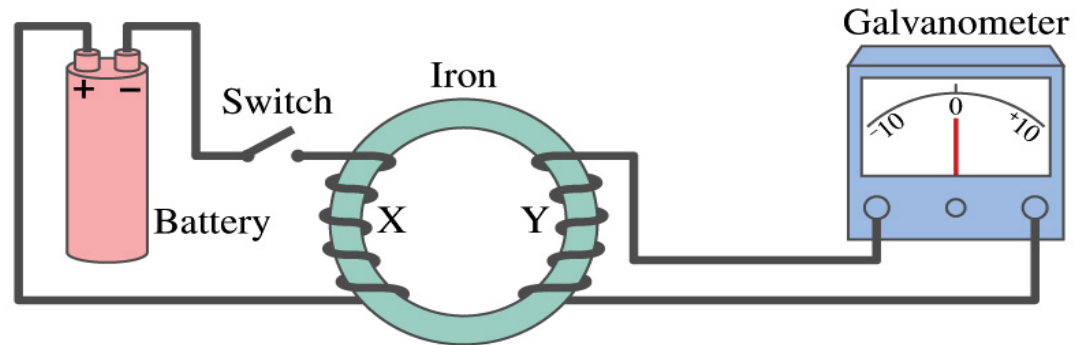
Induced EMF

- It has been discovered by Oersted and company in early 19th century that
 - Magnetic field can be produced by the electric current
 - Magnetic field can exert force on the electric charge
- So if you were scientists at that time, what would you wonder?
 - Yes, you are absolutely right! You would wonder if the magnetic field can create the electric current.
 - An American scientist Joseph Henry and an English scientist Michael Faraday independently found that it was possible
 - Though, Faraday was given the credit since he published his work before Henry did
 - He also did a lot of detailed studies on magnetic induction



Electromagnetic Induction

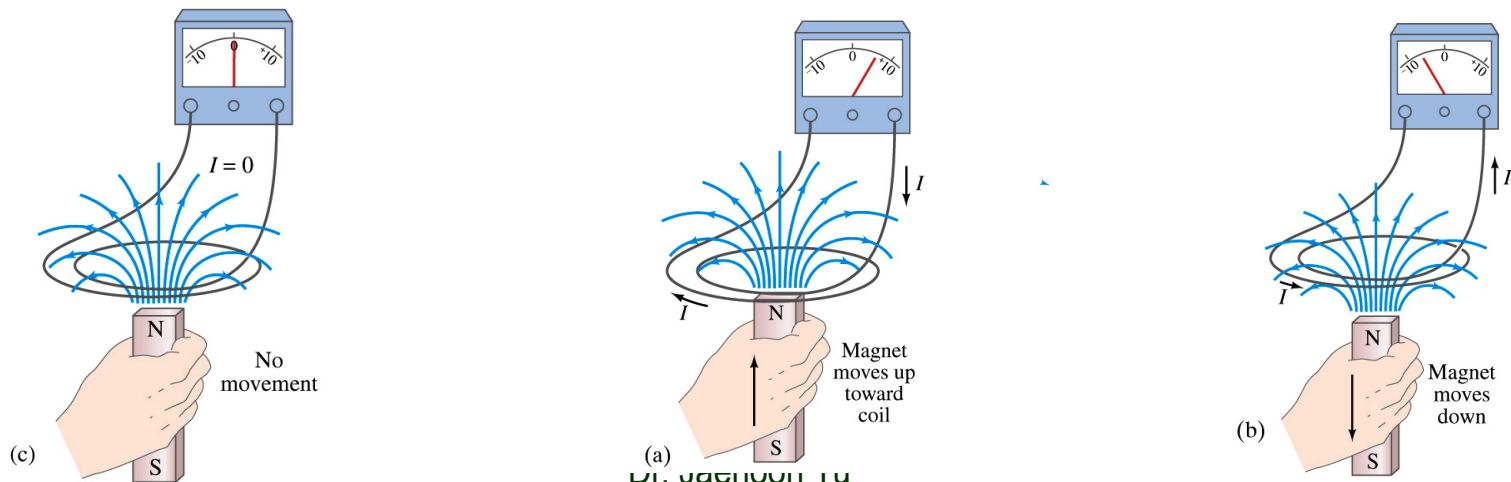
- Faraday used an apparatus below to show that magnetic field can induce current



- Despite his hope he did not see steady current induced on the other side when the switch is thrown
- But he did see that the needle on the Galvanometer turns strongly when the switch is initially thrown and is opened
 - When the magnetic field through coil Y changes, a current flows as if there were a source of emf
- Thus he concluded that **an induced emf is produced by a changing magnetic field** → **Electromagnetic Induction**

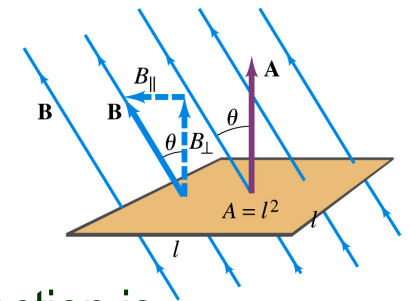
Electromagnetic Induction

- Further studies on electromagnetic induction taught
 - If a magnet is moved quickly into a coil of wire, a current is induced in the wire.
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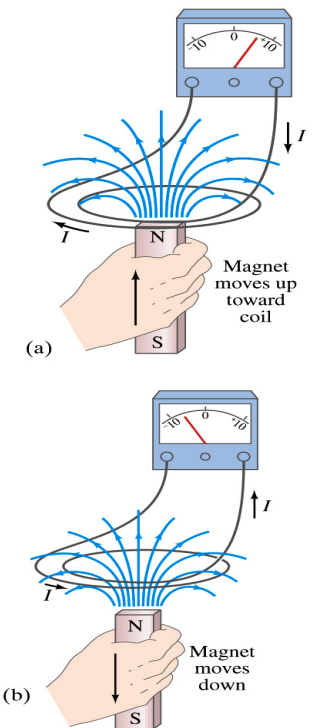
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 - Has got a lot to do with the direction of induced emf...

Lenz's Law

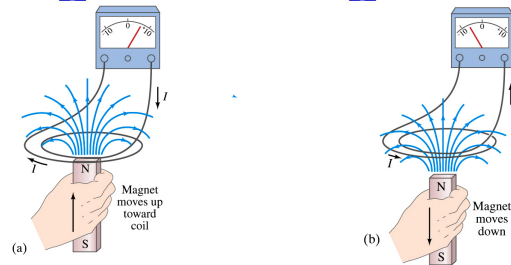
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 - Energy conservation. Why?



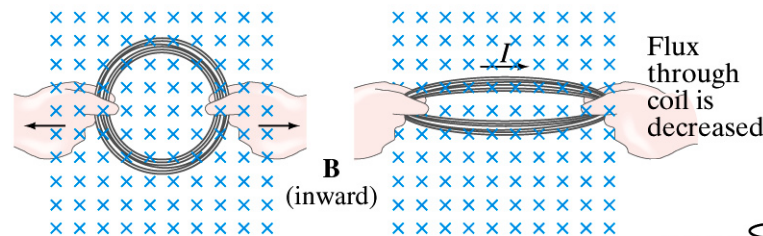
Induction of EMF

- How can we induce emf?
- Let's look at the formula for the magnetic flux
- $\Phi_B = \int \vec{B} \cdot d\vec{A} = \int B \cos \theta dA$
- What do you see? What are the things that can change with time to result in change of magnetic flux?

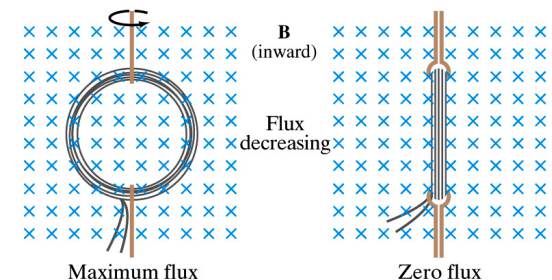
- Magnetic field



- The area of the loop



- The angle θ between the field and the area vector



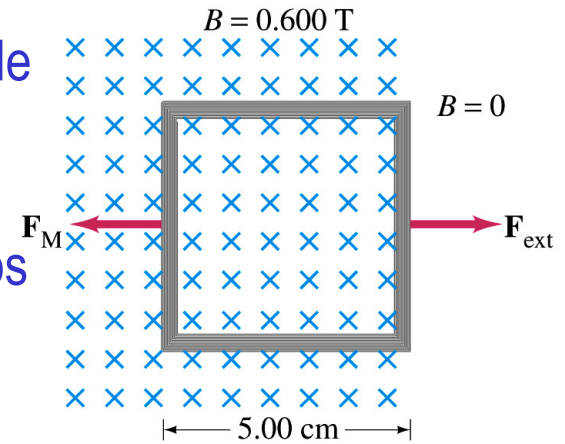
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Example 29 – 5

Pulling a coil from a magnetic field. A square coil of wire with side 5.00cm contains 100 loops and is positioned perpendicular to a uniform 0.600-T magnetic field. It is quickly and uniformly pulled from the field (moving perpendicular to B) to a region where B drops abruptly to zero. At $t=0$, the right edge of the coil is at the edge of the field. It takes 0.100s for the whole coil to reach the field-free region. Find (a) the rate of change in flux through the coil, (b) the emf and the current induced, and (c) how much energy is dissipated in the coil if its resistance is 100Ω . (d) what was the average force required?



What should be computed first? The initial flux at $t=0$.

The flux at $t=0$ is $\Phi_B = \vec{B} \cdot \vec{A} = BA = 0.600T \cdot (5 \times 10^{-2}m)^2 = 1.50 \times 10^{-3} Wb$

The change of flux is $\Delta\Phi_B = 0 - 1.50 \times 10^{-3} Wb = -1.50 \times 10^{-3} Wb$

Thus the rate of change of the flux is

$$\frac{\Delta\Phi_B}{\Delta t} = \frac{-1.50 \times 10^{-3} Wb}{0.100s} = -1.50 \times 10^{-2} Wb/s$$

Example 29 – 5, cnt'd

Thus the total emf induced in this period is

$$\varepsilon = -N \frac{d\Phi_B}{dt} = -100 \cdot (-1.50 \times 10^{-2} \text{ Wb/s}) = 1.5V$$

The induced current in this period is

$$I = \frac{\varepsilon}{R} = \frac{1.5V}{100\Omega} = 1.50 \times 10^{-2} \text{ A} = 15.0 \text{ mA}$$

Which direction would the induced current flow? Clockwise

The total energy dissipated is

$$E = Pt = I^2 R t = (1.50 \times 10^{-2} \text{ A})^2 \cdot 100\Omega \cdot 0.100 \text{ s} = 2.25 \times 10^{-3} \text{ J}$$

Force for each coil is $\vec{F} = I\vec{l} \times \vec{B}$ Force for N coil is $\vec{F} = NI\vec{l} \times \vec{B}$

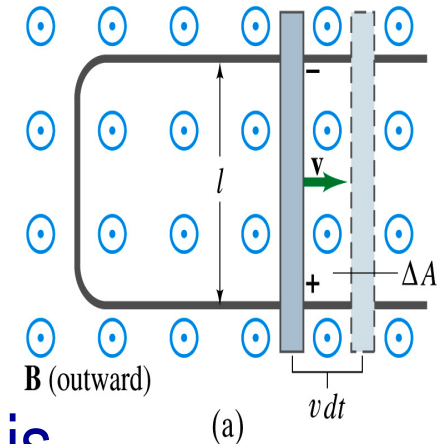
$$|F| = NIlB = 100 \cdot (1.50 \times 10^{-2} \text{ A}) \cdot (4 \times 5 \times 10^{-2}) \cdot 0.600 \text{ T} = 0.045 \text{ N}$$

EMF Induced on a Moving Conductor

- Another way of inducing emf is using a U shaped conductor with a movable rod resting on it.
- As the rod moves at a speed v , it travels vdt in time dt , changing the area of the loop by $dA = \ell v dt$.
- Using Faraday's law, the induced emf for this loop is

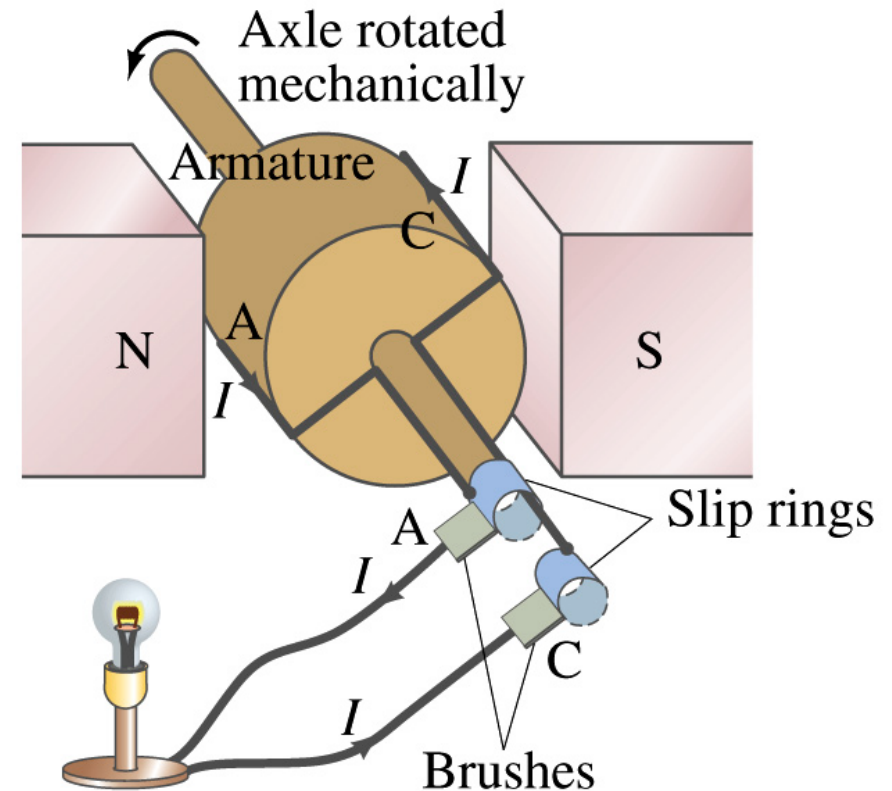
$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

- This equation is valid as long as B , ℓ and v are perpendicular to each other. What do we do if not?
 - Use the scalar product of vector quantities
- An emf induced on a conductor moving in a magnetic field is called a **motional emf**



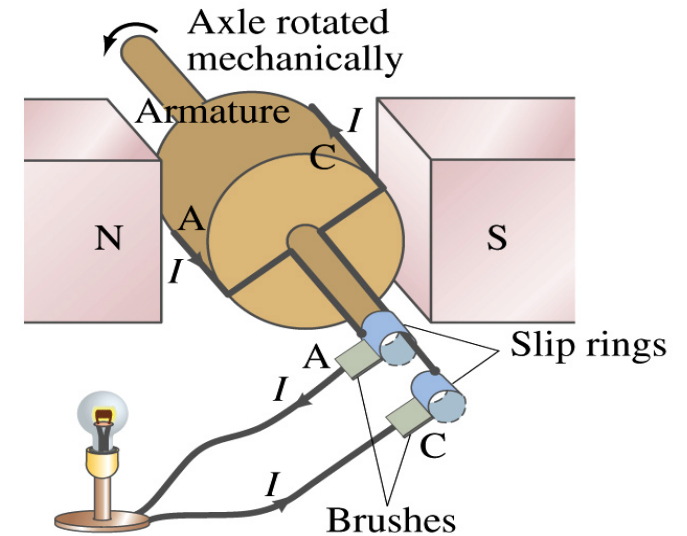
Electric Generators

- What does a generator do?
 - Transforms mechanical energy into the electrical energy
 - What does this look like?
 - An inverse of an electric motor which transforms electrical energy to mechanical energy
 - An electric generator is also called a dynamo
- Whose law does the generator based on?
 - Faraday's law of induction



How does an Electric Generator work?

- An electric generator consists of
 - Many coils of wires wound on an armature that can rotate by mechanical means in a magnetic field
- An emf is induced in the rotating coil
- Electric current is the output of a generator
- Which direction does the output current flow when the armature rotates counterclockwise?
 - The conventional current flows outward on wire A toward the brush
 - After half the revolution the wire A will be where the wire C is and the current flow on A is reversed
- Thus the current produced is alternating its direction



How does an Electric Generator work?

- Let's assume the loop is rotating in a uniform B field w/ a constant angular velocity ω . The induced emf is

- $$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} = -\frac{d}{dt} [BA \cos \theta]$$

- What is the variable that changes above?

- The angle θ . What is $d\theta/dt$?

- The angular speed ω .

- So $\theta = \theta_0 + \omega t$

- If we choose $\theta_0 = 0$, we obtain

- $$\varepsilon = -BA \frac{d}{dt} [\cos \omega t] = BA\omega \sin \omega t$$

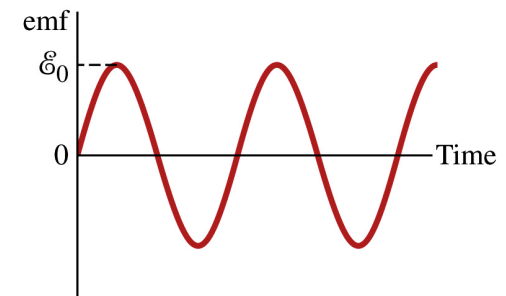
- If the coil contains N loops:
$$\varepsilon = -N \frac{d\Phi_B}{dt} = NBA\omega \sin \omega t = \varepsilon_0 \sin \omega t$$

- What is the shape of the output?

- Sinusoidal w/ the amplitude $\varepsilon_0 = NBA\omega$

- USA frequency is 60Hz. Europe is at 50Hz

- Most the U.S. power is generated at steam plants



Example 29 – 9

An AC generator. The armature of a 60-Hz AC generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0 \times 10^{-2} \text{m}^2$, how many loops must the coil contain if the peak output is to be $\epsilon_0 = 170 \text{V}$?

The maximum emf of a generator is $\epsilon_0 = NBA\omega$

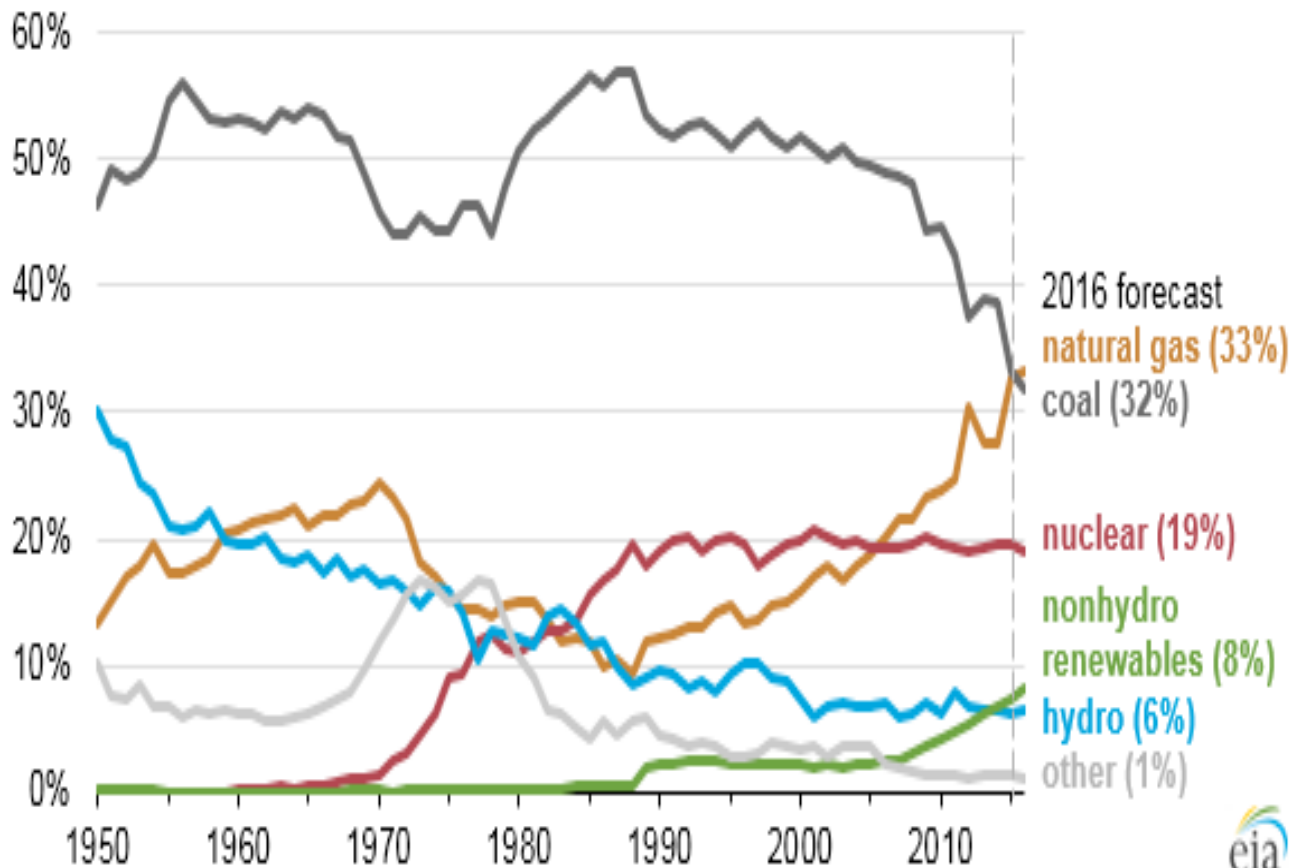
Solving for N $N = \frac{\epsilon_0}{BA\omega}$

Since $\omega = 2\pi f$ We obtain

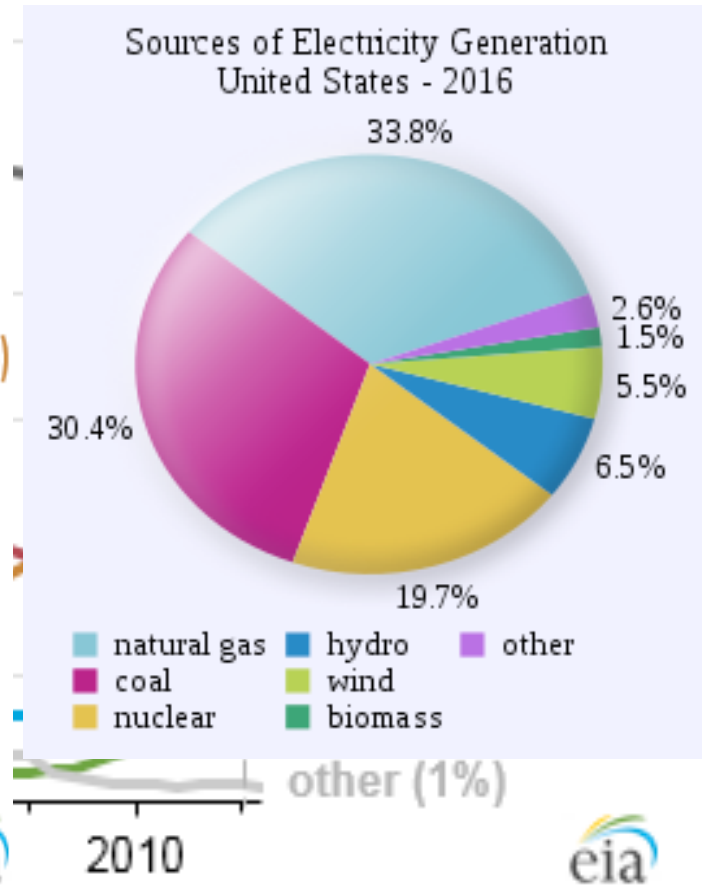
$$N = \frac{\epsilon_0}{2\pi BAf} = \frac{170 \text{V}}{2\pi \cdot (0.15 \text{T}) \cdot (2.0 \times 10^{-2} \text{m}^2) \cdot (60 \text{s}^{-1})} = 150 \text{turns}$$

US Electricity Sources

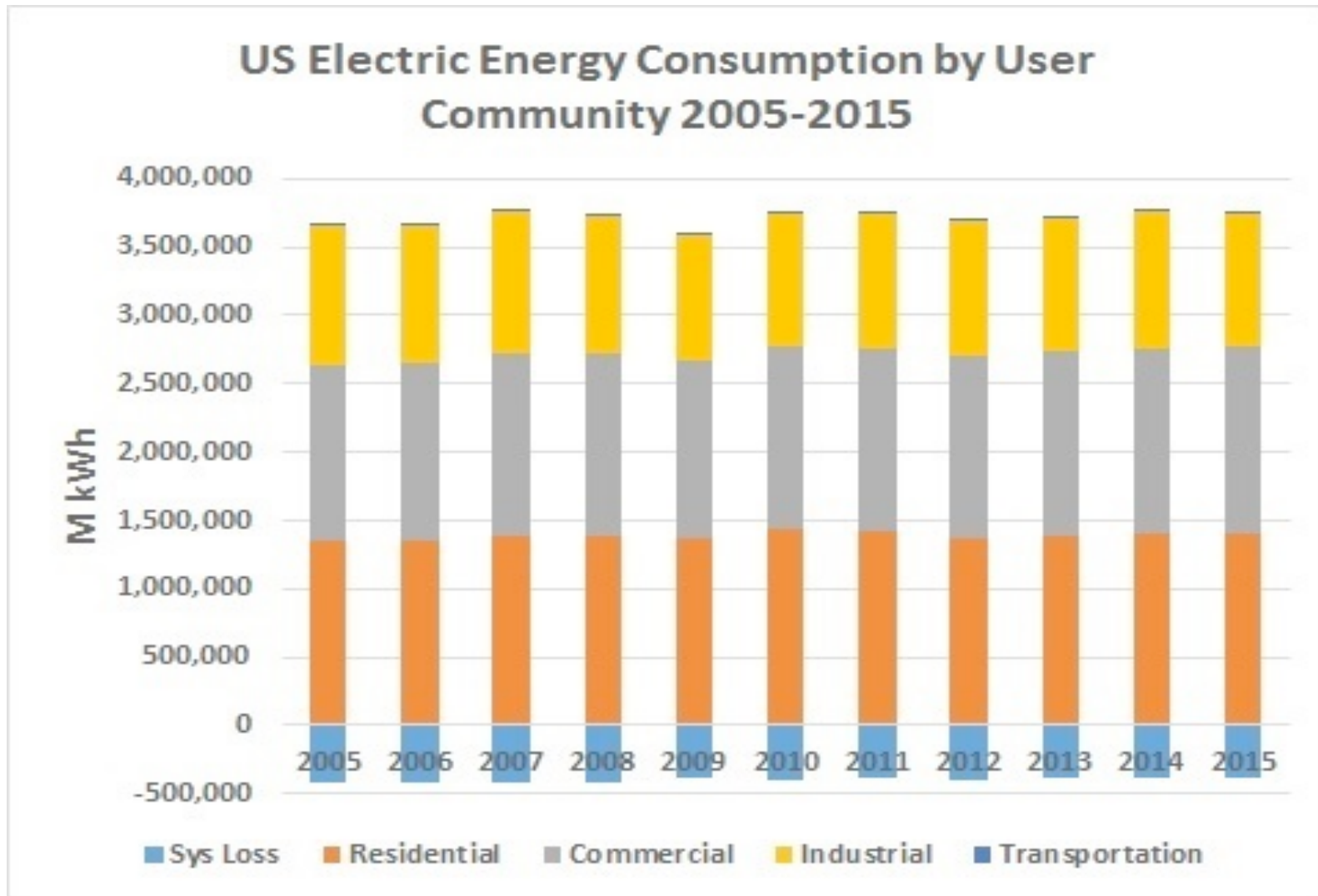
Annual share of total U.S. electricity generation by source (1950-2016)
percent of total



1950-2016)



US Electric E Consumption by Users



US Energy Information Administration <http://www.eia.gov/electricity/>

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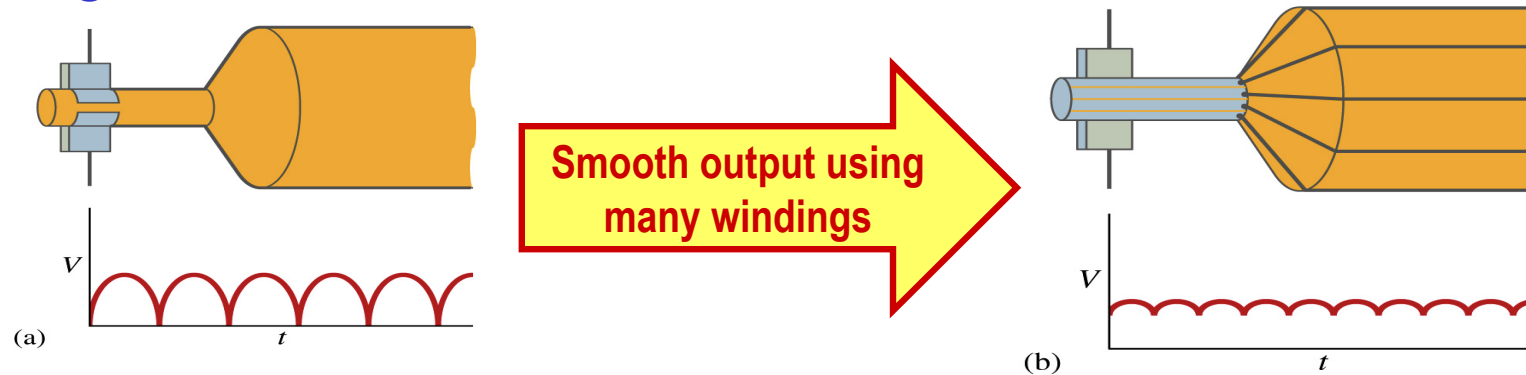
The World Energy Consumption

- In 2013, total worldwide energy consumption was 567 EJ (567×10^{18} J = 157 PWh) → expected >1000 EJ by 2050
 - Equivalent to an average energy consumption rate of 18 terawatts (1.8×10^{13} W)
 - US uses 39.1 PWh (1.38 kWh/person, as of 2014)
- The potential for renewable energy
 - solar energy 1600 EJ (444,000 TWh)
 - wind power 600 EJ (167,000 TWh)
 - geothermal energy 500 EJ (139,000 TWh),
 - biomass 250 EJ (70,000 TWh)
 - hydropower 50 EJ (14,000 TWh) and
 - ocean energy 1 EJ (280 TWh)
 - Read [this paper if you want to learn more](#)



A DC Generator

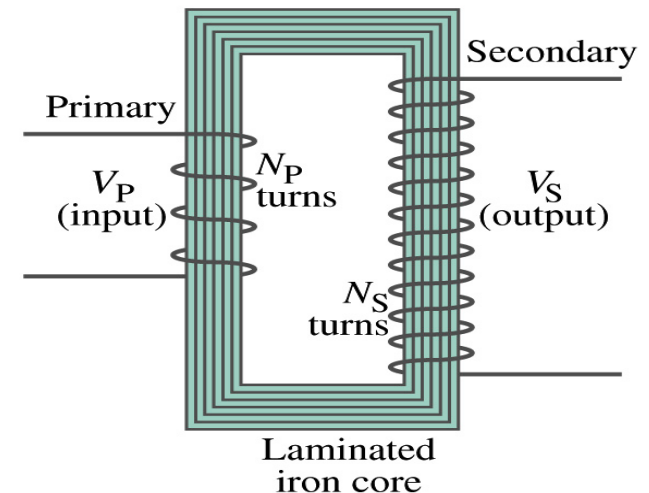
- A DC generator is almost the same as an AC generator except the slip rings are replaced by split-ring commutators



- Output can be smoothed out by placing a capacitor in parallel to the output
 - More commonly done using many armature windings

Transformer

- What is a transformer?
 - A device for increasing or decreasing an AC voltage
 - A few examples?
 - TV sets to provide the high voltage to picture tubes, portable electronic device converters, transformers on the pole, etc
- A transformer consists of two coils of wires known as the primary and the secondary
 - The two coils can be interwoven or linked by a laminated soft iron core to reduce losses due to Eddy current
- Transformers are designed so that all magnetic flux produced by the primary coil pass through the secondary



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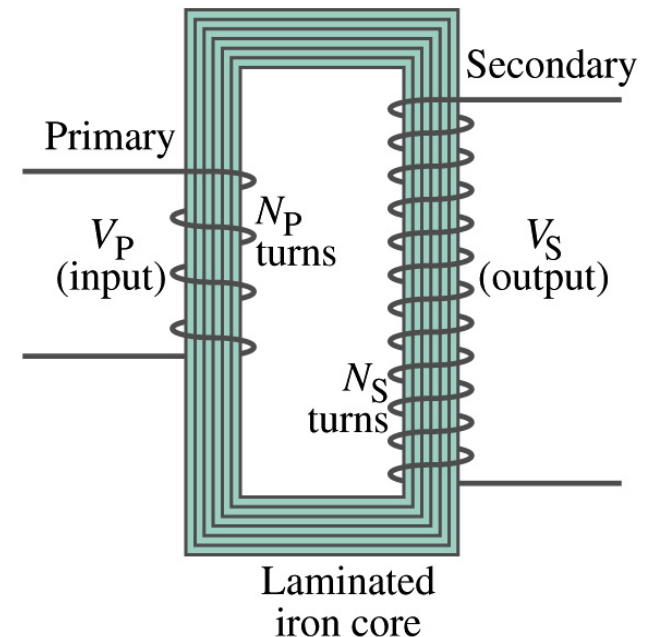
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How does a transformer work?

- When an AC voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary wire
- So how would we make the voltage different?
 - By varying the number of loops in each coil
 - From Faraday's law, the induced emf in the secondary is
 - $V_S = N_S \frac{d\Phi_B}{dt}$
 - The input primary voltage is
 - $V_P = N_P \frac{d\Phi_B}{dt}$
 - Since $d\Phi_B/dt$ is the same, we obtain

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Transformer
Equation



Transformer Equation

- The transformer equation does not work for DC current
 - Since there is no change of magnetic flux!!
- If $N_S > N_P$, the output voltage is greater than the input so it is called a step-up transformer while $N_S < N_P$ is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?
 - Wrong! Wrong! Wrong! Energy is always conserved!
 - A well designed transformer can be more than 99% efficient
 - The power output is the same as the input:

- $V_P I_P = V_S I_S$

- $\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}$

The output current for a step-up transformer will be lower than the input, while it is larger for a step-down x-former than the input.

Example for A Transformer

Portable radio transformer. A transformer for home use of a portable radio reduces 120-V AC to 9.0V AC. The secondary contains 30 turns, and the radio draws 400mA. Calculate (a) the number of turns in the primary (b) the current in the primary and (c) the power transformed.

(a) What kind of a transformer is this? A step-down x-former

Since $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ We obtain $N_P = N_S \frac{V_P}{V_S} = 30 \frac{120V}{9V} = 400 \text{ turns}$

(b) Also from the transformer equation $\frac{I_S}{I_P} = \frac{V_P}{V_S}$ We obtain $I_P = I_S \frac{V_S}{V_P} = 0.4A \frac{9V}{120V} = 0.03A$

(c) Thus the power transformed is

$$P = I_S V_S = (0.4A) \cdot (9V) = 3.6W$$

How about the input power? The same assuming 100% efficiency.

Example 29 – 13: Power Transmission

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω . Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$.

(a) If 120kW is sent at 240V, the total current is $I = \frac{P}{V} = \frac{120 \times 10^3}{240} = 500 A$.

Thus the power loss due to transmission line is

$$P = I^2 R = (500 A)^2 \cdot (0.4 \Omega) = 100 kW$$

(b) If 120kW is sent at 24,000V, the total current is $I = \frac{P}{V} = \frac{120 \times 10^3}{24 \times 10^3} = 5.0 A$.

Thus the power loss due to transmission line is

$$P = I^2 R = (5 A)^2 \cdot (0.4 \Omega) = 10 W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
 - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field

- $\mathcal{E} = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$

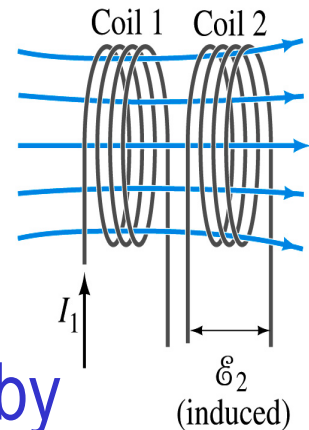
- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
 - A changing current in one circuit must induce an emf in a nearby circuit → Mutual inductance
 - Or induce an emf in itself → Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2 , in coil2 proportional to?
 - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $M_{21} = N_2\Phi_{21}/I_1$.
- The emf induced in coil2 due to the changing current in coil1 is



$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d(N_2\Phi_{21})}{dt} = -M_{21} \frac{dI_1}{dt}$$

Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21} ,
 - is a constant and does not depend on I_1 .
 - depends only on “geometric” factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

What? Does this make sense?

- The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
 - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $\varepsilon_1 = -M_{12} \frac{dI_2}{dt}$
 - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
 - We can put $M = M_{12} = M_{21}$ and obtain
 - SI unit for mutual inductance is henry (H)

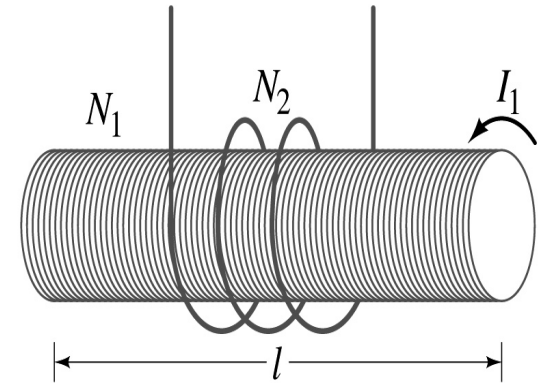
$$\varepsilon_1 = -M \frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M \frac{dI_1}{dt}$$

$$1H = 1V \cdot s / A = 1\Omega \cdot s$$



Example 30 – 1

Solenoid and coil. A long thin solenoid of length l and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \frac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is $M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
 - When a changing current passes through a coil
 - A changing magnetic flux is produced inside the coil
 - The changing magnetic flux in turn induces an emf in the same coil
 - This emf opposes the change in flux. Whose law is this?
 - Lenz's law
- What would this do?
 - When the current through the coil is increasing?
 - The increasing magnetic flux induces an emf that opposes the original current
 - This tends to impede its increase, trying to maintain the original current
 - When the current through the coil is decreasing?
 - The decreasing flux induces an emf in the same direction as the current
 - This tends to increase the flux, trying to maintain the original current



Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
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- The induced emf in a coil of self-inductance $\mathcal{L}$ is
 - $\varepsilon = -N \frac{d\Phi_B}{dt} = -L \frac{dI}{dt}$
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- What does magnitude of $\mathcal{L}$ depend on?
 - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

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- So what?
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- But it also provides means to store energy, just like the capacitance



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 - Would have both resistance and inductance
 - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
 - This is called a “non-inductive winding”
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
 - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
 - The quality of an inductor is indicated by the term reactance or impedance

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length l and whose cross-sectional area is A . (b) Calculate the value of $\mathcal{L}$ if $N=100$, $l=5.0\text{cm}$, $A=0.30\text{cm}^2$ and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

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Using the formula for self inductance: $L = \frac{N\Phi_B}{I} = \frac{N \cdot \mu_0 NIA / l}{I} = \frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) 100^2 (0.30 \times 10^{-4} \text{ m}^2)}{5.0 \times 10^{-2} \text{ m}} = 7.5 \mu\text{H}$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI / l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000 (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) 100^2 (0.30 \times 10^{-4} \text{ m}^2)}{5.0 \times 10^{-2} \text{ m}} = 0.030 \text{ H} = 30 \text{ mH}$$

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
 - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field

- $\mathcal{E} = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi_B}{dt}$$

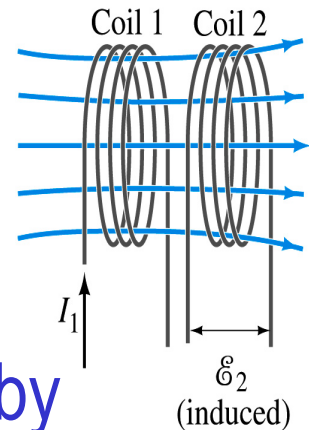
- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
 - A changing current in one circuit must induce an emf in a nearby circuit → Mutual inductance
 - Or induce an emf in itself → Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2 , in coil2 proportional to?
 - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $M_{21} = N_2\Phi_{21}/I_1$.
- The emf induced in coil2 due to the changing current in coil1 is



$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d(N_2\Phi_{21})}{dt} = -M_{21} \frac{dI_1}{dt}$$

Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21} ,
 - is a constant and does not depend on I_1 .
 - depends only on “geometric” factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

What? Does this make sense?

- The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
 - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $\varepsilon_1 = -M_{12} \frac{dI_2}{dt}$
 - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
 - We can put $M = M_{12} = M_{21}$ and obtain
 - SI unit for mutual inductance is henry (H)

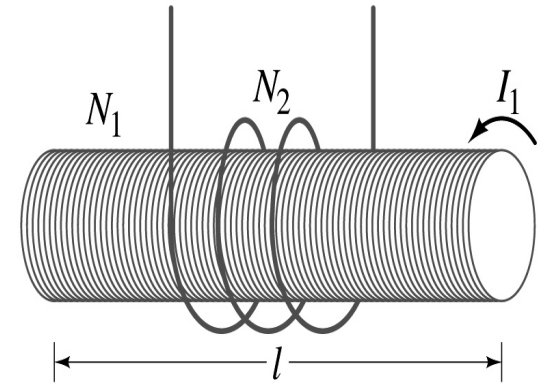
$$\varepsilon_1 = -M \frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M \frac{dI_1}{dt}$$

$$1H = 1V \cdot s / A = 1\Omega \cdot s$$



Example 30 – 1

Solenoid and coil. A long thin solenoid of length l and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \frac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is $M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
 - When a changing current passes through a coil
 - A changing magnetic flux is produced inside the coil
 - The changing magnetic flux in turn induces an emf in the same coil
 - This emf opposes the change in flux. Whose law is this?
 - Lenz's law
- What would this do?
 - When the current through the coil is increasing?
 - The increasing magnetic flux induces an emf that opposes the original current
 - This tends to impede its increase, trying to maintain the original current
 - When the current through the coil is decreasing?
 - The decreasing flux induces an emf in the same direction as the current
 - This tends to increase the flux, trying to maintain the original current



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Energy Stored in the Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt , energy is supplied to the inductor at a rate

- $P = I\varepsilon = IL \frac{dI}{dt}$

- What is the work needed to increase the current in an inductor from 0 to I ?

- The work, dW , done in time dt is $dW = Pdt = LI dI$

- Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LI dI = L \left[\frac{1}{2} I^2 \right]_0^I = \frac{1}{2} LI^2$$



Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I

- $$U = \frac{1}{2} LI^2$$

Energy Stored in a magnetic field inside an inductor

- This is compared to the energy stored in a capacitor, C , when the potential difference across it is V : $U = \frac{1}{2} CV^2$
- Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
- The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?

- Inductance of an ideal solenoid without a fringe effect

$$L = \mu_0 N^2 A / l$$

- The magnetic field in a solenoid is $B = \mu_0 NI / l$

- Thus the energy stored in an inductor is

$$U = \frac{1}{2} LI^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} \underbrace{Al}$$

$$U = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

E

- Thus the energy density is

$$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

What is this?

Volume V

$$u = \frac{1}{2} \frac{B^2}{\mu_0}$$

E density

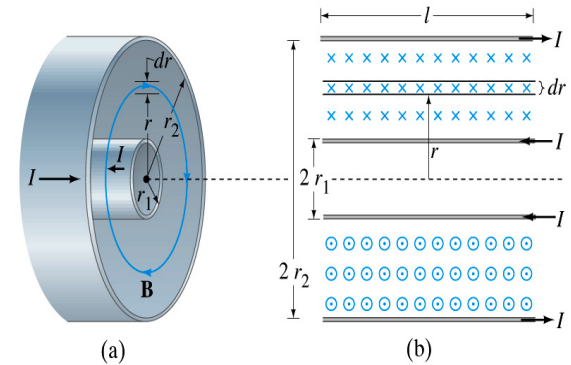
- This formula is valid in any region of space
- If a ferromagnetic material is present, μ_0 becomes μ .

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I ? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int B l dr = \frac{\mu_0 I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}$

Thus the energy stored per unit length is

$$\frac{U}{l} = \frac{1}{2} \frac{L I^2}{l} = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}$$

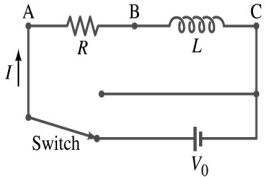
(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2} \frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

LR Circuits

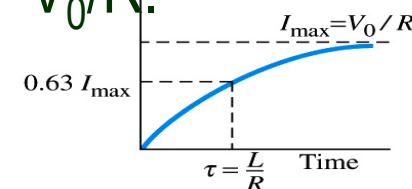
- What happens when an emf is applied to an inductor?
 - An inductor has some resistance, however negligible



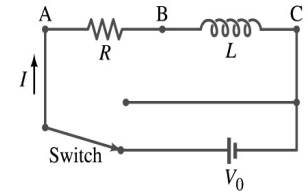
- So an inductor can be drawn as a circuit of separate resistance and coil. What is the name this kind of circuit? **LR Circuit**

- What happens at the instance the switch is thrown to apply emf to the circuit?

- The current starts to flow, gradually increasing from 0
- This change is opposed by the induced emf in the inductor → the emf at point B is higher than point C
- However there is a voltage drop at the resistance which reduces the voltage across inductance
- Thus the current increases less rapidly
- The overall behavior of the current is a gradual increase, reaching to the maximum current $I_{\max} = V_0/R$.



LR Circuits



- This can be shown w/ Kirchhoff loop rules

- The emfs in the circuit are the battery voltage V_0 and the emf $\varepsilon = -\mathcal{L}(dI/dt)$ in the inductor opposing the current increase

- The sum of the potential changes through the circuit is

$$V_0 + \varepsilon - IR = V_0 - L dI/dt - IR = 0$$

- Where I is the current at any instance

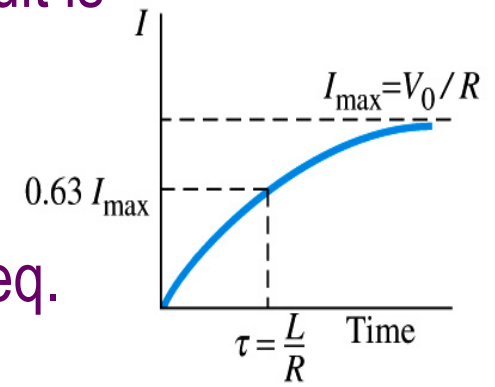
- By rearranging the terms, we obtain a differential eq.

- $L dI/dt + IR = V_0$

- We can integrate just as in RC circuit

- So the solution is $-\frac{1}{R} \ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$

- Where $\tau = L/R$



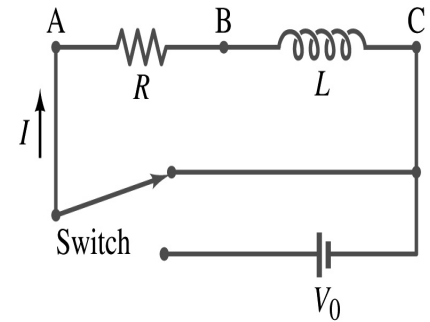
$$\int_{I=0}^I \frac{dI}{V_0 - IR} = \int_{t=0}^t \frac{dt}{L}$$

$$I = V_0 (1 - e^{-t/\tau}) / R = I_{\max} (1 - e^{-t/\tau})$$

- This is the time constant τ of the LR circuit and is the time required for the current I to reach 0.63 of the maximum

Discharge of LR Circuits

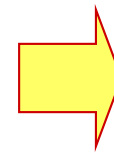
- If the switch is flipped away from the battery



- The differential equation becomes

- $L \frac{dI}{dt} + IR = 0$

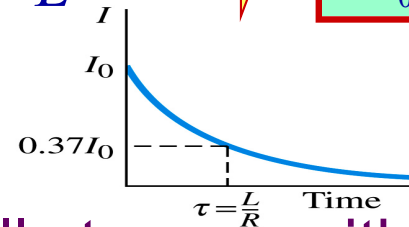
- So the integration is $\int_{I_0}^I \frac{dI}{IR} = \int_{t=0}^t \frac{dt}{L}$



$$\ln \frac{I}{I_0} = -\frac{R}{L} t$$

- Which results in the solution

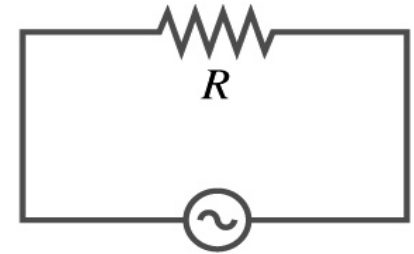
$$I = I_0 e^{-\frac{R}{L} t} = I_0 e^{-t/\tau}$$



- The current decays exponentially to zero with the time constant $\tau = L/R$
- So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.
- The current in LR circuit behaves almost the same as that in RC circuit but the time constant is inversely proportional to R in LR circuit unlike the RC circuit

AC Circuit w/ Resistance only

- What do you think will happen when an AC source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain



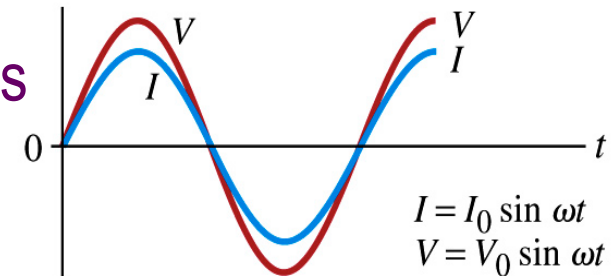
- Thus $V - IR = 0$

$$V = I_0 R \sin \omega t = V_0 \sin \omega t$$

– where $V_0 = I_0 R$

- What does this mean?

- Current is 0 when voltage is 0 and current is in its peak when voltage is in its peak.
- Current and voltage are “in phase”



- Energy is lost via the transformation into heat at an average rate

$$\bar{P} = \bar{I} \bar{V} = I_{rms}^2 R = V_{rms}^2 / R$$

AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

$$V - L \frac{dI}{dt} = 0$$

- Thus

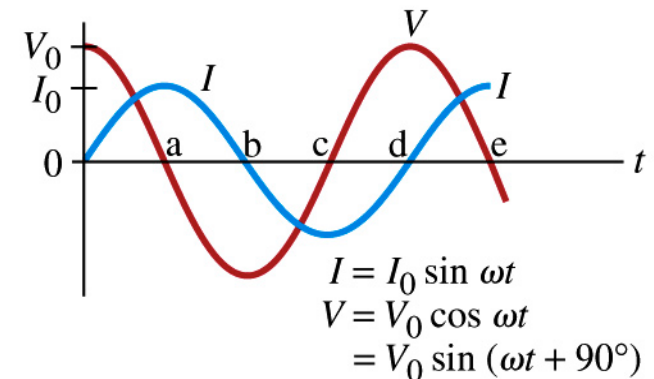
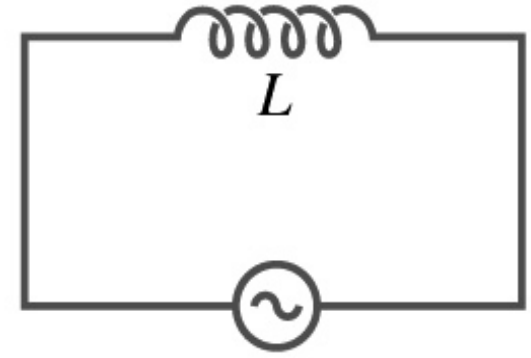
$$V = L \frac{dI}{dt} = L \frac{d(I_0 \sin \omega t)}{dt} = \omega L I_0 \cos \omega t$$

- Using the identity $\cos \theta = \sin(\theta + 90^\circ)$
- $V = \omega L I_0 \sin(\omega t + 90^\circ) = V_0 \sin(\omega t + 90^\circ)$
 - where $V_0 = \omega L I_0$
- What does this mean?

- Current and voltage are “out of phase by $\pi/2$ or 90° ”. In other words the current reaches its peak $1/4$ cycle after the voltage

- What happens to the energy?

- No energy is dissipated
- The average power is 0 at all times
- The energy is stored temporarily in the magnetic field
- Then released back to the source



AC Circuit w/ Inductance only

- How are the resistor and inductor different in terms of energy?
 - Inductor Stores the energy temporarily in the magnetic field and then releases it back to the emf source
 - Resistor Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they the same?
 - They both impede the flow of charge
 - For a resistance R , the peak voltage and current are related to $V_0 = I_0 R$
 - Similarly, for an inductor we may write $V_0 = I_0 X_L$
 - Where X_L is the inductive reactance of the inductor $X_L = \omega L$ 0 when $\omega=0$.
 - What do you think is the unit of the reactance? Ω
 - The relationship $V_0 = I_0 X_L$ is not valid at a particular instance. Why not?
 - Since V_0 and I_0 do not occur at the same time



$$V_{rms} = I_{rms} X_L$$

is valid! 91

Example 30 – 9

Reactance of a coil. A coil has a resistance $R=1.00\Omega$ and an inductance of 0.300H . Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since $X_L = \omega L = 0$

So for DC power, the current is from Kirchhoff's rule ^{$\omega=0$} , $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120\text{V}}{1.00\Omega} = 120\text{A}$$

For an AC power with $f=60\text{Hz}$, the reactance is

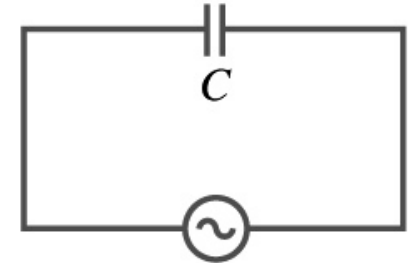
$$X_L = \omega L = 2\pi fL = 2\pi \cdot (60.0\text{s}^{-1}) \cdot 0.300\text{H} = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120\text{V}}{113\Omega} = 1.06\text{A}$$

AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a DC power source?
 - The capacitor quickly charges up.
 - There is no steady current flow in the circuit
 - Since the capacitor prevents the flow of the DC current
- What do you think will happen if it is connected to an AC power source?
 - The current flows continuously. Why?
 - When the AC power turns on, charge begins to flow one direction, charging up the plates
 - When the direction of the power reverses, the charge flows in the opposite direction



AC Circuit w/ Capacitance only

- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- The current at any instance is $I = \frac{dQ}{dt} = I_0 \sin \omega t$

- The charge Q on the plate at any instance is

$$Q = \int_{Q=0}^Q dQ = \int_{t=0}^t I_0 \sin \omega t dt = -\frac{I_0}{\omega} \cos \omega t$$

- Thus the voltage across the capacitor is

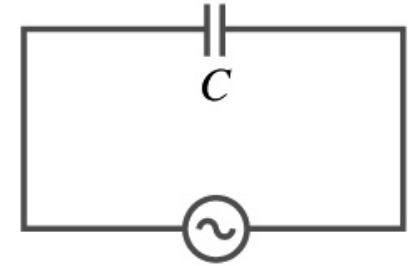
$$V = \frac{Q}{C} = -I_0 \frac{1}{\omega C} \cos \omega t$$

- Using the identity $\cos \theta = -\sin(\theta - 90^\circ)$

$$V = I_0 \frac{1}{\omega C} \sin(\omega t - 90^\circ) = V_0 \sin(\omega t - 90^\circ)$$

- Where

- $V_0 = \frac{I_0}{\omega C}$

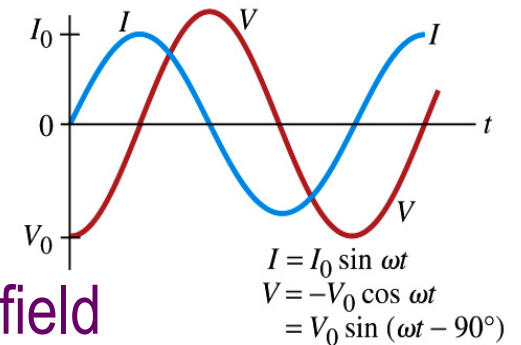


AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin(\omega t - 90^\circ)$
- What does this mean?
 - Current and voltage are “out of phase by $\pi/2$ or 90° ” but in this case, the voltage reaches its peak $1/4$ cycle after the current

- What happens to the energy?

- No energy is dissipated
- The average power is 0 at all times
- The energy is stored temporarily in the electric field
- Then released back to the source



- Applied voltage and the current in the capacitor can be written as $V_0 = I_0 X_C$

- Where the capacitive reactance X_C is defined as
- Again, this relationship is only valid for rms quantities

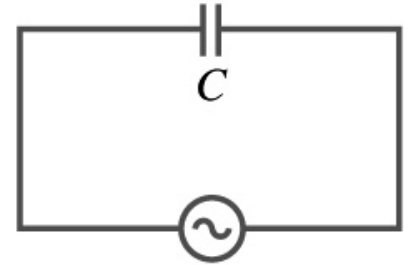
$$X_C = \frac{1}{\omega C}$$

Infinite
when
 $\omega=0$.

$$V_{rms} = I_{rms} X_C$$

Example 30 – 10

Capacitor reactance. What are the peak and rms current in the circuit in the figure if $C=1.0\mu\text{F}$ and $V_{\text{rms}}=120\text{V}$? Calculate for (a) $f=60\text{Hz}$, and then for (b) $f=6.0\times 10^5\text{Hz}$.



The peak voltage is $V_0 = \sqrt{2}V_{\text{rms}} = 120\text{V} \cdot \sqrt{2} = 170\text{V}$

The capacitance reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot (60\text{s}^{-1}) \cdot 1.0 \times 10^{-6}\text{F}} = 2.7\text{k}\Omega$$

Thus the peak current is

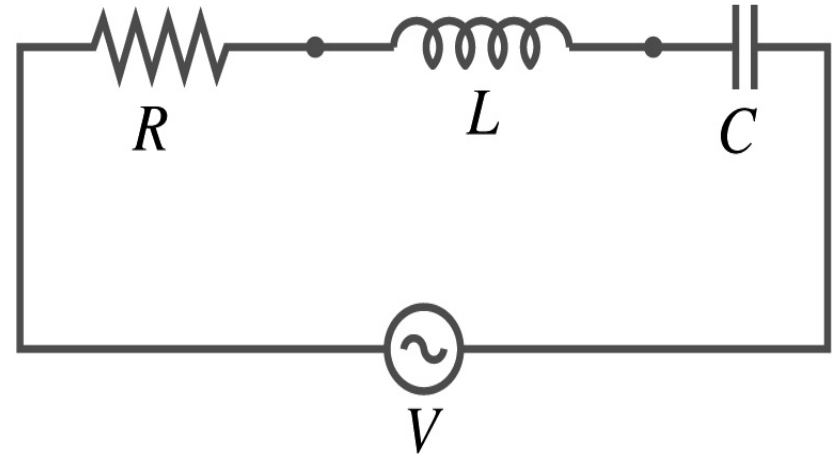
$$I_0 = \frac{V_0}{X_C} = \frac{170\text{V}}{2.7\text{k}\Omega} = 63\text{mA}$$

The rms current is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{120\text{V}}{2.7\text{k}\Omega} = 44\text{mA}$$

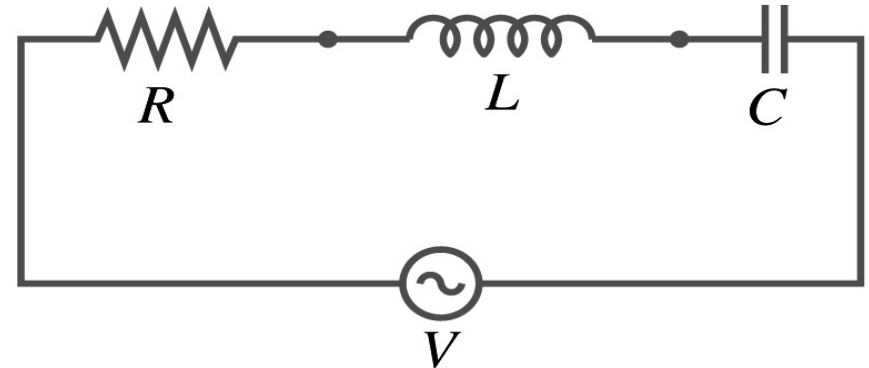
AC Circuit w/ LRC

- The voltage across each element is
 - V_R is in phase with the current
 - V_L leads the current by 90°
 - V_C lags the current by 90°
- From Kirchhoff's loop rule
- $V = V_R + V_L + V_C$
 - However since they do not reach the peak voltage at the same time, the peak voltage of the source V_0 will not equal $V_{R0} + V_{L0} + V_{C0}$
 - The rms voltage also will not be the simple sum of the three
- Let's try to find the total impedance, peak current I_0 and the phase difference between I_0 and V_0 .



AC Circuit w/ LRC

- The current at any instance is the same at all point in the circuit
 - The currents in each elements are in phase
 - Why?
 - Since the elements are in series
 - How about the voltage?
 - They are not in phase.



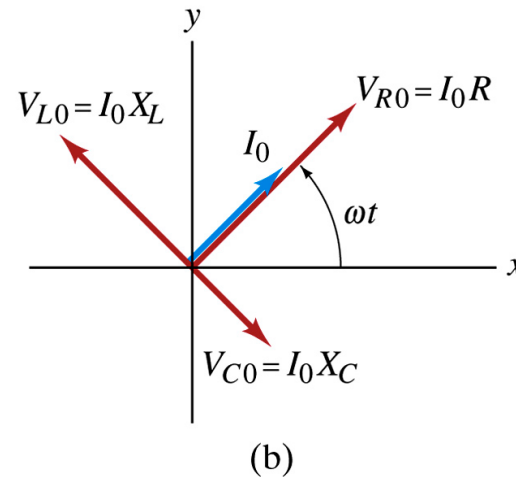
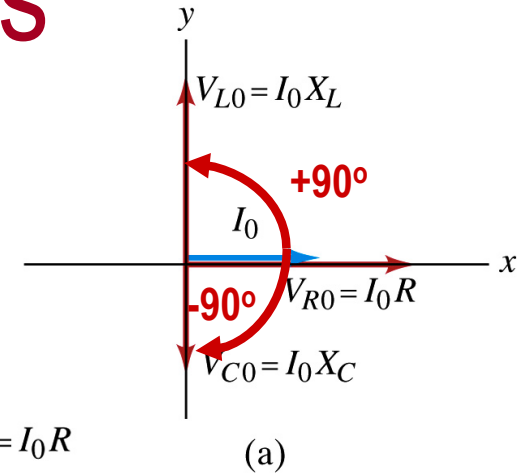
- The current at any given time is

$$I = I_0 \sin \omega t$$

- The analysis of LRC circuit is done using the “phasor” diagram in which arrows are drawn in an xy plane to represent the amplitude of each voltage, just like vectors
 - The lengths of the arrows represent the magnitudes of the peak voltages across each element; $V_{R0} = I_0 R$, $V_{L0} = I_0 X_L$ and $V_{C0} = I_0 X_C$
 - The angle of each arrow represents the phase of each voltage relative to the current, and the arrows rotate at the angular frequency ω to take into account the time dependence.
 - The projection of each arrow on y axis represents voltage across each element at any given time

Phasor Diagrams

- At $t=0$, $I=0$.
 - Thus $V_{R0}=0$, $V_{L0}=I_0X_L$, $V_{C0}=I_0X_C$
- At $t=t$, $I = I_0 \sin \omega t$

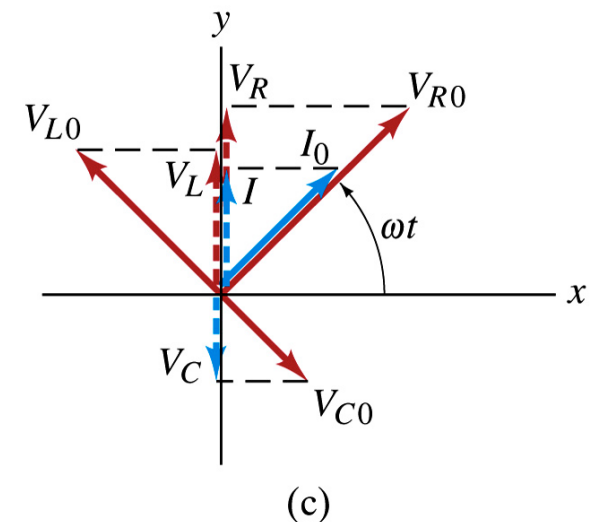


- Thus, the voltages (y-projections) are

$$V_R = V_{R0} \sin \omega t$$

$$V_L = V_{L0} \sin(\omega t + 90^\circ)$$

$$V_C = V_{C0} \sin(\omega t - 90^\circ)$$



Thursday, July 5, 2018

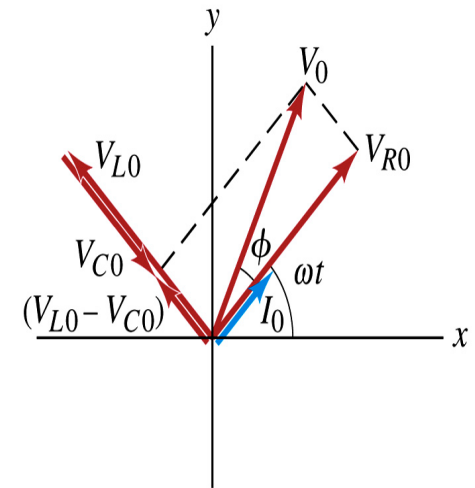


PHYS 1444-001, Summer 2018
Dr. Jaehoon Yu

AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.

- The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
- So we can use the sum of all vectors as the representation of the peak source voltage V_0 .



- V_0 forms an angle ϕ to V_{R0} and rotates together with the other vectors as a function of time, $V = V_0 \sin(\omega t + \phi)$
- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0 (X_L - X_C)}{I_0 R} = \frac{(X_L - X_C)}{R}$$

- or

$$\cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

- What is the power dissipated in the circuit?

- Which element dissipates the power?
- Only the resistor

- The average power is $\bar{P} = I_{rms}^2 R$

- Since $R = Z \cos \phi$

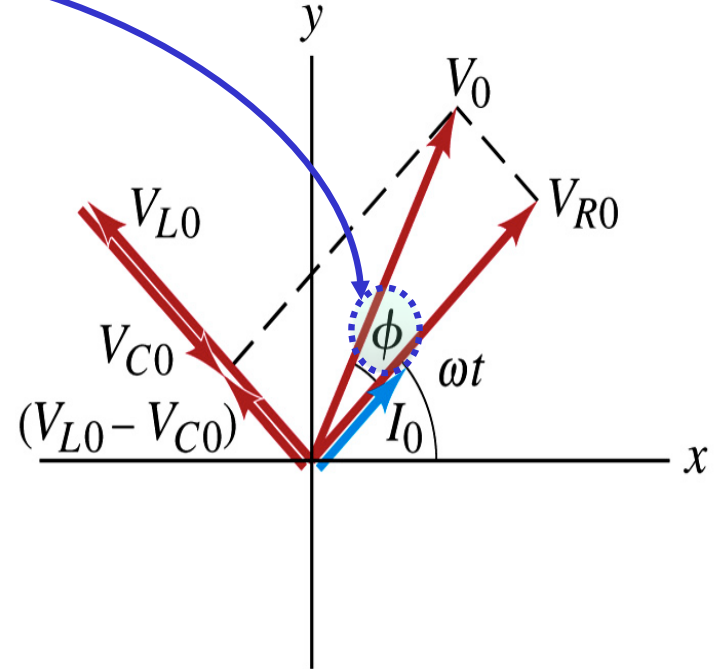
- We obtain

$$\bar{P} = I_{rms}^2 Z \cos \phi = I_{rms} V_{rms} \cos \phi$$

- The factor $\cos \phi$ is referred as the power factor of the circuit

- For a pure resistor, $\cos \phi = 1$ and $\bar{P} = I_{rms} V_{rms}$

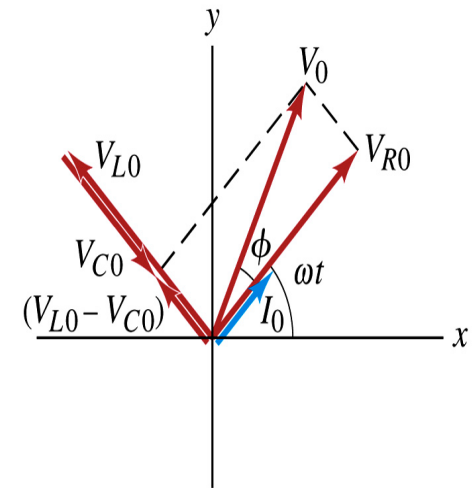
- For a capacitor or inductor alone $\phi = -90^\circ$ or $+90^\circ$, so $\cos \phi = 0$ and $\bar{P} = 0$.



AC Circuit w/ LRC

- Since the sum of the projections of the three vectors on the y axis is equal to the projection of their sum.

- The sum of the projections represents the instantaneous voltage across the whole circuit which is the source voltage
- So we can use the sum of all vectors as the representation of the peak source voltage V_0 .



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- We determine the total impedance Z of the circuit defined by the relationship $V_{rms} = I_{rms} Z$ or $V_0 = I_0 Z$
- From Pythagorean theorem, we obtain

$$V_0 = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = \sqrt{I_0^2 R^2 + I_0^2 (X_L - X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} = I_0 Z$$

- Thus the total impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

AC Circuit w/ LRC

- The phase angle ϕ is

$$\tan \phi = \frac{V_{L0} - V_{C0}}{V_{R0}} = \frac{I_0 (X_L - X_C)}{I_0 R} = \frac{(X_L - X_C)}{R}$$

- or

$$\cos \phi = \frac{V_{R0}}{V_0} = \frac{I_0 R}{I_0 Z} = \frac{R}{Z}$$

- What is the power dissipated in the circuit?

- Which element dissipates the power?
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- The average power is $\bar{P} = I_{rms}^2 R$

- Since $R = Z \cos \phi$

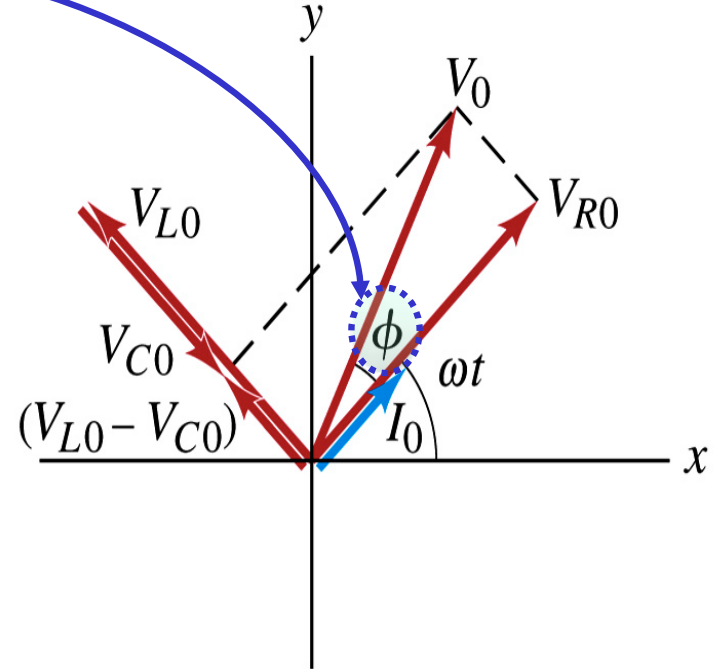
- We obtain

$$\bar{P} = I_{rms}^2 Z \cos \phi = I_{rms} V_{rms} \cos \phi$$

- The factor $\cos \phi$ is referred as the power factor of the circuit

- For a pure resistor, $\cos \phi = 1$ and $\bar{P} = I_{rms} V_{rms}$

- For a capacitor or inductor alone $\phi = -90^\circ$ or $+90^\circ$, so $\cos \phi = 0$ and $\bar{P} = 0$.



Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
 - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
 - This theory provided the prediction of EM waves
 - As important as Newton's law since it provides dynamics of electromagnetism
 - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
 - What do you think this accomplishment did?
 - Open a new world of communication
 - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect



Ampere's Law

- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

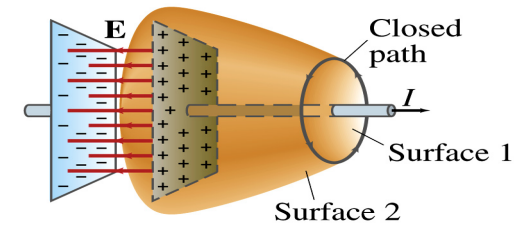
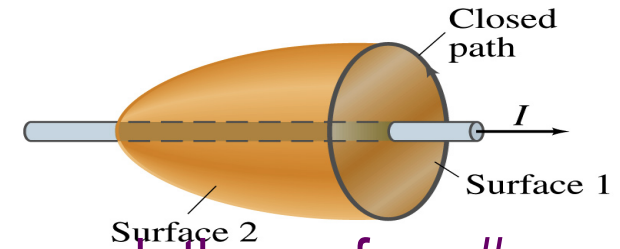
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field
- Then can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
 - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism



Expanding Ampere's Law

- Let's consider a wire carrying current I
 - The current that is enclosed in the loop passes through the surface # 1 in the figure
 - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
 - Still I .
 - So the Ampere's law still works
- We could then consider a capacitor being charged up or being discharged.
 - The current I enclosed in the loop passes through the surface #1
 - However the surface #2 that shares the same closed loop do not have any current passing through it.
 - There is magnetic field present since there is current $\rightarrow$ In other words there is a changing electric field in between the plates
 - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field



Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
 - The charge Q on the capacitor with capacitance C is $Q=CV$
 - Where V is the potential difference between the plates
 - Since $V=Ed$
 - Where E is the uniform field between the plates, and d is the separation of the plates
 - And for parallel plate capacitor $C=\epsilon_0 A/d$
 - We obtain

$$Q = CV = \left(\epsilon_0 \frac{A}{d} \right) Ed = \epsilon_0 AE$$

Modifying Ampere's Law

- If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt}$$

- Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d(AE)}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

- Where $\Phi_E = EA$ is the electric flux through the surface between the plates

- So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Extra term
by Maxwell

- This equation represents the general form of Ampere's law
 - This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 31 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area $A=100\text{cm}^2$. It is charged by a 70-V battery through a $2.0\text{-}\Omega$ resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume $\mathbf{E}$ is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0(1 - e^{-t/RC})$

For the initial current ($t=0$), we differentiate the charge with respect to time.

$$I_0 = \left. \frac{dQ}{dt} \right|_{t=0} = \left. \frac{CV_0}{RC} e^{-t/RC} \right|_{t=0} = \frac{V_0}{R} = \frac{70\text{V}}{2.0\Omega} = 35\text{A}$$

The electric field is $E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\epsilon_0} = \frac{35\text{A}}{(8.85 \times 10^{-12} \text{C}^2/\text{N} \cdot \text{m}^2) \cdot (1.0 \times 10^{-2} \text{m}^2)} = 4.0 \times 10^{14} \text{V}/\text{m} \cdot \text{s}$

Example 31 – 1

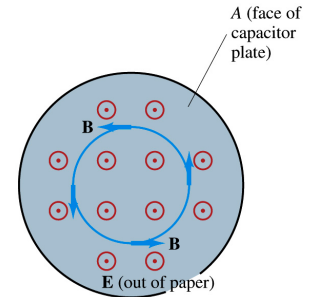
(c) Determine the magnetic field induced between the plates. Assume $\mathbf{E}$ is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to $\mathbf{E}$ and is circular due to symmetry

Whose law can we use to determine B ?

Extended Ampere's Law w/ $I_{\text{encl}}=0$!

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



We choose a circular path of radius r , centered at the center of the plane, following the B .

For $r < r_{\text{plate}}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r^2)}{dt} = \mu_0 \epsilon_0 \pi r^2 \frac{dE}{dt}$

Solving for B

$$B = \mu_0 \epsilon_0 \frac{r}{2} \frac{dE}{dt}$$

For $r < r_{\text{plate}}$

Since we assume $E=0$ for $r > r_{\text{plate}}$, the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{\text{plate}}^2$$

So from Ampere's law, we obtain $B \cdot (2\pi r) = \mu_0 \epsilon_0 \frac{d(E\pi r_{\text{plate}}^2)}{dt} = \mu_0 \epsilon_0 \pi r_{\text{plate}}^2 \frac{dE}{dt}$

Solving for B

$$B = \frac{\mu_0 \epsilon_0 r_{\text{plate}}^2}{2r} \frac{dE}{dt}$$

For $r > r_{\text{plate}}$

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Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
 - He called this term as the displacement current, I_D
 - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_{encl} + I_D)$$

- Where

$$I_D = \epsilon_0 \frac{d\Phi_E}{dt}$$

- While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field $\vec{B}$, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

- Where the integration is over the area of either an open or a closed surface

- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
 - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ϵ_0 .

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss' Law
for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for
magnetism

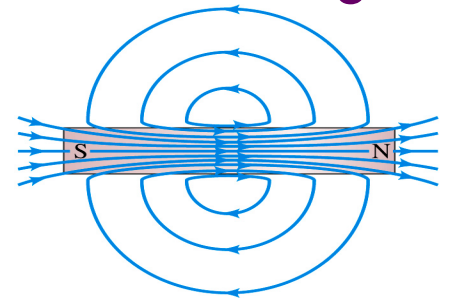
- Why is result of the integral zero?
 - There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- There are as many magnetic flux lines that enter the enclosed volume as leave it
- If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
 - Electricity do have the source and the sink
- Magnetic field lines must be continuous
- Even for bar magnets, the field lines exist both inside and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Ampère's Law

A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

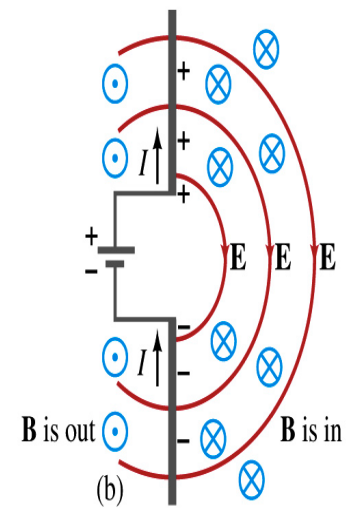
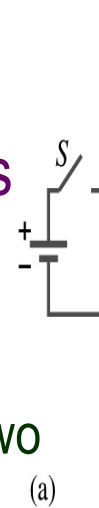
- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
 - He then took this concept one step further and concluded that
 - If a changing magnetic field produces an electric field, the electric field is also changing in time.
 - This changing electric field in turn produces the magnetic field that also changes.
 - This changing magnetic field then in turn produces the electric field that changes.
 - This process continues.
 - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space

Production of EM Waves

- Consider two conducting rods that will serve as an antenna are connected to a DC power source

- What do you think will happen when the switch is closed?

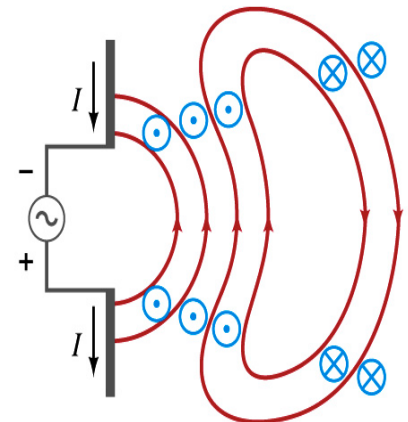
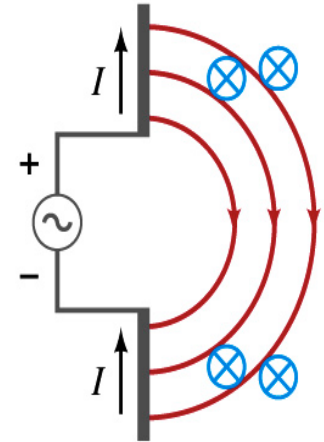
- The rod connected to the positive terminal is charged positive and the other negatively
- Then the electric field will be generated between the two rods
- Since there is current that flows through, the rods generates a magnetic field around them



- How far would the electric and magnetic fields extend?
 - In static case, the field extends indefinitely
 - When the switch is closed, the fields are formed nearby the rods quickly but
 - The stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an ac power source?
 - When the connection was initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
 - The field lines form as in the dc case
 - The field lines propagate away from the antenna
 - Then the direction of the voltage reverses
 - The new field lines with the opposite direction forms
 - While the original field lines still propagates away from the rod reaching out far
 - Since the original field propagates through an empty space, the field lines must form a closed loop (no charge exist)
 - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward is self supporting and do not need antenna with flowing charge
 - The fields far from the antenna is called the **radiation field**
 - Both electric and magnetic fields form closed loops perpendicular to each other



Properties of Radiation Fields

- The fields travel on the other side of the antenna as well
- The field strength are the greatest in the direction perpendicular to the oscillating charge while along the direction is 0
- The magnitude of E and B in the radiation field decrease with distance as $1/r$
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
 - So the intensity of wave decreases as $1/r^2$

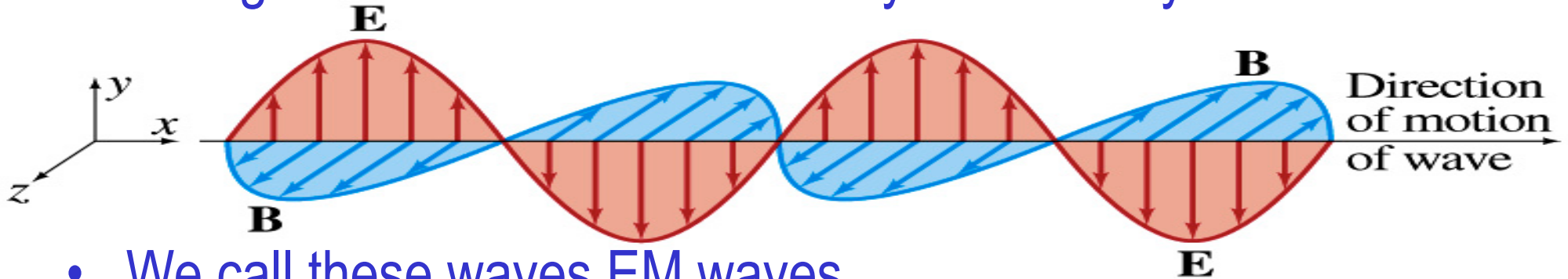
Properties of Radiation Fields

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
 - The field strengths vary from maximum in one direction, to 0 and to max in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
 - Called plane waves



EM Waves

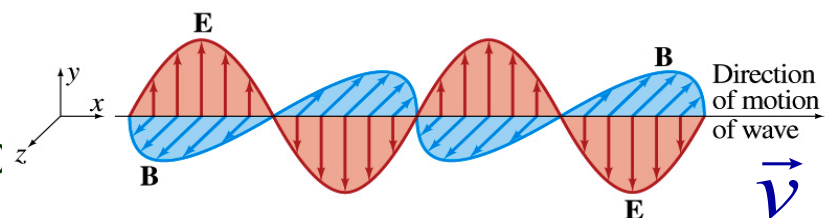
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
 - Since these are fields, they can propagate through an empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally verified by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
 - An area of space where there is no charges or conduction currents
 - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
 - What are these flat waves called?
 - Plane waves
 - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
 - So we can also assume that the wave is traveling in the x-direction w/ velocity, $\mathbf{v} = v\mathbf{i}$, and that **E** is parallel to y axis and **B** is parallel to z axis



Maxwell's Equations w/ $Q=I=0$

- In this region of free space, $Q=0$ and $I=0$, thus the four Maxwell's equations become

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$	$Q_{encl}=0$	$\oint \vec{E} \cdot d\vec{A} = 0$
$\oint \vec{B} \cdot d\vec{A} = 0$	No Changes	$\oint \vec{B} \cdot d\vec{A} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	No Changes	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$I_{encl}=0$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency f , such traveling wave can be written as

$$E = E_y = E_0 \sin(kx - \omega t)$$

$$B = B_z = B_0 \sin(kx - \omega t)$$

– Where

$$k = \frac{2\pi}{\lambda} \quad \omega = 2\pi f \quad \text{Thus} \quad f\lambda = \frac{\omega}{k} = v$$

– What is v ?

- It is the speed of the traveling wave

– What are E_0 and B_0 ?

- The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law

- Let's apply Faraday's law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- to the rectangular loop of height Δy and width dx

- $\vec{E} \cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?

- Since $\vec{E}$ is perpendicular to $d\vec{l}$

- So the result of the integral through the loop counterclockwise becomes $\oint \vec{E} \cdot d\vec{l} = \vec{E} \cdot d\vec{x} + (\vec{E} + d\vec{E}) \cdot \Delta\vec{y} + \vec{E} \cdot d\vec{x} + \vec{E} \cdot \Delta\vec{y} =$

$$= 0 + (E + dE)\Delta y - 0 - E\Delta y = dE\Delta y$$

- For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt} dx \Delta y$$

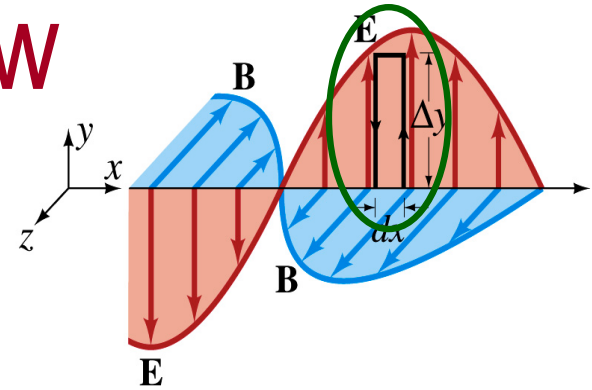
Thus

$$dE\Delta y = -\frac{dB}{dt} dx \Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$



From Modified Ampère's Law

- Let's apply Maxwell's 4th equation

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

- to the rectangular loop of length Δz and width dx

- $\vec{B} \cdot d\vec{l}$ along the x-axis of the loop is 0

- Since **B** is perpendicular to $d\vec{l}$
- So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{B} \cdot d\vec{l} = B\Delta z - (B + dB)\Delta z = -dB\Delta z$$

- For the right-hand side of the equation is

$$\mu_0 \epsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \epsilon_0 \frac{dE}{dt} dx \Delta z$$

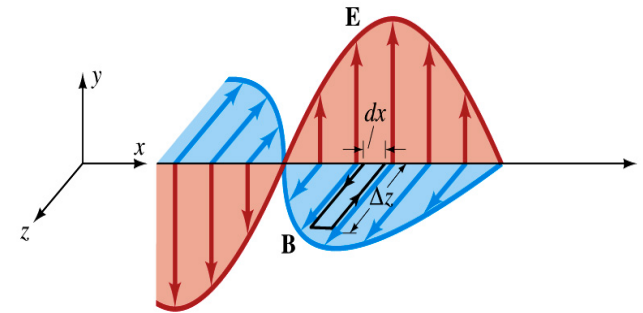
Thus

$$-dB\Delta z = \mu_0 \epsilon_0 \frac{dE}{dt} dx \Delta z$$

$$-\frac{dB}{dx} = -\mu_0 \epsilon_0 \frac{dE}{dt}$$

Since E and B depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$



Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$
- Taking the derivatives of **E** and **B** as given their traveling wave form, we obtain

$$\frac{\partial E}{\partial x} = \frac{\partial}{\partial x} (E_0 \sin(kx - \omega t)) = kE_0 \cos(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial t} (B_0 \sin(kx - \omega t)) = -\omega B_0 \cos(kx - \omega t)$$

Since $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$  **We obtain** $kE_0 \cos(kx - \omega t) = \omega B_0 \cos(kx - \omega t)$

 **Thus** $\frac{E_0}{B_0} = \frac{\omega}{k} = v$

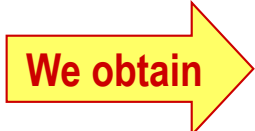
- Since **E** and **B** are in phase, we can write $E/B = v$
 - This is valid at any point and time in space. What is v ?
 - The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Ampere's law $\frac{\partial B}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x} (B_0 \sin(kx - \omega t)) = kB_0 \cos(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} (E_0 \sin(kx - \omega t)) = -\omega E_0 \cos(kx - \omega t)$$

Since $\frac{\partial B}{\partial x} = -\epsilon_0 \mu_0 \frac{\partial E}{\partial t}$  **We obtain** $kB_0 \cos(kx - \omega t) = \epsilon_0 \mu_0 \omega E_0 \cos(kx - \omega t)$

 **Thus** $\frac{B_0}{E_0} = \frac{\epsilon_0 \mu_0 \omega}{k} = \epsilon_0 \mu_0 v$

– However, from the previous page we obtain $E_0/B_0 = v = \frac{1}{\epsilon_0 \mu_0 v}$

– Thus $v^2 = \frac{1}{\epsilon_0 \mu_0}$

$$v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{1}{\sqrt{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A})}} = 3.00 \times 10^8 \text{ m/s}$$

The speed of EM waves is the same as the speed of light. EM waves behaves like the light.

Speed of Light w/o Sinusoidal Wave Forms

- Taking the time derivative on the relationship from Ampere's laws, we obtain $\frac{\partial^2 B}{\partial x \partial t} = -\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$

- By the same token, we take position derivative on the relationship from Faraday's law $\frac{\partial^2 E}{\partial x^2} = -\frac{\partial^2 B}{\partial x \partial t}$

- From these, we obtain

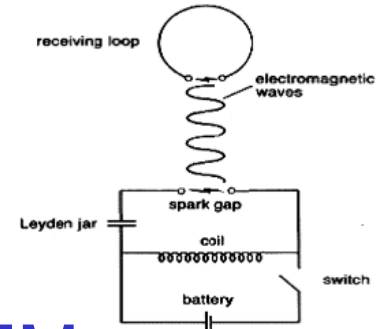
$$\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 E}{\partial x^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial t^2} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B}{\partial x^2}$$

- Since the equation for traveling wave is $\frac{\partial^2 x}{\partial t^2} = v^2 \frac{\partial^2 x}{\partial x^2}$
- By correspondence, we obtain $v^2 = \frac{1}{\epsilon_0 \mu_0}$

- A natural outcome of Maxwell's equations is that E and B obey the wave equation for waves traveling w/ speed $v = 1/\sqrt{\epsilon_0 \mu_0}$
 - Maxwell predicted the existence of EM waves based on this

Light as EM Wave

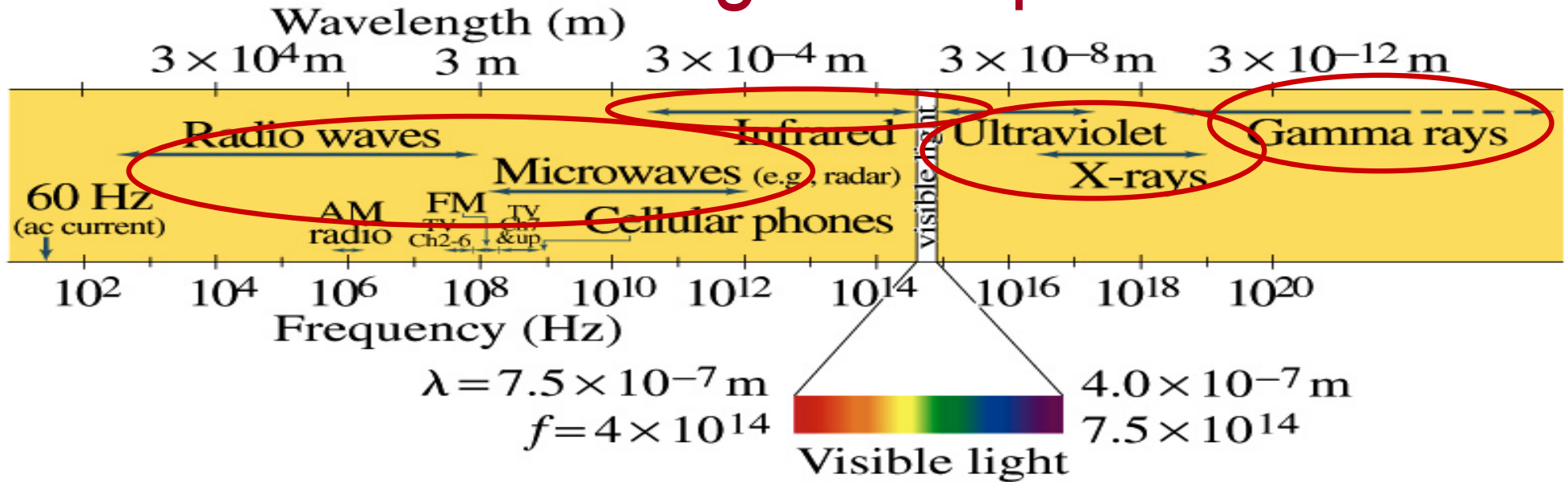
- People knew some 60 years before Maxwell that light behaves like a wave, but ...
 - They did not know what kind of waves they are.
 - Most importantly what is it that oscillates in light?
- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
 - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
 - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
 - These waves were later shown to travel at the speed of light and behave exactly like the light just not visible



Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19th century
 - The visible light wave length were found to be between $4.0 \times 10^{-7} \text{m}$ (400nm) and $7.5 \times 10^{-7} \text{m}$ (750nm)
 - The frequency of visible light is $f\lambda = c$
 - Where f and λ are the frequency and the wavelength of the wave
 - What is the range of visible light frequency?
 - $4.0 \times 10^{14} \text{Hz}$ to $7.5 \times 10^{14} \text{Hz}$
 - c is $3 \times 10^8 \text{m/s}$, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum

Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
 - The Sun emits visible lights, IR and UV
 - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus warm up

Example 31 – 3

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14} Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f \lambda$

Thus, we obtain $\lambda = \frac{c}{f}$

For $f=60\text{Hz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{60 \text{ s}^{-1}} = 5 \times 10^6 \text{ m}$$

For $f=93.3\text{MHz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{93.3 \times 10^6 \text{ s}^{-1}} = 3.22 \text{ m}$$

For $f=4.74 \times 10^{14}\text{Hz}$
$$\lambda = \frac{3 \times 10^8 \text{ m/s}}{4.74 \times 10^{14} \text{ s}^{-1}} = 6.33 \times 10^{-7} \text{ m}$$

EM Wave in the Transmission Lines

- Can EM waves travel through a wire?
 - Can it not just travel through the empty space?
 - Nope. It sure can travel through a wire.
- When a source of emf is connected to a transmission line, the electric field within the wire does not set up immediately at all points along the line
 - When two wires are separated via air, the EM wave travel through the air at the speed of light, c .
 - However, through medium w/ permittivity ϵ and permeability μ , the speed of the EM wave is given $v = 1/\sqrt{\epsilon\mu} < c$
 - Is this faster than c ? **Nope! It is slower.**

Example 31 – 5

Phone call time lag. You make a telephone call from New York to London. Estimate the time the electrical signal to travel to reach London (a) carried on a 5000km telephone cable under the Atlantic Ocean and (b) sent via satellite 36,000km above the ocean. Would this cause a noticeable delay in either case?

Time delay via the cable: $t = \frac{d}{c} = \frac{5 \times 10^6}{3 \times 10^8} = 0.017s$

Delay via satellite $t = \frac{2d_s}{c} = \frac{2 \times 3.6 \times 10^7}{3.0 \times 10^8} = 0.24s$

So in case of satellite, the delay is likely noticeable!!

Energy in EM Waves

- Since $B=E/c$ and $c = 1/\sqrt{\epsilon_0\mu_0}$, we can rewrite the energy density

$$u = u_E + u_B = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\frac{\epsilon_0\mu_0 E^2}{\mu_0} = \epsilon_0 E^2 \quad \boxed{u = \epsilon_0 E^2}$$

- Note that the energy density associated with B field is the same as that associated with E
- So each field contributes half to the total energy

- By rewriting in B field only, we obtain

$$u = \frac{1}{2}\epsilon_0 \frac{B^2}{\epsilon_0\mu_0} + \frac{1}{2}\frac{B^2}{\mu_0} = \frac{B^2}{\mu_0}$$

$$\boxed{u = \frac{B^2}{\mu_0}}$$

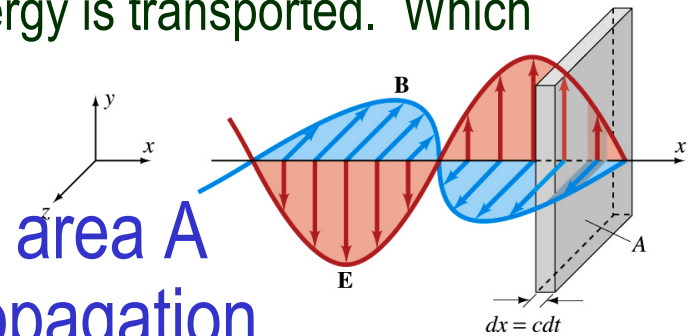
- We can also rewrite to contain both E and B

$$u = \epsilon_0 E^2 = \epsilon_0 E c B = \frac{\epsilon_0 E B}{\sqrt{\epsilon_0\mu_0}} = \sqrt{\frac{\epsilon_0}{\mu_0}} E B$$

$$\boxed{u = \sqrt{\frac{\epsilon_0}{\mu_0}} E B}$$

Energy Transport

- What is the energy the wave transport per unit time per unit area?
 - This is given by the vector **S**, the Poynting vector
 - The unit of **S** is **W/m²**.
 - The direction of **S** is the direction in which the energy is transported. Which direction is this?
 - The direction the wave is moving
- Let's consider a wave passing through an area **A** perpendicular to the x-axis, the axis of propagation
 - How much does the wave move in time dt ?
 - $dx = cdt$
 - The energy that passes through **A** in time dt is the energy that occupies the volume dV , $dV = A dx = A c dt$
 - Since the energy density is $u = \epsilon_0 E^2$, the total energy, dU , contained in the volume V is $dU = u dV = \epsilon_0 E^2 A c dt$



Energy Transport

- Thus, the energy crossing the area A per time dt is

$$S = \frac{1}{A} \frac{dU}{dt} = \epsilon_0 c E^2$$

- Since $E=cB$ and $c = 1/\sqrt{\epsilon_0 \mu_0}$, we can also rewrite

$$S = \epsilon_0 c E^2 = \frac{c B^2}{\mu_0} = \frac{EB}{\mu_0}$$

- Since the direction of S is along $\mathbf{v}$, perpendicular to $\mathbf{E}$ and $\mathbf{B}$, the Poynting vector $\mathbf{S}$ can be written

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

What is the unit? W/m^2

- This gives the energy transported per unit area per unit time at any instant

Average Energy Transport

- The average energy transport in an extended period of time is useful since sometimes we do not detect the rapid variation with respect to time because the frequency is so high.
- If E and B are sinusoidal, $\overline{E^2} = E_0^2 / 2$
- Thus we can write the magnitude of the average Poynting vector as

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0}$$

- This time averaged value of S is the intensity, defined as the average power transferred across unit area. E_0 and B_0 are maximum values.
- We can also write
$$\bar{S} = \frac{E_{rms} B_{rms}}{\mu_0}$$
 - Where E_{rms} and B_{rms} are the rms values ($E_{rms} = \sqrt{\overline{E^2}}$, $B_{rms} = \sqrt{\overline{B^2}}$)

Example 31 – 6

E and B from the Sun. Radiation from the Sun reaches the Earth (above the atmosphere) at a rate of about 1350 W/m^2 . Assume that this is a single EM wave and calculate the maximum values of E and B.

What is given in the problem? The average S!!

$$\bar{S} = \frac{1}{2} \epsilon_0 c E_0^2 = \frac{1}{2} \frac{c}{\mu_0} B_0^2 = \frac{E_0 B_0}{2 \mu_0}$$

For E_0 ,
$$E_0 = \sqrt{\frac{2\bar{S}}{\epsilon_0 c}} = \sqrt{\frac{2 \cdot 1350 \text{ W/m}^2}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \cdot (3.00 \times 10^8 \text{ m/s})}} = 1.01 \times 10^3 \text{ V/m}$$

For B_0
$$B_0 = \frac{E_0}{c} = \frac{1.01 \times 10^3 \text{ V/m}}{3 \times 10^8 \text{ m/s}} = 3.37 \times 10^{-6} \text{ T}$$