PHYS 1441 – Section 001
Lecture #4

Thursday, June 6, 2019

Dr. Jaehoon Yu

- Chapter 21
  - The Field Lines
  - Electric Fields and Conductors
  - Motion of a Charged Particle in an Electric Field
  - Electric Dipole and Its Electric Field
- Chapter 22
  - Gauss' Law
  - Electric Flux
  - Gauss' Law with Multiple Charges

Today's homework is #3, due 11pm, next Wednesday, June 12!!

Announcements

- 1^{st} Term exam
  - In class, coming Monday, June 10: DO NOT MISS THE EXAM!
  - CH21.1 to what we learn on today (CH22.2?) + Appendices A1 – A8
  - You can bring your calculator but it must not have any relevant formula pre-input
    - Cell phones or any types of computers cannot replaced a calculator!
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, no setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Quiz 1 results
  - Class average: 15.5/35
    - Equivalent to 44.3/100
  - Top score: 26/35

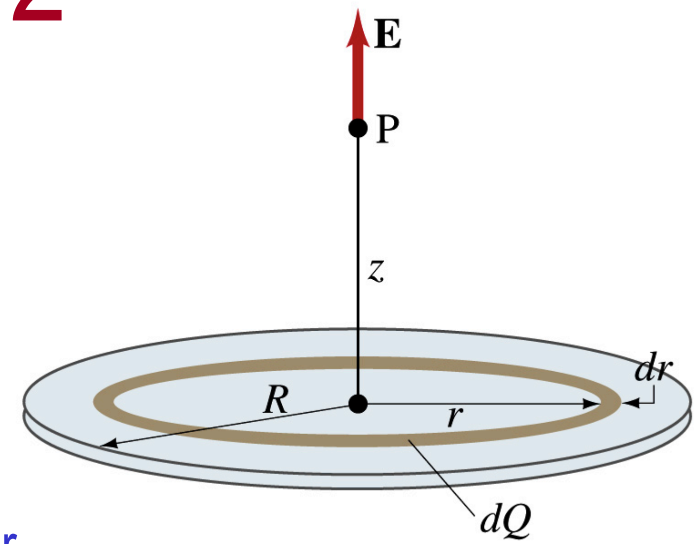
Reminder: Extra Credit Special Project #1

- Compare the Coulomb force to the Gravitational force in the following cases by expressing Coulomb force (F_C) in terms of the gravitational force (F_G)
  - Between the two protons separated by 1m
  - Between the two protons separated by an arbitrary distance R
  - Between the two electrons separated by 1m
  - Between the two electrons separated by an arbitrary distance R
- Five points each, totaling 20 points
- BE SURE to show all the details of your own work, including all formulae, proper references to them and explanations
- Please be sure to staple them before the submission
- Due at the beginning of the class Monday, June 10

SP#2 – Angels & Demons

- Compute the total possible energy released from an annihilation of x-grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class Wednesday, June. 12

Example 21 – 12



- **Uniformly charged disk**: Charge is distributed uniformly over a thin circular disk of radius R. The charge per unit area (C/m^2) is σ. Calculate he electric field at a point P on the axis of the disk, a distance z above its center.

How do we solve this problem?

First, compute the magnitude of the field (dE) at point P due to the charge (dQ) on the ring of infinitesimal width dr.

From the result of example 21 – 11 (please do this problem yourself)

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{zdQ}{\left(z^2+r^2\right)^{3/2}}$$

Since the surface charge density is constant, σ, and the ring has an area of $2\pi rdr$, the infinitesimal charge of dQ is

$$dQ = 2\pi\sigma rdr$$

So the infinitesimal field dE can be written

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{zdQ}{\left(z^2+r^2\right)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi z\sigma}{\left(z^2+r^2\right)^{3/2}}rdr = \frac{\sigma z}{2\varepsilon_0}\frac{r}{\left(z^2+r^2\right)^{3/2}}dr$$

Example 21 – 12 cnt'd

Now integrating dE over 0 through R, we get

$$E=\int dE=\int_0^R \frac{1}{4\pi\varepsilon_0}\frac{2\pi z\sigma}{\left(z^2+r^2\right)^{3/2}}r\,dr=\frac{z\sigma}{2\varepsilon_0}\int_0^R \frac{r}{\left(z^2+r^2\right)^{3/2}}dr$$

$$=\frac{\sigma}{2\varepsilon_0}\left[-\frac{z}{\left(z^2+r^2\right)^{1/2}}\right]_0^R=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\left(z^2+R^2\right)^{1/2}}\right]$$

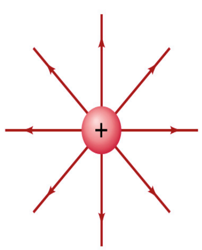
What happens if the disk has infinitely large area?

$$E=\frac{\sigma}{2\varepsilon_0}\left[1-\frac{1}{\left(z^2+R^2\right)^{1/2}}\right]\Rightarrow E=\frac{\sigma}{2\varepsilon_0}$$

So the electric field due to an evenly distributed surface charge with density, σ, is $\boxed{E=\frac{\sigma}{2\varepsilon_0}}$

Field Lines

- The electric field is a vector quantity. Thus, its magnitude can be expressed by the length of an arrow and the direction by the direction the arrowhead points.
- Since the field permeates through the entire space, drawing vector arrows is not a good way of expressing the field.
- Electric field lines are drawn to indicate the direction of the force due to the given field on a **positive test charge**.
  - Number of lines crossing unit area perpendicular to E is proportional to the magnitude of the electric field.
  - The closer the lines are together, the stronger the electric field in that region.
  - Start on positive charges and end on negative charges.



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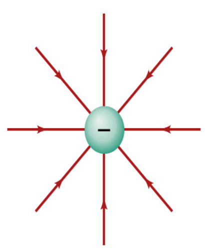

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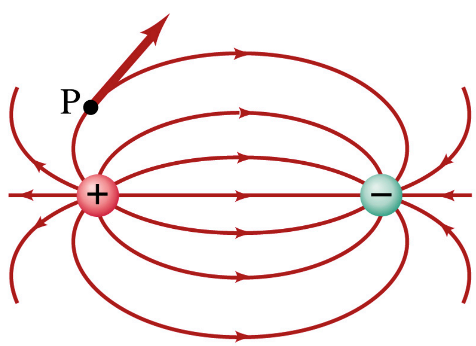
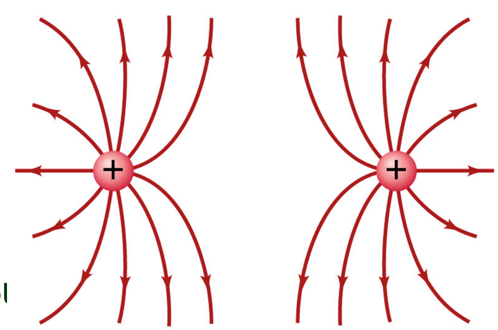
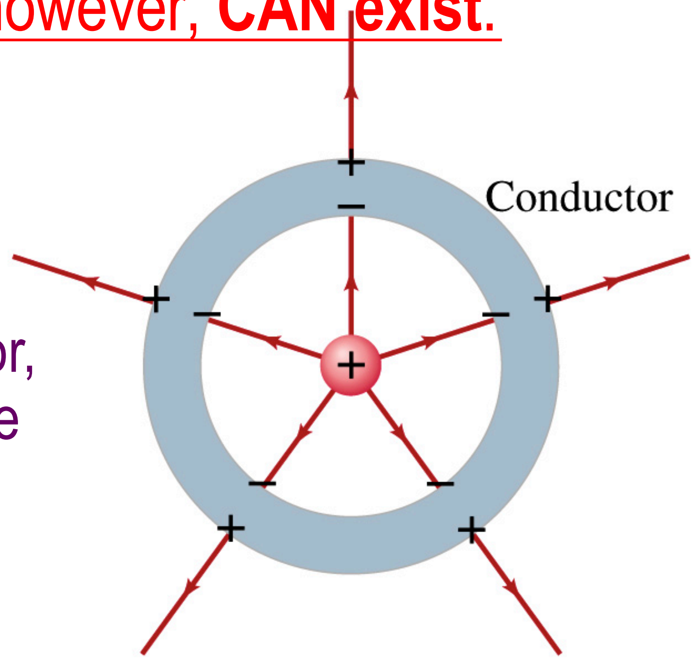




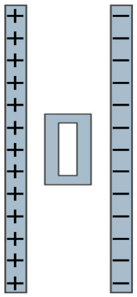
Earth's G-field lines

Electric Field and Conductors

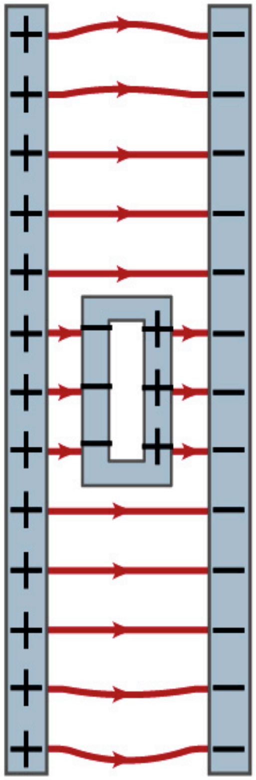
- The electric field **inside a conductor** is **ZERO** in static situation. (If the charge is at rest.) Why?
  - If there were an electric field within a conductor, there would be a force on its free electrons.
  - The electrons will move until they reached the position where the electric field becomes zero.
  - Electric field **inside a non-conductor**, however, **CAN exist**.
- Consequences of the above
  - Any net charge on a conductor distributes itself on the surface.
  - Although no E field exists inside a conductor, the field can exist outside the conductor due to induced charges on the surface
  - The electric field is always perpendicular to the surface outside of a conductor.



Example 21-13



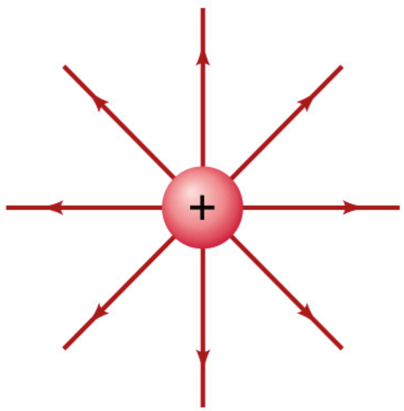
- **Shielding, and safety in a storm.** A hollow metal box is placed between two parallel charged plates. What is the field like in the box?
- If the metal box were solid
  - The free electrons in the box would redistribute themselves along the surface so that the field lines would not penetrate into the metal.



- The free electrons do the same in hollow metal boxes just as well as it did in a solid metal box.
- Thus a conducting box is an effective device for shielding. ➔ Faraday cage
- So what do you think will happen if you were inside a car when the car was struck by a lightening?

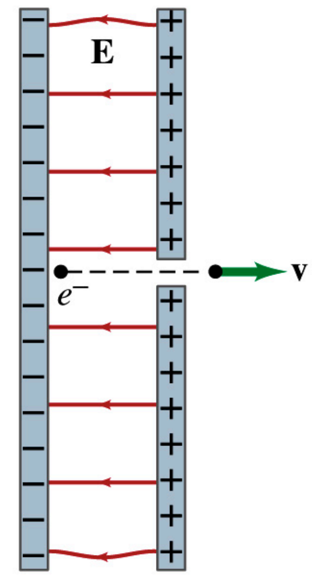
Motion of a Charged Particle in an Electric Field

- If an object with an electric charge q is at a point in space where the electric field is **E**, the force exerting on the object is $\vec{F} = q\vec{E}$.
- What do you think will happen to the charge?
  - Let's think about the cases like these on the right.
  - The object will move along the field line…Which way?
    - Depends on the sign of the charge
  - The charge gets accelerated under an electric field.



(a)

Example 21 – 14



- **Electron accelerated by electric field.** An electron (mass m = 9.1×10^{-31} kg) is accelerated in a uniform field E ($E = 2.0 \times 10^4$ N/C) between two parallel charged plates. The separation of the plates is 1.5cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate. (a) With what speed does it leave the hole? (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $$a = \frac{F}{m} = \frac{qE}{m}$$

Between the plates the field **E** is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{\left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^{4} N / C\right)}{\left(9.1 \times 10^{-31} kg\right)} = 3.5 \times 10^{15}\, m/s^2$$

Example 21 – 14

Since the travel distance is $1.5\text{x}10^{-2}$m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \quad \therefore v = \sqrt{2ax} = \sqrt{2 \cdot 3.5 \times 10^{15} \cdot 1.5 \times 10^{-2}} = 1.0 \times 10^7\, m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

- (b) Show that the gravitational force can be ignored. Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6 \times 10^{-19} C\right)\left(2.0 \times 10^4 N/C\right) = 3.2 \times 10^{-15} N$$

The magnitude of the gravitational force on the electron is

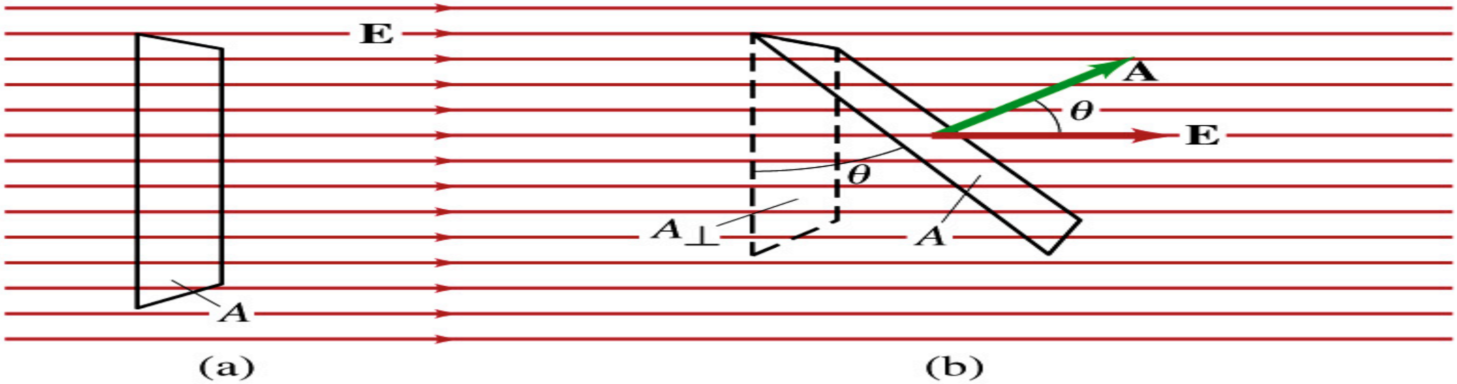
$$F_G = mg = 9.8\, m/s^2 \times \left(9.1 \times 10^{-31} kg\right) = 8.9 \times 10^{-30} N$$

Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Gauss' Law

- Gauss' law states the relationship between the electric charge and the electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

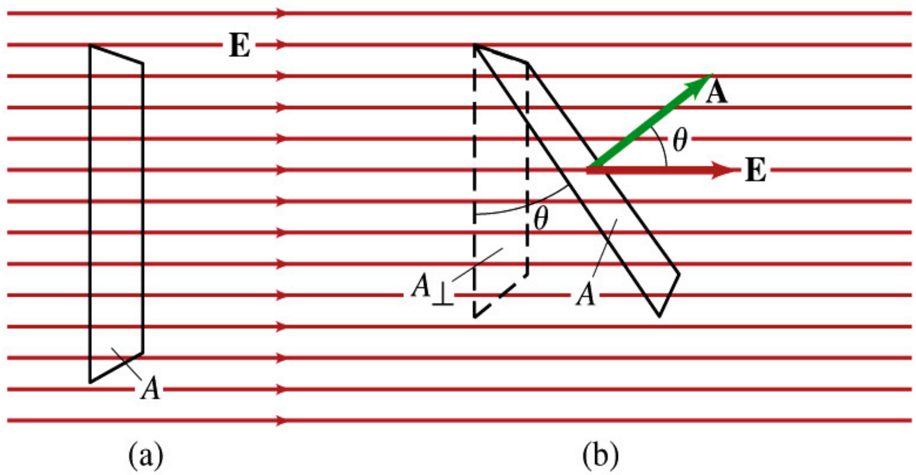
Electric Flux



- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux Φ_E is defined as
  - Φ_E=EA, if the field is perpendicular to the surface
  - Φ_E=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

Example 22 – 1

- **Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux if the angle is 30 degrees?



The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

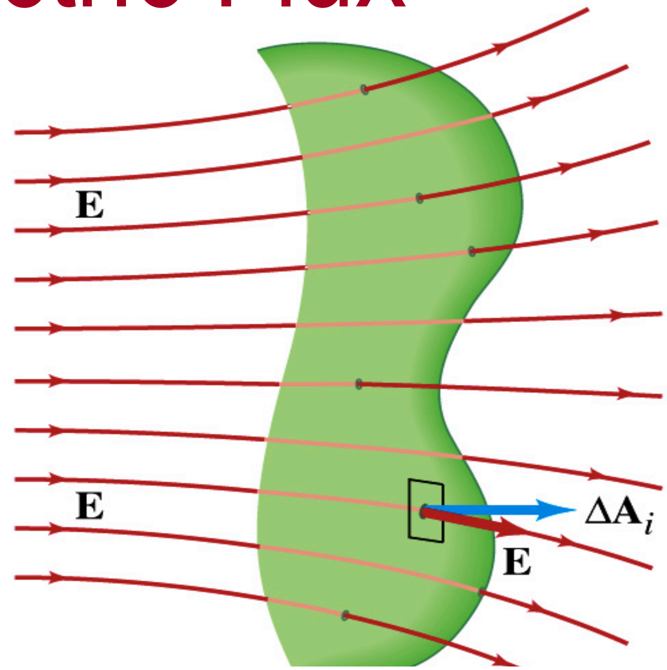
$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot\left(0.1\times0.2m^2\right) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees, we obtain

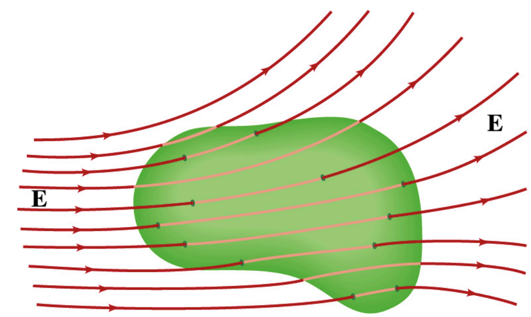
$$\Phi_E = EA\cos30^\circ = (200N/C)\cdot\left(0.1\times0.2m^2\right)\cos30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta \mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is approximately



$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

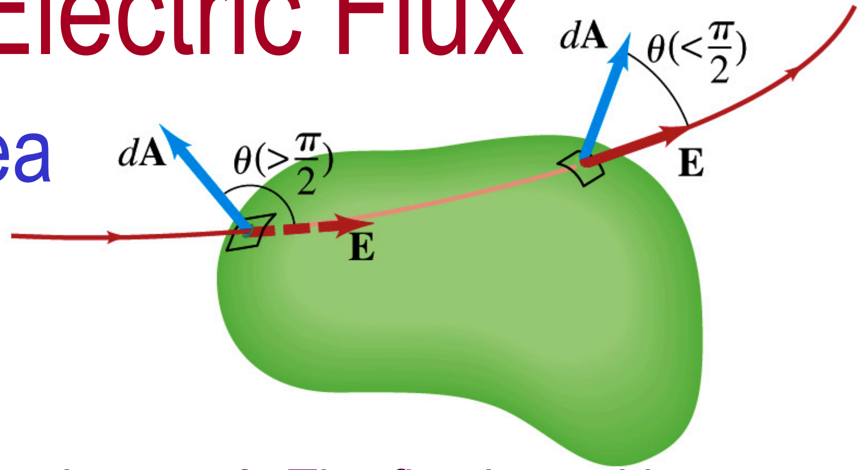


- In the limit where $\Delta \mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

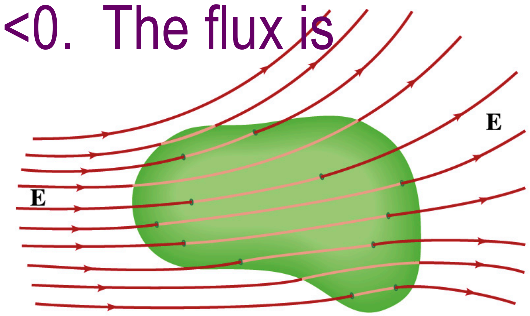
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

Generalization of the Electric Flux



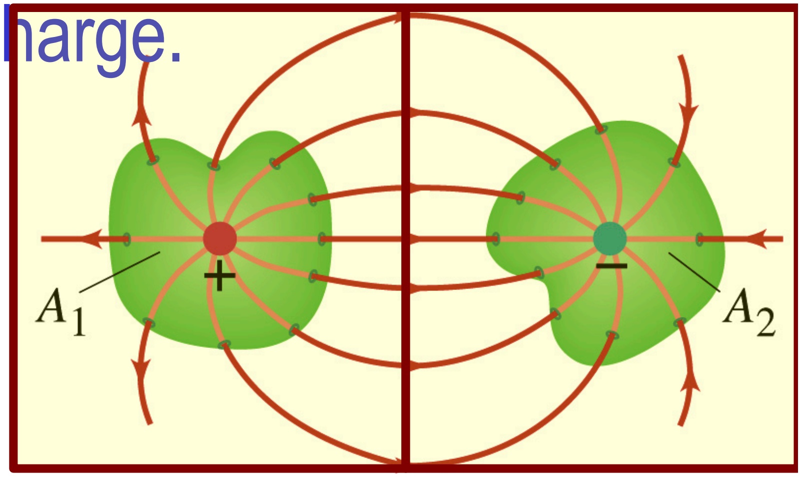
- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $|\theta|<\pi/2$ and $\cos\theta>0$. The flux is positive.
  - For the line coming into the volume, $|\theta|>\pi/2$ and $\cos\theta<0$. The flux is negative.
  - If $\Phi_E>0$, there is net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.



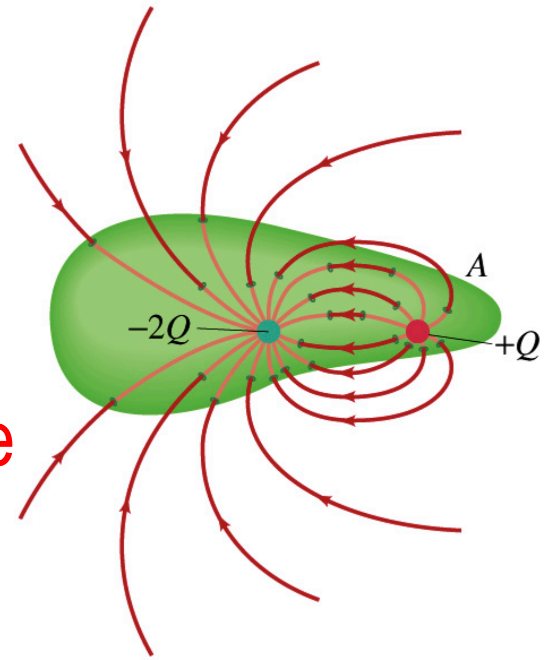
- In the above figures, each field line that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E}\cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

Generalization of the Electric Flux

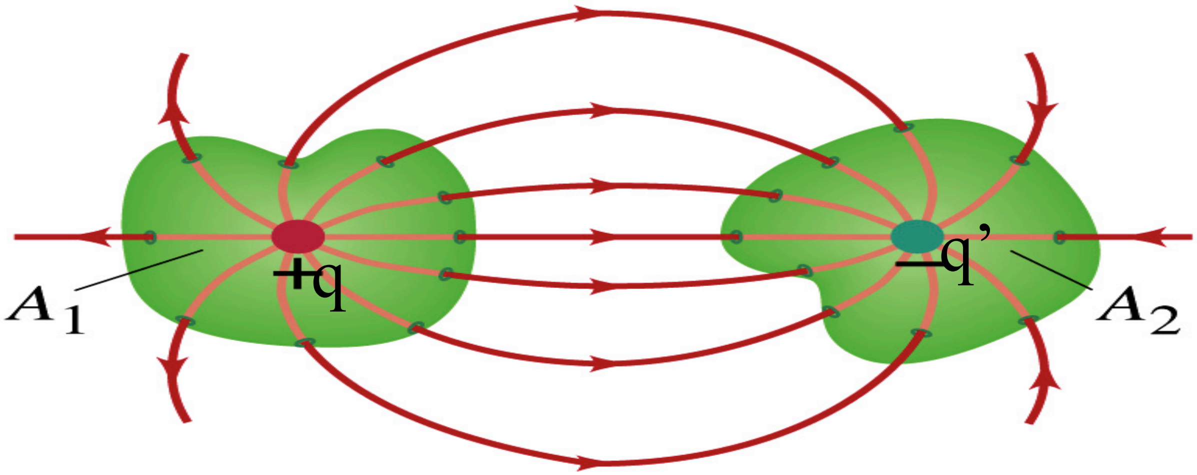
- The field line starts or ends only on a charge.
- Sign of the net flux on the surface A_1?
  - Net outward flux (positive flux)
- How about A_2?
  - Net inward flux (negative flux)



- What is the flux in the figure bottom right?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.

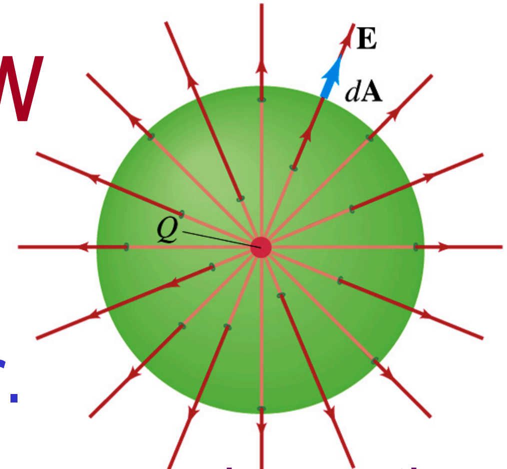


Gauss' Law



- Let's consider the case in the above figure.
- What are the results of the closed integral of the Gaussian surfaces A_1 and A_2?
  - For A_1 $\oint \vec{E} \cdot d\vec{A} = \frac{+q}{\varepsilon_0}$
  - For A_2 $\oint \vec{E} \cdot d\vec{A} = \frac{-q'}{\varepsilon_0}$

Coulomb's Law from Gauss' Law



- Let's consider a charge Q enclosed inside our imaginary Gaussian surface of sphere of radius r.
  - Since we can choose any surface enclosing the charge, we choose the simplest possible one! ☺
- The surface is symmetric about the charge.
  - What does this tell us about the field E?
    - Have the same magnitude (uniform) at any point on the surface
    - Points radially outward parallel to the surface vector d**A**.
- The Gaussian integral can be written as

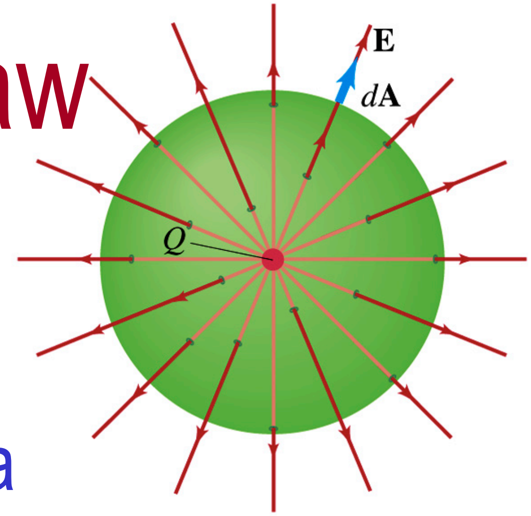
$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E \oint dA = E\left(4\pi r^2\right) = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

Solve for E

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Electric Field of Coulomb's Law

Gauss' Law from Coulomb's Law



- Let's consider a single static point charge Q surrounded by an imaginary spherical surface.
- Coulomb's law tells us that the electric field at a spherical surface of radius r is $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

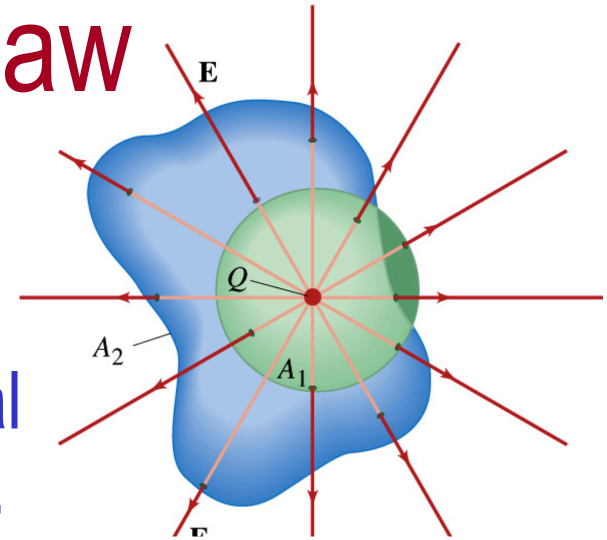
- Performing a closed integral over the surface, we obtain

$$\oint \vec{E}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}\cdot d\vec{A} = \oint \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}dA$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\oint dA = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\left(4\pi r^2\right) = \frac{Q}{\varepsilon_0}$$

Gauss' Law

Gauss' Law from Coulomb's Law Irregular Surface



- Let's consider the same single static point charge Q surrounded by a symmetric spherical surface A_1 and a randomly shaped surface A_2.
- What is the difference in the total number of field lines due to the charge Q, passing through the two surfaces?
  - None. What does this mean?
    - The total number of field lines passing through the surface is the same no matter what the shape of the enclosed surface is.
  - So we can write: $\oint_{A_1} \vec{E}\cdot d\vec{A} = \oint_{A_2} \vec{E}\cdot d\vec{A} = \dfrac{Q}{\varepsilon_0}$
  - What does this mean?
    - The flux due to the given enclosed charge is the same independent of the shape of the surface enclosing it is. ➔ Gauss' law, $\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, is valid for any surface surrounding a single point charge Q.

Gauss' Law w/ more than one charge

- Let's consider several charges inside a closed surface.
- For each charge, Q_i inside the chosen closed surface,

$$\oint \vec{E}_i \cdot d\vec{A} = \frac{Q_i}{\varepsilon_0}$$

What is $\vec{E}_i$?

The electric field produced by Q_i alone!

- Since electric fields can be added vectorially, following the superposition principle, the total field **E** is equal to the sum of the fields due to each charge $\vec{E} = \sum \vec{E}_i$ and any external fields. So

$$\oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_{ext} + \sum \vec{E}_i \right) \cdot d\vec{A} = \frac{\sum Q_i}{\varepsilon_0} = \frac{Q_{encl}}{\varepsilon_0}$$

What is Q_{encl}?

The total enclosed charge!

- The value of the flux depends only on the charge enclosed in the surface!! ➔ Gauss' law.

So what is Gauss' Law good for?

- Derivation of Gauss' law from Coulomb's law is only valid for **static electric charge**.
- Electric field can also be produced by changing magnetic fields.
  - Coulomb's law cannot describe this field while Gauss' law is still valid
- Gauss' law is more general than Coulomb's law.
  - Can be used to obtain electric field, forces or obtain charges

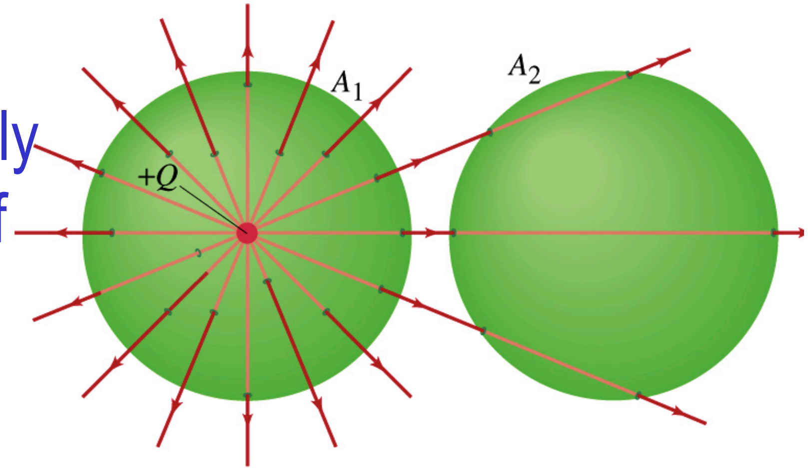
Gauss' Law: Any **differences** between the input and output flux of the electric field over any enclosed surface is due to the charge inside that surface!!!

Solving problems with Gauss' Law

- Identify the symmetry of the charge distributions
- Draw an appropriate Gaussian surface, making sure it passes through the point you want to know the electric field
- Use the symmetry of charge distribution to determine the direction of E at the point of the Gaussian surface
- Evaluate the flux
- Calculate the enclosed charge by the Gaussian surface
    - Ignore all the charges outside the Gaussian surface
- Equate the flux to the enclosed charge and solve for E

Example 22 – 2

Flux from Gauss' Law: Consider two Gaussian surfaces, A_1 and A_2, shown in the figure. The only charge present is the charge +Q at the center of surface A_1. What is the net flux through each surface A_1 and A_2?



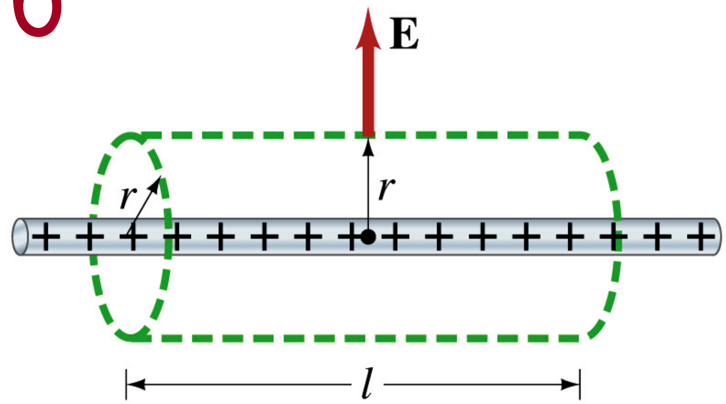
- The surface A_1 encloses the charge +Q, so from Gauss' law we obtain the total net flux

$$\oint \vec{E} \cdot d\vec{A} = \frac{+Q}{\varepsilon_0}$$

- The surface A_2 the charge, +Q, is outside the surface, so the total net flux is 0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{0}{\varepsilon_0} = 0$$

Example 22 – 6



Long uniform line of charge: A very long straight wire possesses a uniform positive charge per unit length, λ. Calculate the electric field a point near but outside the wire, far from the ends.

- Which direction do you think the field due to the charge on the wire is?
  - Radially outward from the wire, the direction of radial vector **r**.
- Due to cylindrical symmetry, the field is the same on the Gaussian surface of a cylinder surrounding the wire.
  - The end surfaces do not contribute to the flux at all. Why?
    - Because the field vector **E** is perpendicular to the surface vector d**A**.
- From Gauss' law

$$\oint \vec{E} \cdot d\vec{A} = E \oint dA = E(2\pi r l) = \frac{Q_{encl}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$$

Solving for E

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

A Brain Teaser of Electric Flux

- What would change the electric flux through a circle lying in the xz plane where the electric field is (10N/C)**j**?
  1. Changing the magnitude of the electric field
  2. Changing the surface area of the circle
  3. Tipping the circle so that it is lying in the xy plane
  4. All of the above
  5. None of the above

Gauss' Law Summary

- The precise relationship between flux and the enclosed charge is given by Gauss' Law $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$
  - ε_0 is the permittivity of free space in the Coulomb's law
- A few important points on Gauss' Law
  - Freedom to choose!!
    - The surface integral is performed over the value of **E** on a closed surface of our choice in any given situation.
  - Test of existence of electrical charge!!
    - The charge Q_{encl} is the net charge enclosed by the arbitrary closed surface of our choice.
  - Universality of the law!
    - It does NOT matter where or how much charge is distributed inside the surface. Gauss' law still applies!
  - The charge outside the surface does not contribute to Q_{encl}. Why?
    - The charge outside the surface might impact field lines but not the total number of lines entering or leaving the surface.