

# PHYS 1441 – Section 001

## Lecture #6

*Tuesday, June 11, 2019*

*Dr. Jaehoon Yu*

- Chapter 23
  - Electric Potential Energy
  - Electric Potential due to Point Charges
  - Shape of the Electric Potential
  - $V$  due to Charge Distributions



# Announcements

- Reading Assignment
  - CH 23.9



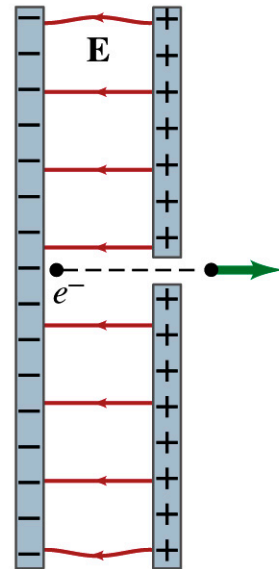
# Reminder: SP#2 – Angels & Demons

- Compute the total possible energy released from an annihilation of  $x$ -grams of anti-matter and the same quantity of matter, where  $x$  is the last two digits of your SS#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in  $x$  ns, where  $x$  is again the first two digits of your SS#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class tomorrow, Wednesday, June. 12



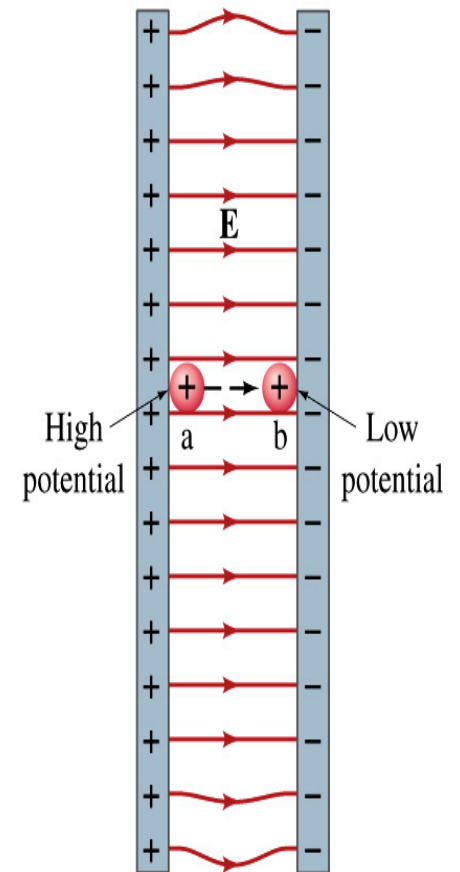
# Reminder: Special Project #3

- **Particle Accelerator.** A charged particle of mass  $M$  with charge  $-Q$  is accelerated in the uniform field  $E$  between two parallel charged plates whose separation is  $D$  as shown in the figure on the right. The charged particle is accelerated from an initial speed  $v_0$  near the negative plate and passes through a tiny hole in the positive plate as shown in the figure right.
  - Derive the formula for the electric field  $E$  to accelerate the charged particle to a fraction  $f$  of the speed of light  $c$ . Express  $E$  in terms of  $M$ ,  $Q$ ,  $D$ ,  $f$ ,  $c$  and  $v_0$ .
  - (a) Using the Coulomb force and kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. ( 8 points)
  - (c) Using the formula above, evaluate the strength of the electric field  $E$  to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Please do NOT copy but have your own handwritten answer!
- Due beginning of the class Monday, June 17



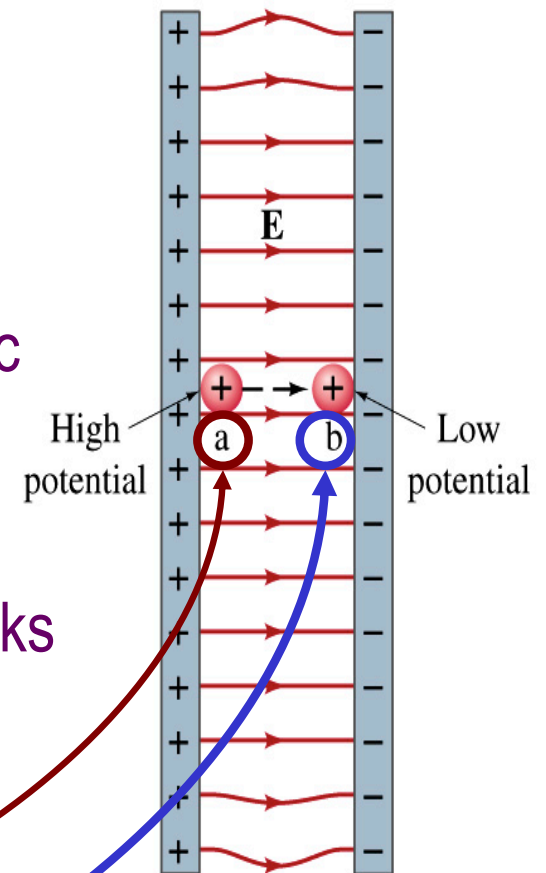
# Electric Potential Energy

- How would you define the change in electric potential energy  $U_b - U_a$ ?
  - The potential to work gained by the charge as it moves from point  $a$  to point  $b$ .
  - The negative work done on the charge by the electric force to move it from  $a$  to  $b$ .
- Let's consider an electric field between two parallel plates w/ equal but opposite charges
  - The field between the plates is uniform since the gap is small and the plates are infinitely long...
- What happens when we place a small charge,  $+q$ , on a point at the positive plate and let go?
  - The electric force will accelerate the charge toward the negative plate.
  - What kind of energy does this charged particle gain?
    - Kinetic energy



# Electric Potential Energy

- What does this mean in terms of energies?
  - The electric force is a conservative force.
  - Thus, the mechanical energy ( $K+U$ ) is conserved under this force.
  - The positively charged object has only the electric potential energy (no KE) at the positive plate.
  - The electric potential energy decreases and
  - Turns into kinetic energy as the electric force works on the charged object, and the charged object gains speed.



- Point of the greatest potential energy for

– Positively charged object

– Negatively charged object

$PE = U$	0
$KE = 0$	K
$ME = U$	K
$U + K$	

# Electric Potential

- How is the electric field defined?
  - Electric force per unit charge:  $F/q$
- We can define electric potential (potential) as
  - The electric potential energy per unit charge
  - This is like the voltage of a battery...
- Electric potential is written with the symbol  $V$ 
  - If a positive test charge  $q$  has the potential energy  $U_a$  at the point  $a$ , the electric potential of the charge at that point is

$$V_a = \frac{U_a}{q}$$



# Electric Potential, cont'd

- Since only the difference in potential energy is meaningful, only the potential difference between two points is measurable
- What happens when the electric force does a “positive work”?
  - The charge gains kinetic energy
  - Electric potential energy of the charge decreases
- Thus the difference in potential energy is the same as the negative of the work,  $W_{ba}$ , done on the charge by the electric field to move the charge from point a to b.
- The potential difference  $V_{ba}$  is

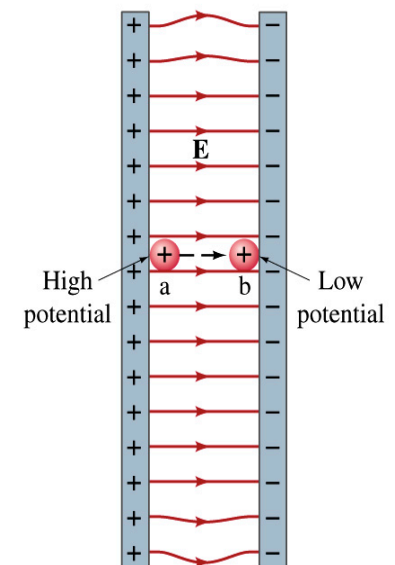
$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = \frac{-W_{ba}}{q}$$

- Electric potential is independent of the test charge!! Unit?



# A Few Things about the Electric Potential

- What does the electric potential depend on?
  - Other charges that creates the field
  - What about the test charge?
    - No, the electric potential is independent of the test charge
    - Test charge gains potential energy by existing in the potential created by other charges
- Which plate is at the higher potential?
  - Positive plate. Why?
    - Since positive charge has the greatest potential energy on it.
  - What happens to the positive charge if it is let go?
    - It moves from higher potential to lower potential
  - How about the negative charge?
    - Its potential energy is higher on the negative plate. Thus, it moves from negative plate to positive. **Potential difference is still the same**
- The unit of the electric potential is Volt (V).
- From the definition,  $1V = 1J/C$ .

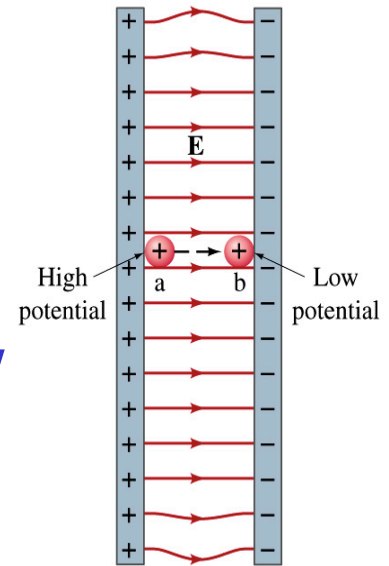


Zero point of electric potential can be chosen arbitrarily.

Often the ground, a conductor connected to Earth, is zero.

# Example 23 – 1

**A negative charge:** Suppose a negative charge, such as an electron, is placed at point *b* in the figure. If the electron is free to move, will its electric potential energy increase or decrease? How will the electric potential change?



- An electron placed at point *b* will move toward the positive plate since it was released at its highest **potential energy** point.
- It will gain kinetic energy as it moves toward left, decreasing its potential energy.
- The electron, however, moves from the point *b* at a lower potential to point *a* at a higher **potential**.  $\Delta V = V_a - V_b > 0$ .
- This is because the **potential is generated by the charges on the plates** not by the electron.

# Electric Potential and Potential Energy

- What is the definition of the electric potential?
  - The potential energy difference per unit charge  $V_{ba} = \frac{U_b - U_a}{q}$
- OK, then, how would you express the potential energy that a charge  $q$  would obtain when it is moved between point  $a$  and  $b$  with the potential difference  $V_{ba}$ ?

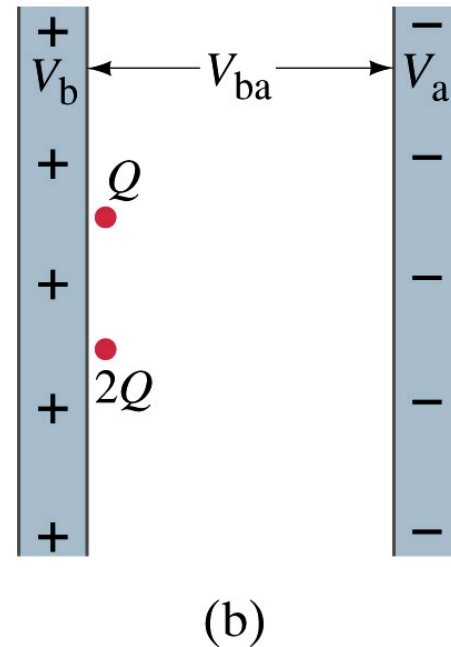
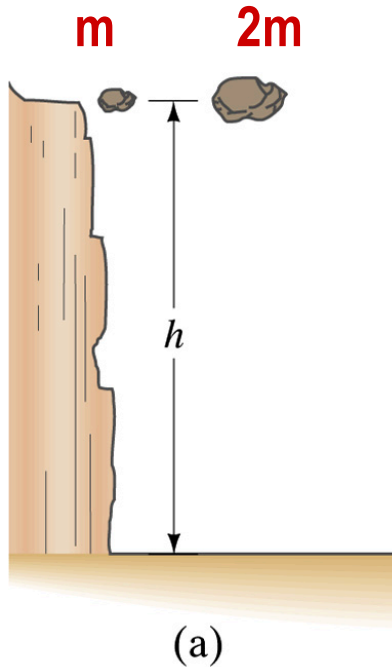
$$U_b - U_a = q(V_b - V_a) = qV_{ba}$$

- In other words, if an object with charge  $q$  moves through a potential difference  $V_{ba}$ , its potential energy changes by  $qV_{ba}$ .
- So based on this, how differently would you describe the electric potential in words?
  - A measure of how much energy an electric charge can acquire in a given situation
  - A measure of how much work a given charge can do.



# Comparisons of Potential Energies

- Let's compare gravitational and electric potential energies



- What are the potential energies of the rocks?
  - $mgh$  and  $2mgh$
- Which rock has a bigger potential energy?
  - The rock with a larger mass
- Why?
  - It's got a bigger mass.
- What are the potential energies of the charges?
  - $QV_{ba}$  and  $2QV_{ba}$
- Which object has a bigger potential energy?
  - The object with a larger charge.
- Why?
  - It's got a bigger charge.

**The potential is the same but the heavier rock or larger charge can do a greater work.**

# Electric Potential and Potential Energy

- The electric potential difference gives potential energy or the possibility to perform work based on the charge of the object.
- So what is happening in a battery or a generator?
  - They maintain a potential difference.
  - The actual amount of energy used or transformed depends on how much charge flows.
  - What is the potential difference maintained by a car's battery?
    - 12Volts
  - If for a given period, 5C charge flows through the headlight lamp, what is the total energy transformed?
    - $E_{\text{tot}} = 5\text{C} \cdot 12\text{V} = 60$  Umm... What is the unit? **Joules**
  - If it is left on twice as long?  $E_{\text{tot}} = 10\text{C} \cdot 12\text{V} = 120\text{J}$ .



# Some Typical Voltages

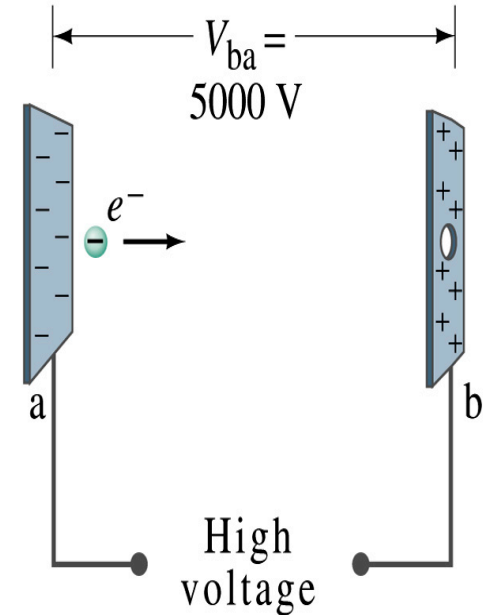
Sources	Approximate Voltage
Thundercloud to ground (~5km)	$10^8 \text{ V}$
High-Voltage Power Lines	$10^6 \text{ V}$
Power supply for TV tube	$10^4 \text{ V}$
Automobile ignition	$10^4 \text{ V}$
Household outlet	$10^2 \text{ V}$
Automobile battery	$12 \text{ V}$
Flashlight battery	$1.5 \text{ V}$
Resting potential across nerve membrane	$10^{-1} \text{ V}$
Potential changes on skin (EKG and EEG)	$10^{-4} \text{ V}$

In a typical lightening strike,  $15\text{C}$  of electrons are released in  $500\mu\text{s}$ . What is the total kinetic energy of these electrons when they strike ground? What is the power released during this strike? What do you think will happen to a tree hit by this lightening?



# Example 23 – 2

**Electrons in TV tube:** Suppose an electron in the picture tube of a television set is accelerated from rest through a potential difference  $V_{ba}=+5000\text{V}$ . (a) What is the change in potential energy of the electron? (b) What is the speed of the electron ( $m=9.1\times 10^{-31}\text{kg}$ ) as a result of this acceleration? (c) Repeat for a proton ( $m=1.67\times 10^{-27}\text{kg}$ ) that accelerates through a potential difference of  $V_{ba}=-5000\text{V}$ .



- (a) What is the charge of an electron?

$$- \quad e = -1.6 \times 10^{-19} \text{ C}$$

- So what is the change of its potential energy?

$$\Delta U = qV_{ba} = eV_{ba} = (-1.6 \times 10^{-19} \text{ C})(+5000 \text{ V}) = -8.0 \times 10^{-16} \text{ J}$$

## Example 23 – 2

- (b) Speed of the electron?
  - The entire potential energy of the electron turns to its kinetic energy. Thus the equation is

$$\Delta K = \frac{1}{2} m_e v_e^2 - 0 = W = -\Delta U = -eV_{ba} = \\ = -(-1.6 \times 10^{-19} \text{ C}) 5000 \text{ V} = 8.0 \times 10^{-16} \text{ J}$$

$$v_e = \sqrt{\frac{2 \times eV_{ba}}{m_e}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{9.1 \times 10^{-31}}} = 4.2 \times 10^7 \text{ m/s}$$

- (C) Speed of the proton?

$$\Delta K = \frac{1}{2} m_p v_p^2 - 0 = W = -\Delta U = -\{(-e)(-V_{ba})\} = -eV_{ba} = 8.0 \times 10^{-16} \text{ J}$$

$$v_p = \sqrt{\frac{2 \times eV_{ba}}{m_p}} = \sqrt{\frac{2 \times 8.0 \times 10^{-16}}{1.67 \times 10^{-27}}} = 9.8 \times 10^5 \text{ m/s}$$



# Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, how are these two quantities related?



# Electric Potential and Electric Field

- **Potential energy** change is expressed in terms of a conservative force (point **a** at a higher potential)

$$U_b - U_a = -\vec{F} \cdot \vec{D} = -W_C$$

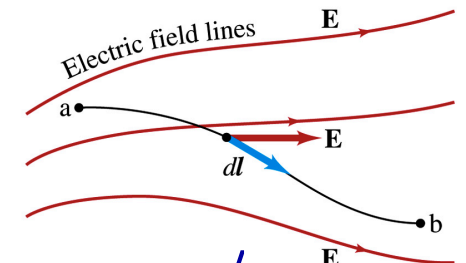
- For the electrical case, we are more interested in the **potential** difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q} \cdot \vec{D} = -\vec{E} \cdot \vec{D} = -ED \cos \theta$$

- This formula can be used to determine  $V_{ba}$  when the electric field is given.

- When the field is uniform

$$V_b - V_a = -\vec{E} \cdot \vec{D} = -ED \cos \theta = -Ed \quad \text{so} \quad E = -V_{ba} / d$$



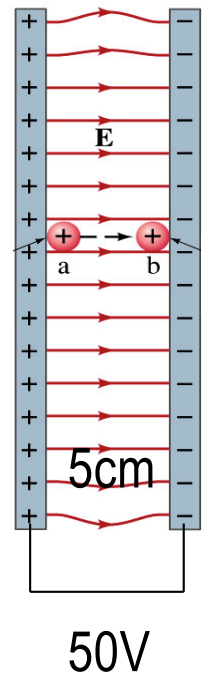
What does “-” sign mean? The direction of  $\vec{E}$  is along that of decreasing potential.

Unit of the electric field in terms of potential?  $\text{V/m}$  Can you derive this from  $\text{N/C}$ ?

# Example

## Uniform electric field obtained from voltage:

Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.



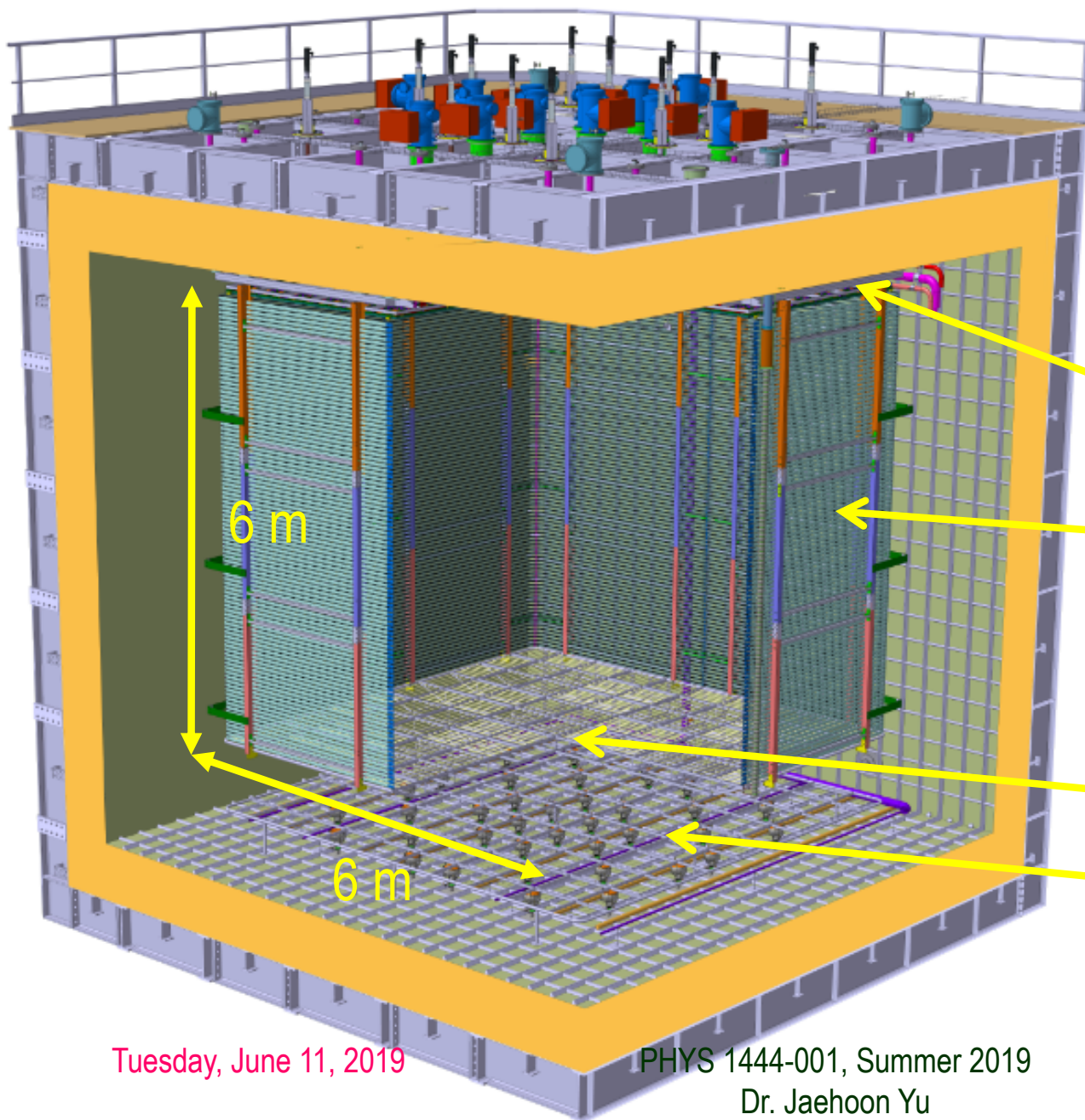
What is the relationship between electric field and the potential for a uniform field?

$$V = Ed$$

**Solving for E**  $E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5 \times 10^{-2}m} = 1000V/m$

Which direction is the field? Direction of decreasing potential!

# E Field in ProtoDUNE DP



- What is the electric field between the cathode and the ground grid?
- What do you need to know?
  - $V_{\text{cathode}} = -300\text{kV}$
  - $d = 70\text{cm}$

Charge Readout Planes

Field Cage (common structural elements with SP)

Cathode

Ground Grid

# Electric Potential due to Point Charges

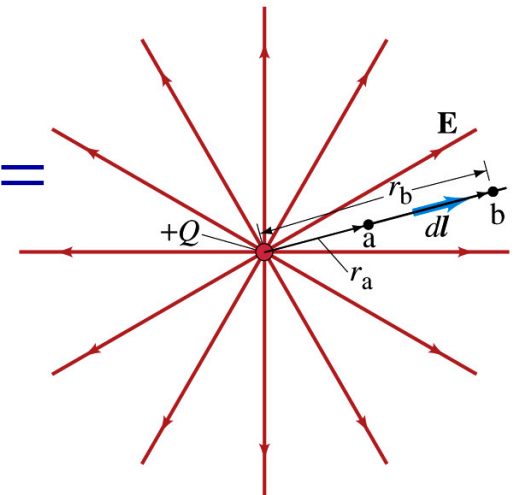
- What is the electric field by a single point charge  $Q$  at a distance  $r$ ?

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2}$$

- Electric potential due to the field  $E$  for moving from point  $r_a$  to  $r_b$  in the radial direction away from the charge  $Q$  is

$$V_b - V_a = - \int_{r_a}^{r_b} \vec{E} \cdot d\vec{l} = - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{\hat{r}}{r^2} \cdot \hat{r} dr =$$

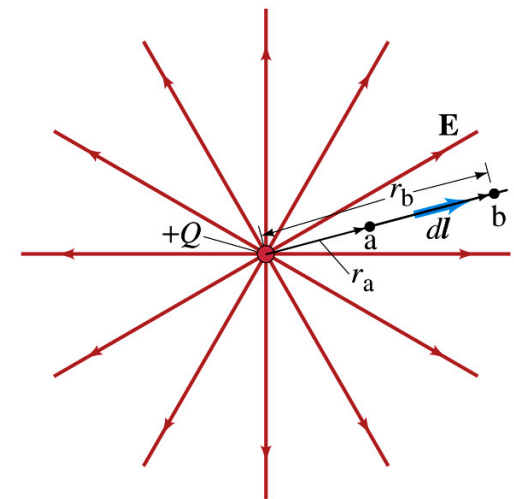
$$= - \frac{Q}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr = \frac{Q}{4\pi\epsilon_0} \left( \frac{1}{r_b} - \frac{1}{r_a} \right)$$



# Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose  $V_b = 0$  at  $r_b = \infty$ .
- The electrical potential  $V$  at a distance  $r$  from a single point charge  $Q$  is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of the potential difference by a single point charge between  $r$  and infinity