PHYS 1444 – Section 001
Lecture #9

Monday, June 17, 2019

Dr. Jaehoon Yu

- Chapter 24 Capacitance etc..
  - Determination of Capacitance
  - Capacitors in Series or Parallel
  - Electric Energy Storage
  - Effect of Dielectric
  - Molecular description of Dielectric Material
- Chapter 25
  - Electric Current and Resistance

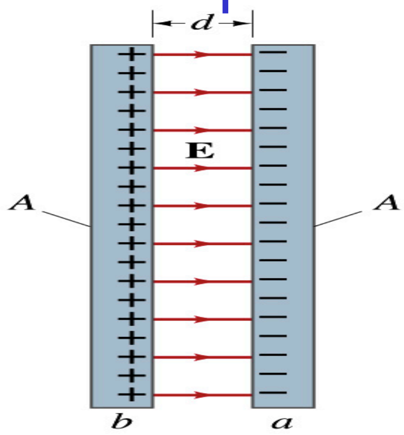
Today's homework is #7, due 11pm, Saturday, June 22!!

Announcements

- Quadruple extra credit
  - Meeting with the Nobel Laureate from 12pm – 1pm this Wednesday in CPB303
  - Sign in sheet will be available for the extra credit
  - Lunch may be provided
- Mid-term exam
  - In class tomorrow, Tuesday, June 18
  - Comprehensive exam which covers CH21.1 through CH24.6 plus appendices A and B the math refresher
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Quiz 2 results
  - Class Average: 31.3/60
    - Equivalent to 52.2/100
    - Previous results: 44.3/100
  - Top score: 49/60

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.
- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, and so E is uniform.
  - E for parallel plates is E=σ/ε_0, σ is the surface charge density.



- E and V are related $V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l}$
- Since we take the integral from lower potential (a) to higher potential (b) along the field lines, we obtain
- $V_{ba} = V_b - V_a = -\int_a^b E\,dl\cos 180^\circ = +\int_a^b E dl = \int_a^b \frac{\sigma}{\varepsilon_0} dl = \int_a^b \frac{Q}{\varepsilon_0 A} dl = \frac{Q}{\varepsilon_0 A}\int_a^b dl = \frac{Q}{\varepsilon_0 A}(b-a) = \frac{Qd}{\varepsilon_0 A}$
- So from the formula (recall Q=CV!): $C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$
  - What do you notice?

C only depends on the area and the distance of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20cmx3.0cm and are separated by a 1.0mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12-V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85 \times 10^{-12}\, C^2/N \cdot m^2\right) \frac{0.2 \times 0.03 m^2}{1 \times 10^{-3}\, m} = 53 \times 10^{-12}\, C^2/N \cdot m = 53\, pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53 \times 10^{-12}\, C^2/N \cdot m\right)\left(12V\right) = 6.4 \times 10^{-10}\, C = 640\, pC$$

Example 24 – 1

(C) Using the formula for the electric field in two parallel plates

$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{A\varepsilon_0}=\frac{6.4\times10^{-10}C}{6.0\times10^{-3}m^2\times8.85\times10^{-12}C^2/N\cdot m^2}=1.2\times10^4\,N/C=1.2\times10^4\,V/m$$

Or, since $V=Ed$ we can obtain $E=\frac{V}{d}=\frac{12V}{1.0\times10^{-3}m}=1.2\times10^4\,V/m$

(d) Solving the capacitance formula for A, we obtain

$$C=\frac{\varepsilon_0 A}{d}$$

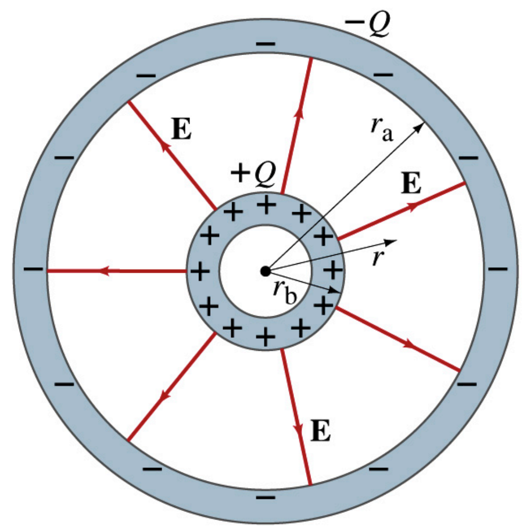
Solve for A

$$A=\frac{Cd}{\varepsilon_0}=\frac{1F\cdot1\times10^{-3}m}{\left(9\times10^{-12}C^2/N\cdot m^2\right)}\approx10^8m^2\approx100km^2$$

About 40% the area of Arlington (256km^2).

Example 24 – 3

Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell an equal but opposite charge –Q. Determine the capacitance of the two shells.



Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l} =$$

$$= -\int_a^b E\cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a - r_b}{r_b r_a}\right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a - r_b}$$

Capacitor Cont'd

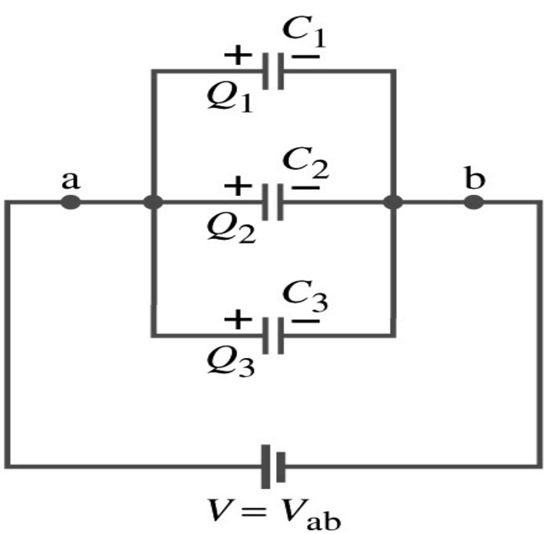
- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to the absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

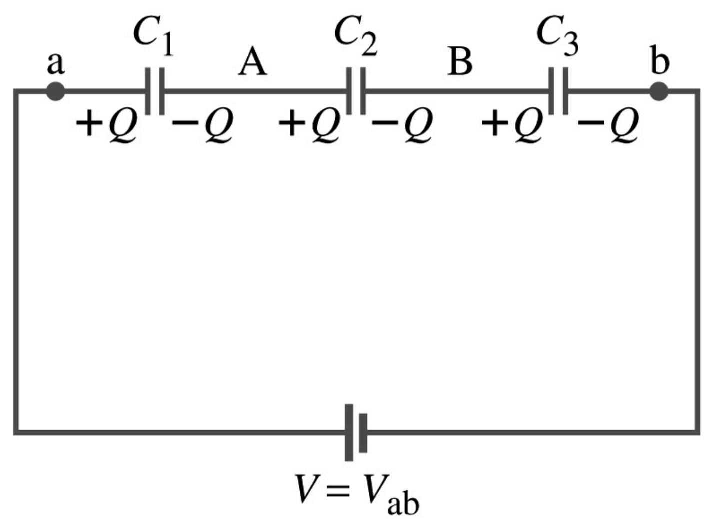
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \qquad \text{where} \quad r_a \to \infty$$

- So its capacitance is $C = \frac{Q}{V} = 4\pi\varepsilon_0 r_b$

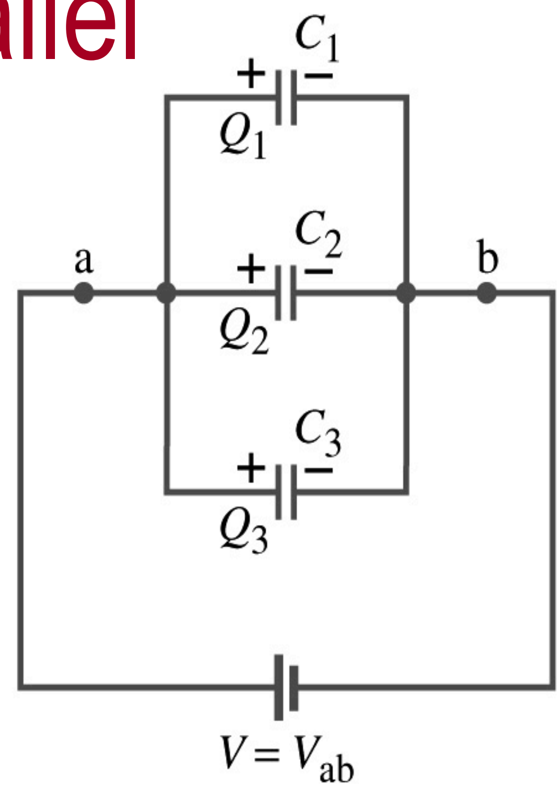
Capacitors in Series or Parallel

- Capacitors may be used in electric circuits
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - And includes a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel, in series or in combination





Capacitors in Parallel



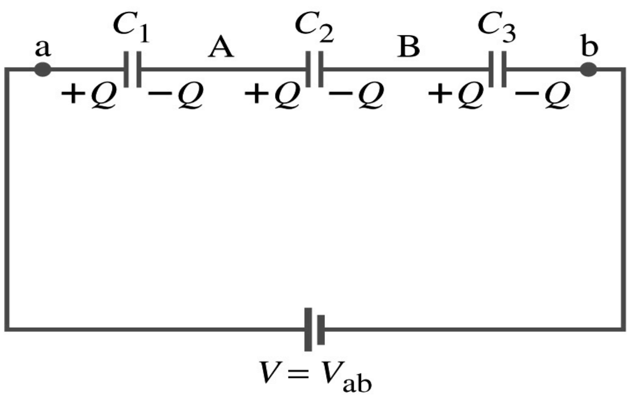
- Parallel arrangement provides the **same voltage** across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave the battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$
- Thus the equivalent capacitance in parallel is $C_{eq} = C_1 + C_2 + C_3$

What is the net effect?

The capacitance increases!!!

Capacitors in Series

- Series arrangement is more interesting
  - When the battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3.
  - Since capacitors in between were originally neutral, charges get induced to neutralize the ones in the middle.
  - So the charge on each capacitor plate is the same value, Q. (**Same charge**)



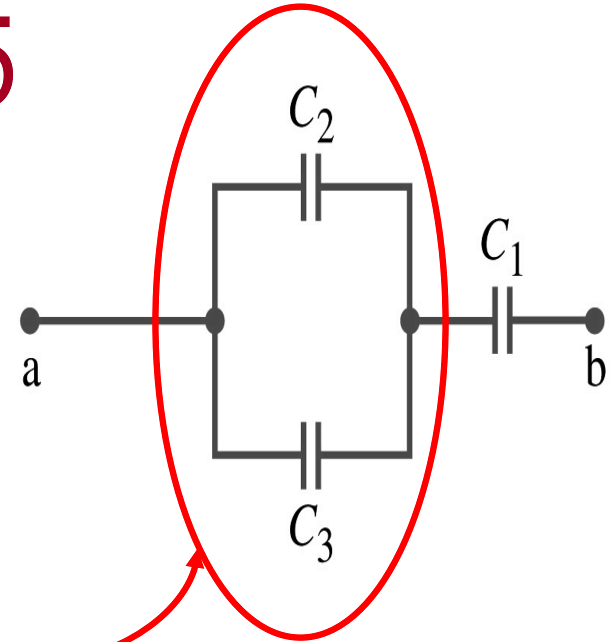
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=Q/C_1+Q/C_2+Q/C_3$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

What is the net effect?

The capacitance smaller than the smallest C!!!

Example 24 – 5



Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.

We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq1} = C_1 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the amount of the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge from one plate and put them on to the other.
  - Battery does this when it is connected to a capacitor.
- Capacitors do not get charged immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external work needs to overcome.
  - When some charge is on each plate, it requires work to add more charge due to the electric repulsion.

Electric Energy Storage

- The work needed to add a small amount of charge, dq, when a potential difference across the plate is V: dW=Vdq.
- Since V=q/C, the work needed to store total charge Q is

$$W = \int_0^Q V\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries the charges +Q and –Q is

$$U = \frac{Q^2}{2C}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24 – 8

Energy store in a capacitor: A camera flash unit stores energy in a 150μF capacitor at 200V. How much electric energy can be stored?

Using the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150\times10^{-6}F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$

Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, V=Ed and $C=\varepsilon_0 A/d$
- Thus the stored energy is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Ad$$

- Since Ad is the gap volume V, we can obtain the energy density, stored energy per unit volume, as

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Valid for any space that is vacuum

Electric energy stored per unit volume in any region of space is proportional to the square of E in that region.

Dielectrics

- Capacitors have an insulating sheet of material, called dielectric, between the plates to
  - Increase breakdown voltage greater than that in air (3MV/m)
  - Apply higher voltage to the gap without the charge passing across
  - Allow the plates get closer together without touching
    - Increases capacitance (recall $C=\varepsilon_0 A/d$)
  - Increase the capacitance by the dielectric constant

$$C = KC_0$$

  - Where C_0 is the intrinsic capacitance when the gap is vacuum

Dielectrics

- The value of **dielectric constant K** varies depending on the material (Table 24 – 1)
  – K for vacuum is 1.0000
  – K for air is 1.0006 (this is why the permittivity of air and vacuum are used interchangeably.)
  – K for paper is 3.7
- **Maximum electric field before breakdown** occurs is called the **dielectric strength**. What is its unit?
  – V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$C = KC_0 = K\varepsilon_0 \frac{A}{d}$$

Dielectrics

- A new quantity of the **permittivity of a dielectric material** is defined as $\varepsilon = K\varepsilon_0$
- The capacitance of a parallel plate capacitor with a dielectric medium filling the gap is

$$C = \varepsilon \frac{A}{d}$$

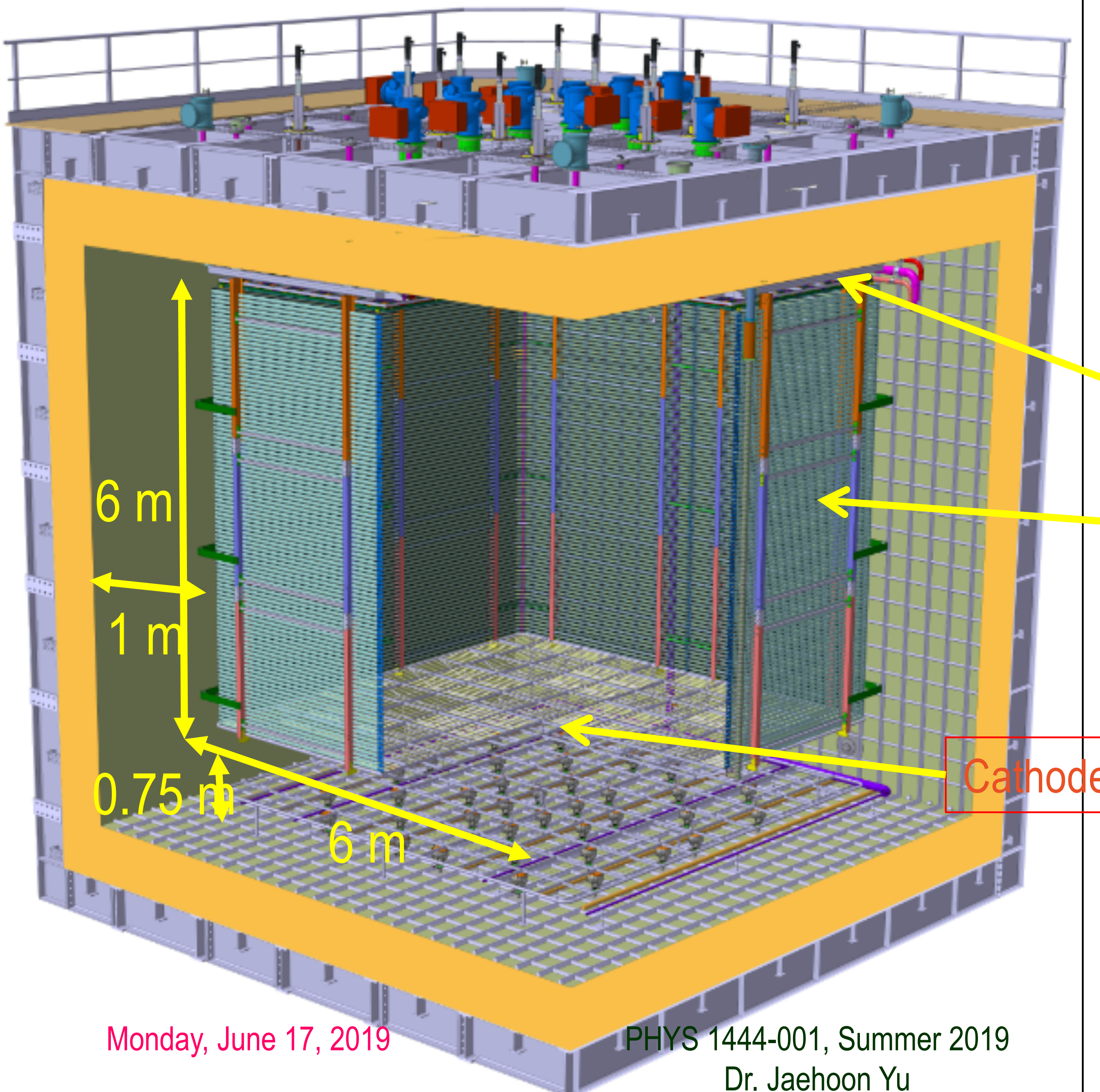
- The energy density stored in an electric field E in a dielectric is

$$u = \frac{1}{2} K \varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

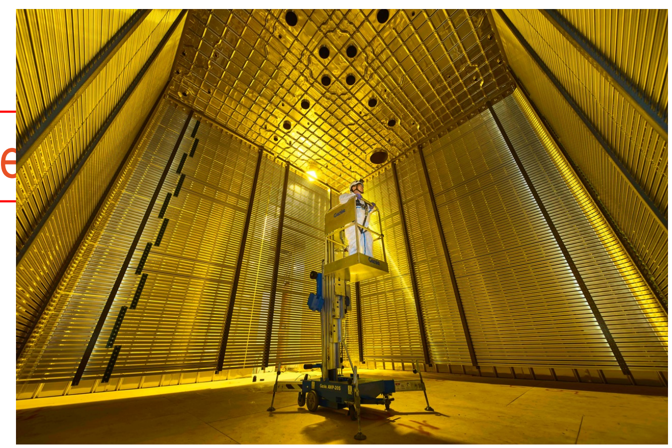
Valid for any space w/ dielectric w/ permittivity ε.

What is the stored energy in between ProtoDUNE cathode (-300kV) plate (6mx6m) and the cryostat wall (0V), d=1.5m, LAr K=1.5?

ProtoDUNE Dual Phase



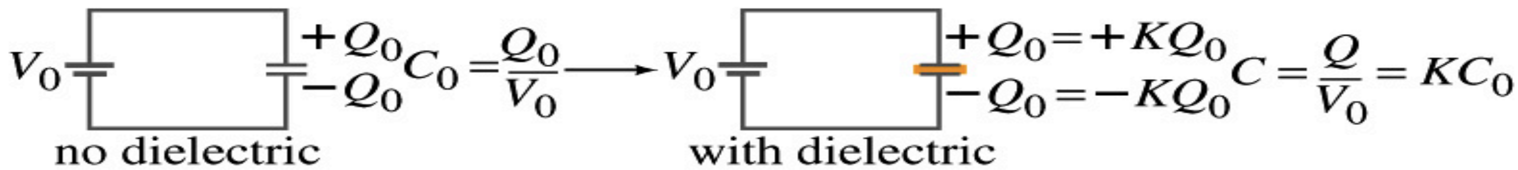
- Calculate the capacitance between each of the side wall to the FC & the cathode to the ground grid
- Calculate the total energy stored in the capacitors
- $\varepsilon_{LAr} = 1.5\ \varepsilon_0$



Effect of a Dielectric Material on Capacitance

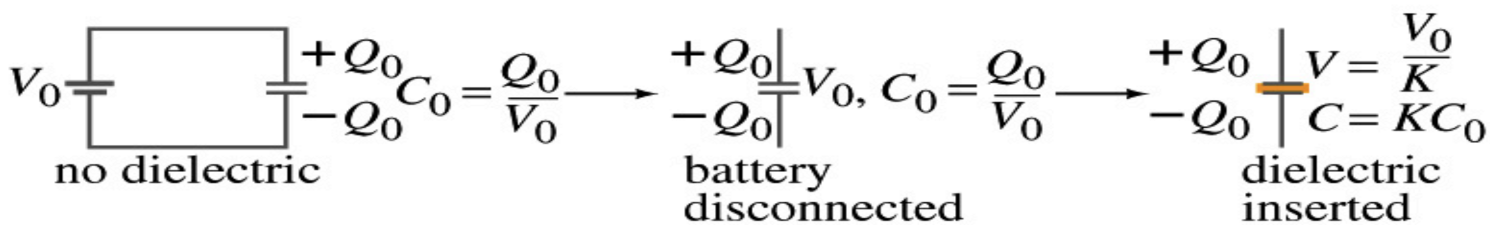
- Let's consider the two cases below:

Case #1 : constant V



(a) Voltage constant

Case #2 : constant Q



(b) Charge constant

- Constant voltage: Experimentally observed that the total charge on the each plate of the capacitor increases by K as a dielectric material is inserted between the gap ➔ $Q=KQ_0$
  - The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K ➔ $V=V_0/K$
  - The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$

Effect of a Dielectric Material on Field

- What happens to the electric field within a dielectric?
- Without a dielectric, the field is

$$E_0 = \frac{V_0}{d}$$

  - What are V_0 and d?
    - V_0: Potential difference between the two plates
    - d: separation between the two plates
- For the constant voltage, the electric field remains the same
- For the constant charge: the voltage drops to $V=V_0/K$, thus the field in the dielectric is

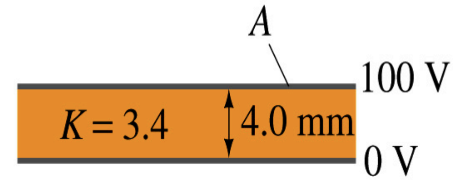
$$E = E_D = \frac{V}{d} = \frac{V_0}{dK} = \frac{E_0}{K}$$

  - The field in the dielectric is reduced.

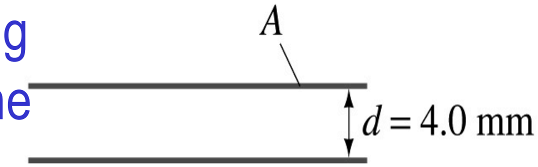
$$E_D = \frac{E_0}{K}$$

Example 24 – 11

Dielectric Removal: A parallel-plate capacitor, filled with the dielectric of K=3.4, is connected to a 100-V battery. After the capacitor is fully charged, the battery is disconnected. The plates have area $A=4.0m^2$, and are separated by d=4.0mm. (a) Find the capacitance, the charge on the capacitor, the electric field strength, and the energy stored in the capacitor. (b) The dielectric is carefully removed, without changing the plate separation nor does any charge leave the capacitor. Find the new value of capacitance, electric field strength, voltage between the plates and the energy stored in the capacitor.



(a)



(b)

(a) $$C = \frac{\varepsilon A}{d} = \frac{K\varepsilon_0 A}{d} = \left(3.4 \times 8.85 \times 10^{-12}\, C^2/N \cdot m^2\right)\frac{4.0m^2}{4.0 \times 10^{-3}\, m} = 3.0 \times 10^{-8}\, F = 30nF$$

$$Q = CV = \left(3.0 \times 10^{-8}\, F\right) \times 100V = 3.0 \times 10^{-6}\, C = 3.0\mu C$$

$$E = \frac{V}{d} = \frac{100V}{4.0 \times 10^{-3}\, m} = 2.5 \times 10^4\, V/m$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(3.0 \times 10^{-8}\, F\right)\left(100V\right)^2 = 1.5 \times 10^{-4}\, J$$

Example 24 – 11 cont'd

(b) Since the dielectric has been removed, the effect of dielectric constant must be removed as well.

$$C_0=\frac{C}{K}=\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\frac{4.0m^2}{4.0\times10^{-3}m}=8.8\times10^{-9}F=8.8nF$$

Since charge is the same ($Q_0=Q$) before and after the removal of the dielectric, we obtain

$$V_0=Q/C_0=K\,Q/C=KV=3.4\times100V=340V$$

$$E_0=\frac{V_0}{d}=\frac{340V}{4.0\times10^{-3}m}=8.5\times10^4\,V/m=84\,kV/m$$

$$U_0=\frac{1}{2}C_0V_0^2=\frac{1}{2}\frac{C}{K}(KV)^2=\frac{1}{2}KCV^2=KU=3.4\times1.5\times10^{-4}J=5.1\times10^{-4}J$$

Where did the extra energy come from?

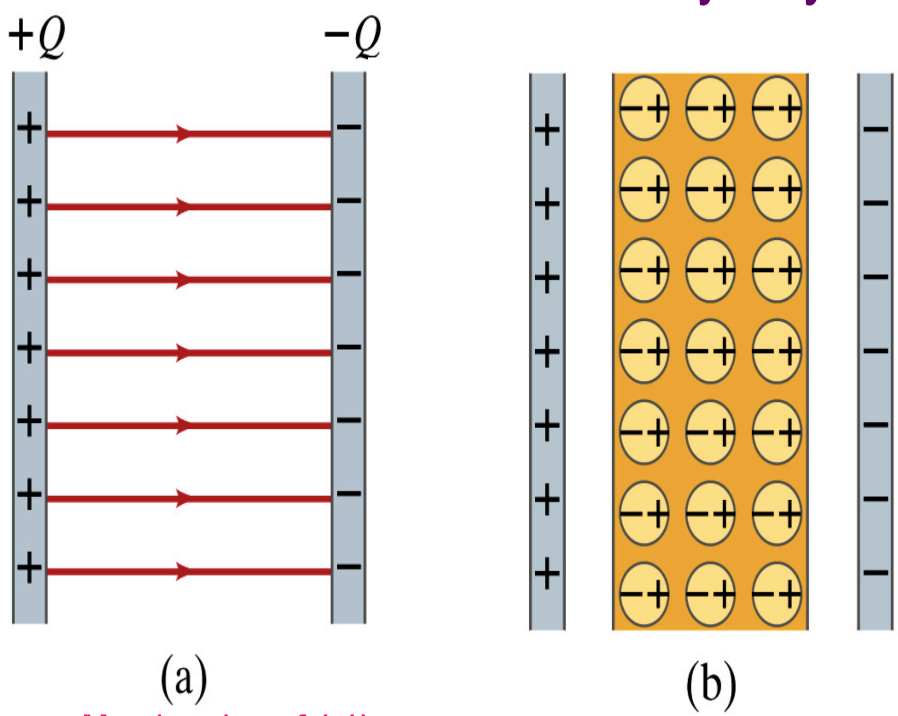
~~T[illegible]ergy conservation law is violated in elec[illegible]??~~

Wrong!
Wrong!
Wrong!

An external force has done the work of 3.6×10^{-4}J on the system to remove dielectric!!

Molecular Description of Dielectric

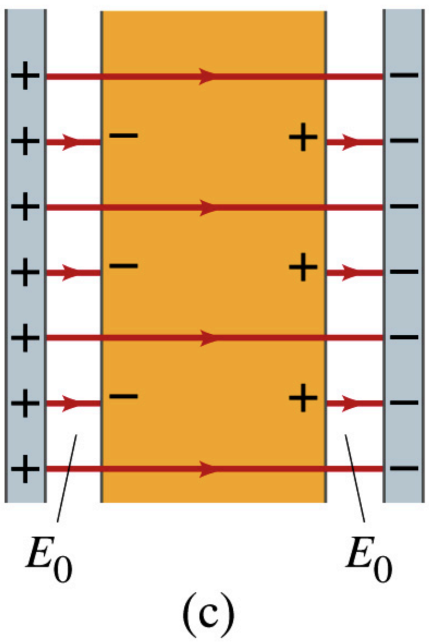
- So what in the world makes dielectrics behave the way they do?
- We need to examine this in a microscopic scale.
- Let's consider a parallel plate capacitor that is charged up $+Q(=C_0V_0)$ and $-Q$ with air in between.
  - Assume there is no way any charge can flow in or out



- Now insert a dielectric
  - Dielectric can be polar ➔ could have permanent dipole moment. What will happen?
- Due to the electric field molecules will be aligned.

Molecular Description of Dielectric

- OK. Then what happens?
- Then effectively, there will be some negative charges close to the surface of the positive plate and positive charge on the negative plate
  - Some electric field do not pass through the whole dielectric but stops at the negative charge



  - So the field inside dielectric is smaller than the air
- Since electric field is smaller, the force is smaller
  - The work need to move a test charge inside the dielectric is smaller
  - Thus the potential difference across the dielectric is smaller than across the air