PHYS 1444 – Section 001
Lecture #13

Tuesday, June 25, 2019

Dr. Jaehoon Yu

- Chapter 26
  - Kirchhoff's Rules
  - EMFs in Series and Parallel
  - RC Circuits
- Chapter 27: Magnetism and Magnetic Field
  - Electric Current and Magnetism
  - Magnetic Forces on Electric Current
  - About Magnetic Field

Announcements

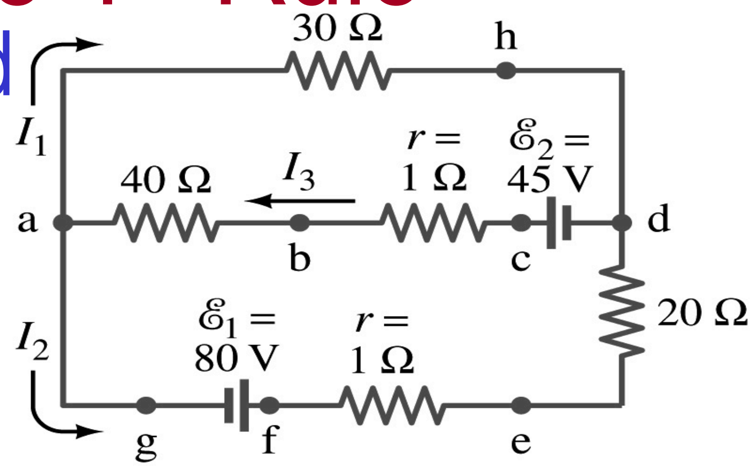
- 2nd Term Exam
  - First 80min this Wednesday, June 26, tomorrow
  - Covers CH25.1 through what we learn today
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Quiz 3 results
  - Class average: 40.7/60
    - Equivalent to 67.8/100
    - Previous quizzes: 44.3/100 and 54.2/100
  - Top score: 60/60

Reminder: Special Project #4

- Make a list of the power consumption and the resistance of all electric and electronic devices at your home and compile them in a table. (10 points total for the first 10 items and 0.5 points each additional item.)
- Estimate the cost of electricity for each of the items on the table using your own electric cost per kWh (if you don't find your own, use $0.12/kWh) and put them in the relevant column. (5 points total for the first 10 items and 0.2 points each additional items)
- Estimate the the total amount of energy in Joules and the total electricity cost per day, per month and per year for your home. (8 points)
- Due: Beginning of the class this Thursday, June 27

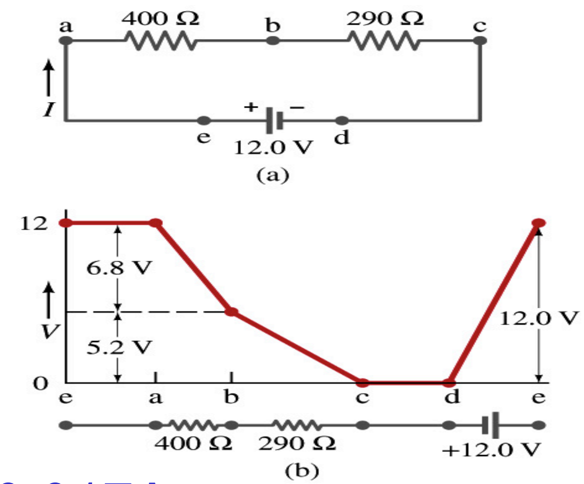
Item Name	Rated power (W)	Number of devices	Number of Hours per day	Daily Power Consumption (kWh)	Energy Cost per kWh (cents)	Daily Energy Consumption (J).	Daily Energy Cost ($)	Monthly Energy Consumption (J)	Monthly Energy Cost ($)	Yearly Energy Consumption (J)	Yearly Energy Cost ($)
Light Bulbs	30	4									
	40	6									
	60	15									
Heaters	1000	2									
	1500	1									
	2000	1									
Fans											
Air Conditioners											
Fridgers, Freezers											
Computers (desktop, laptop, ipad)											
Game consoles											
Total				0		0	0	0	0	0	0

Kirchhoff's Rules – the 1st Rule



- Some circuits are very complicated to do the analysis using the simple combinations of resistors
  - G. R. Kirchhoff devised two rules to deal with complicated circuits.
- Kirchhoff's rules are based on **conservation of charge and energy**
  - Kirchhoff's 1st rule: The junction rule, **charge conservation**.
    - At any junction point, the sum of all currents entering the junction must equal to the sum of all currents leaving the junction.
    - In other words, what goes in must come out.
    - At junction *a* in the figure, I_3 comes into the junction while I_1 and I_2 leaves: $I_3 = I_1 + I_2$

Kirchhoff's Rules – the 2nd Rule

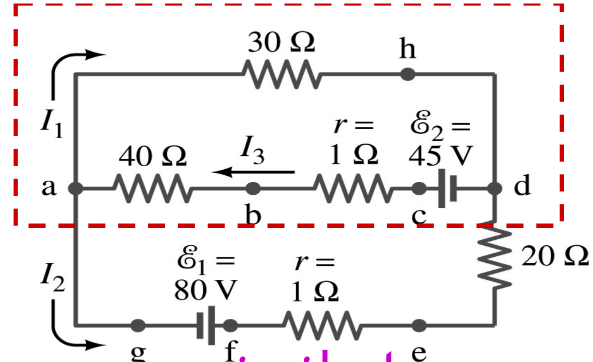


- Kirchoff's 2nd rule: The loop rule, uses <u>**conservation of energy**</u>.
  - The sum of the changes in potential in any closed path of a circuit must be zero.
- The current in the circuit in the figure is I=12/690=0.017A.
  - Point e is the high potential point while point d is the lowest potential.
  - When the test charge starts at e and returns to e, the total potential change is 0.
  - Between point e and a, no potential change since there is no source of potential nor any resistance.
  - Between a and b, there is a 400Ω resistance, causing IR=0.017*400 =6.8V drop.
  - Between b and c, there is a 290Ω resistance, causing IR=0.017*290 =5.2V drop.
  - Since these are voltage drops, we use negative sign for these, -6.8V and -5.2V.
  - No change between c and d while from d to e there is +12V change.
  - Thus the total change of the voltage through the loop is: -6.8V-5.2V+12V=0V.

Using Kirchhoff's Rules

1. Determine the flow of currents at the junctions and label each and everyone of the currents.
   - It does not matter which direction, you decide but keep it!
   - If the value of the current after completing the calculations are negative, you just need to flip the direction of the current flow.
2. Write down the current equation based on Kirchhoff's 1st rule at various junctions.
   - Be sure to see if any of equations are the same.
3. Choose closed loops in the circuit
4. Write down the potential in each interval of the junctions, keeping the sign properly.
5. Write down the potential equations for each loop.
6. Solve the equations for unknowns.

Example 26 – 9



Use Kirchhoff's rules. Calculate the currents I_1, I_2 and I_3 in each of the branches of the circuit in the figure.

The directions of the current through the circuit is not known *a priori* but since the current tends to move away from the positive terminal of a battery, we arbitrarily choose the direction of the currents as shown.

We have three unknowns so we need three equations.

Using Kirchhoff's junction rule at point *a*, we obtain $I_3 = I_1 + I_2$

This is the same for junction *d* as well, so no additional information.

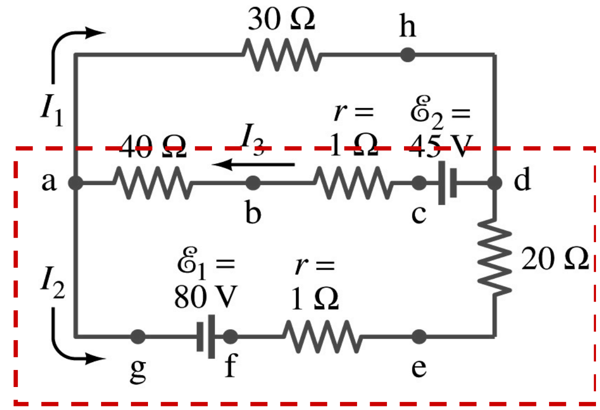
Now the second rule on the loop *ahdcba*.

$$V_{ah} = -I_1 30 \qquad V_{hd} = 0 \qquad V_{dc} = +45 \qquad V_{cb} = -I_3 \qquad V_{ba} = -40 I_3$$

The total voltage change in the loop *ahdcba* is.

$$V_{ahdcba} = -30 I_1 + 45 - 1 \cdot I_3 - 40 I_3 = 45 - 30 I_1 - 41 I_3 = 0$$

Example 26 – 9, cnt'd



Now the second rule on the other loop *agfedcba*.

$$V_{ag} = 0 \quad V_{gf} = +80 \quad V_{fe} = -I_2 \cdot 1 \quad V_{ed} = -I_2 \cdot 20$$

$$V_{dc} = +45 \quad V_{cb} = -I_3 \cdot 1 \quad V_{ba} = -40 \cdot I_3$$

The total voltage change in loop *agfedcba* is. $V_{agfedcba} = -21I_2 + 125 - 41I_3 = 0$

So the three equations become

$$I_3 = I_1 + I_2$$

$$45 - 30I_1 - 41I_3 = 0$$

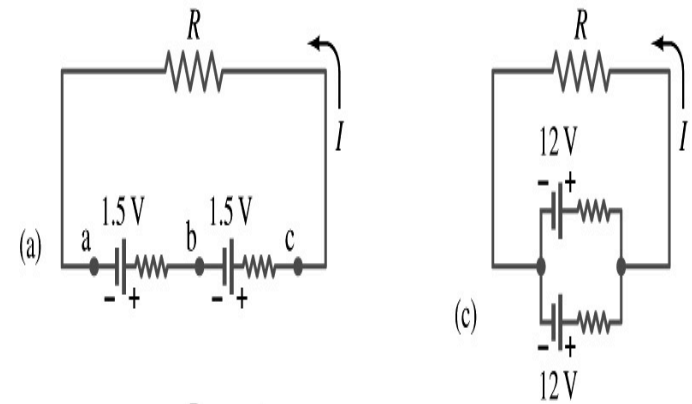
$$125 - 21I_2 - 41I_3 = 0$$

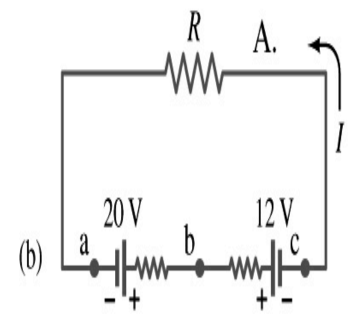
We can obtain the three current by solving these equations for I_1, I_2 and I_3.

Do this yourselves!!

EMFs in Series and Parallel: Charging a Battery

- When two or more sources of emfs, such as batteries, are connected in series
  - The total voltage is the algebraic sum of their voltages, if their direction is the same
    - V_{ab}=1.5 + 1.5=3.0V in figure (a).
  - If the batteries are arranged in the opposite direction, the total voltage is the difference between them
    - V_{ac}=20 – 12=8.0V in figure (b)
    - Connecting batteries in opposite direction is wasteful.
    - This, however, is the way a battery charger works.
    - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
    - Some battery are rechargeable since their chemical reactions are reversible but most the batteries do not reverse their chemical reactions

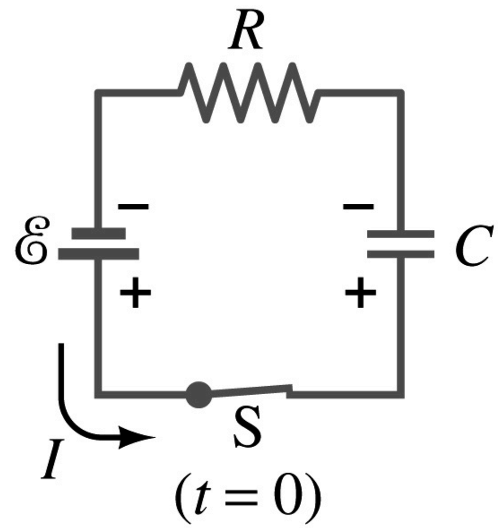




Parallel arrangements (c) are used only to increase currents. An example?

RC Circuits

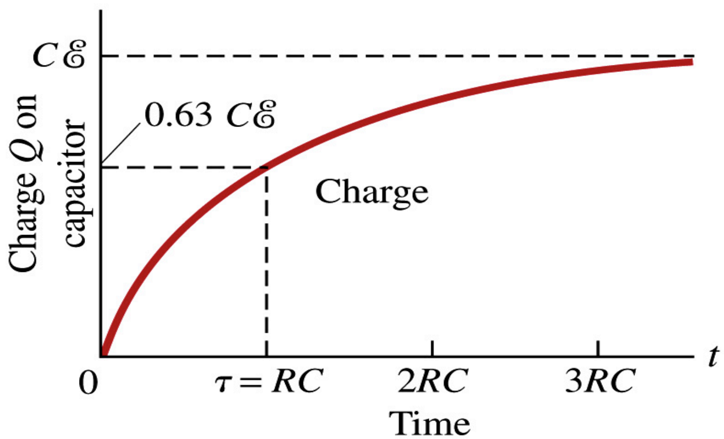
- Circuits containing both resistors and capacitors
  - RC circuits are used commonly in everyday life
    - Control windshield wiper
    - Timing of traffic light from red to green
    - Camera flashes and heart pacemakers
- How does an RC circuit look?
  - There should be a source of emf, capacitors and resisters

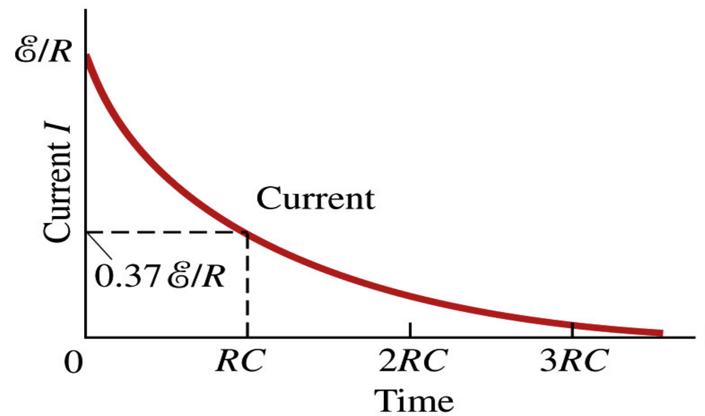


- What happens when the switch S is closed?
  - Current immediately starts flowing through the circuit.
  - Electrons flow out of negative terminal of the emf source, through the resister R and accumulates on the upper plate of the capacitor.
  - The electrons from the bottom plate of the capacitor will flow into the positive terminal of the battery, leaving only the positive charge on the bottom plate.
  - As the charge accumulates on the capacitor, the potential difference across it increases
  - The current reduces gradually to 0 till the voltage across the capacitor is the same as the emf.
  - The charge on the capacitor increases till it reaches to its maximum C$\mathcal{E}$.

RC Circuits

- How does all this look like in graphs?
  - The charge and the current on the capacitor as a function of time





  - From energy conservation (Kirchhoff's 2nd rule), the emf $\mathcal{E}$ must be equal to the sum of voltage drop across the capacitor and the resister
    - $\mathcal{E} = IR + Q/C$
    - R includes all resistance in the circuit, **including the internal resistance of the battery**, I is the current in the circuit at any instance, and Q is the charge of the capacitor at that same instance.

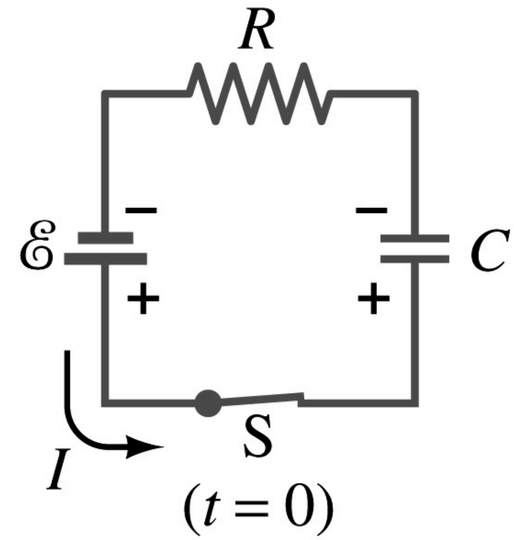
Analysis of RC Circuits

- In an RC circuit $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$ and $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$
- What can we see from the above equations?
  - Q and V_C increase from 0 at t=0 to maximum value $Q_{max}=C\mathcal{E}$ and $V_C=\mathcal{E}$.
- In how much time?
  - The quantity RC is called the **time constant** of the circuit, τ
    - τ=RC, What is the unit? Sec.
  - What is the physical meaning?
    - The time required for the capacitor to reach $(1 - e^{-1})=0.63$ or 63% of the full charge
- The current is

$$I = \frac{dQ}{dt} = \frac{\varepsilon}{R} e^{-t/RC}$$

Example 26 – 12

RC circuit, with emf. The capacitance in the circuit of the figure is C=0.30μF, the total resistance is 20kΩ, and the battery emf is 12V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current I when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when, the current I is 0.20 its maximum value.



(a) Since $\tau = RC$ We obtain $\tau = 20\times10^{3}\cdot0.30\times10^{-6} = 6.0\times10^{-3}\text{ sec}$

(b) Maximum charge is $Q_{\max} = C\varepsilon = 0.30\times10^{-6}\cdot12 = 3.6\times10^{-6}\,C$

(c) Since $Q = C\varepsilon\left(1-e^{-t/RC}\right)$ For 99% we obtain $0.99C\varepsilon = C\varepsilon\left(1-e^{-t/RC}\right)$

$$e^{-t/RC} = 0.01;\quad -t/RC = -2\ln10;\quad t = RC\cdot2\ln10 = 4.6RC = 28\times10^{-3}\text{ sec}$$

(d) Since $\varepsilon = IR + Q/C$ We obtain $I = \left(\varepsilon - Q/C\right)/R$

The current when Q is $0.5Q_{max}$ $I = \left(12-1.8\times10^{-6}/0.30\times10^{-6}\right)/20\times10^{3} = 3\times10^{-4}\,A$

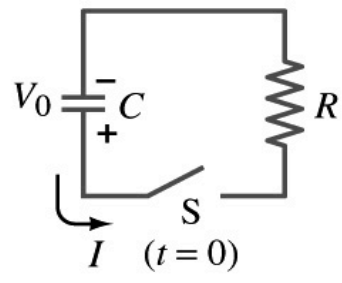
(e) When is I maximum? when Q=0: $I = 12/20\times10^{3} = 6\times10^{-4}\,A$

(f) What is Q when I=120mA? $Q = C\left(\varepsilon - IR\right) =$

$$= 0.30\times10^{-6}\left(12-1.2\times10^{-4}\cdot2\times10^{4}\right) = 2.9\times10^{-6}\,C$$

Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance R.
  - When the switch S is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus IR=Q/C.
  - The rate at which the charge leaves the capacitor equals the negative of the current flows through the resistor
    - $I = -dQ/dt$
    - This is because the current is leaving the capacitor
  - Thus the voltage equation becomes a differential equation



(a)

$$-\frac{dQ}{dt}R = \frac{Q}{C}$$

Rearrange terms

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

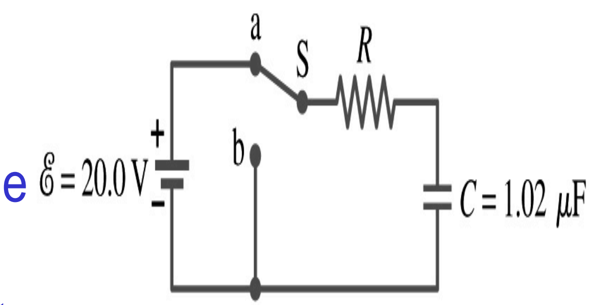
Discharging RC Circuits

- Now, let's integrate from t=0 when the charge is Q_0 to t when the charge is Q $$\int_{Q_0}^{Q}\frac{dQ}{Q}=-\int_{0}^{t}\frac{dt}{RC}$$
- The result is $$\ln Q\Big|_{Q_0}^{Q}=\ln\frac{Q}{Q_0}=-\frac{t}{RC}$$
- Thus, we obtain $$\boxed{Q(t)=Q_0e^{-t/RC}}$$
- What does this tell you about the charge on the capacitor?
  - It decreases exponentially w/ time at the time constant RC
  - Just like the case of charging

What is this?

- The current is: $$I=-\frac{dQ}{dt}=\frac{Q_0}{RC}e^{-t/RC}$$ $$\boxed{I(t)=I_0e^{-t/RC}}$$
  - The current also decreases exponentially w/ time w/ the time constant RC

Example 26 – 13



Discharging RC circuit. In the RC circuit shown in the figure the battery has fully charged the capacitor, so $Q_0=C\mathscr{E}$. Then at t=0, the switch is thrown from position a to b. The battery emf is 20.0V, and the capacitance C=1.02μF. The current I is observed to decrease to 0.50 of its initial value in 40μs. (a) what is the value of R? (b) What is the value of Q, the charge on the capacitor, at t=0? (c) What is Q at t=60μs?

(a) Since the current reaches to 0.5 of its initial value in 40μs, we can obtain

$I(t) = I_0 e^{-t/RC}$ **For $0.5I_0$** → $0.5I_0 = I_0 e^{-t/RC}$ **Rearrange terms** → $-t/RC = \ln 0.5 = -\ln 2$

Solve for R → $R = t/(C\ln 2) = 40\times10^{-6}/(1.02\times10^{-6}\cdot\ln 2) = 56.6\Omega$

(b) The value of Q at t=0 is

$$Q_0 = Q_{\max} = C\varepsilon = 1.02\times10^{-6}\cdot 20.0 = 20.4\mu C$$

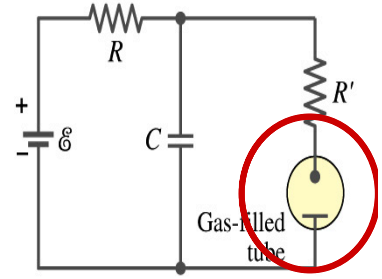
(c) What do we need to know first for the value of Q at t=60μs?

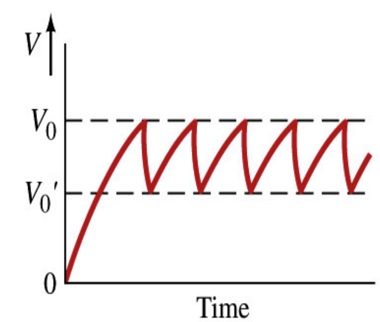
The RC time $\tau = RC = 56.6\cdot1.02\times10^{-6} = 57.7\mu s$

Thus $Q(t=60\mu s) = Q_0 e^{-t/RC} = 20.4\times10^{-6}\cdot e^{-60\mu s/57.7\mu s} = 7.2\mu C$

Application of RC Circuits

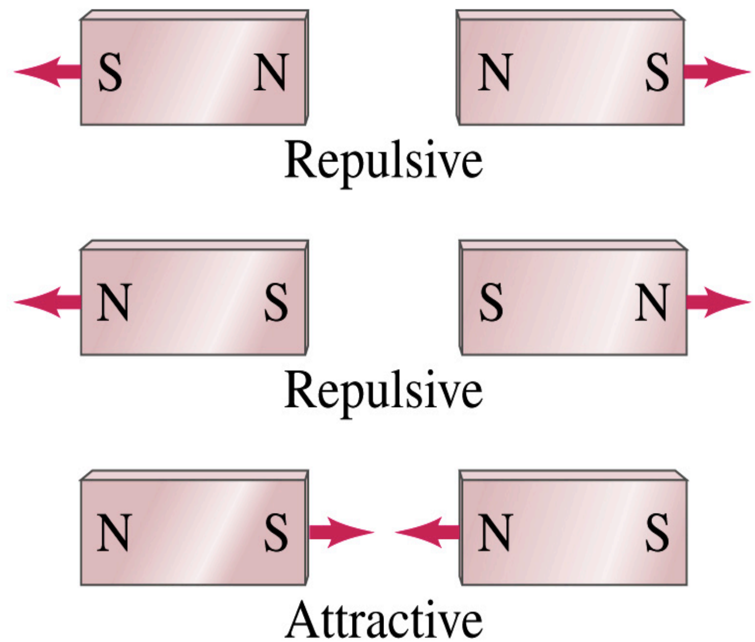
- What do you think the charging and discharging characteristics of RC circuits can be used for?
  - To produce voltage pulses at a regular frequency
  - How?
    - The capacitor charges up to a particular voltage and discharges
    - A simple way of doing this is to use breakdown of voltage in a gas filled tube
      - The discharge occurs when the voltage breaks down at V_0
      - After the completion of discharge, the tube no longer conducts
      - Then the voltage is at V_0' and it starts charging up
      - How do you think the voltage as a function of time look?
        - » A sawtooth shape
    - Pace maker, intermittent windshield wiper, etc





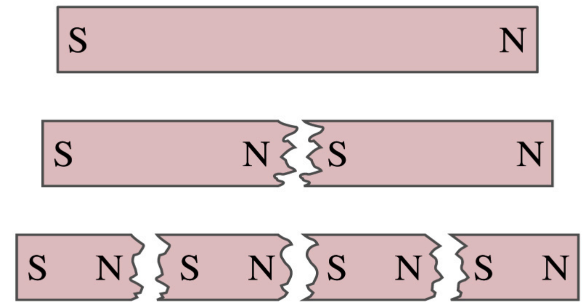
Magnetism

- What are magnets?
  - Objects with two poles, North and South poles
    - The pole that points to the geographical North is the North pole and the other is the South pole
      - Principle of compass
  - These are called the magnet due to the name of the region, Magnesia, where the rocks that attract each other were found
- What happens when two magnets are brought to each other?
  - They exert force onto each other
  - What kind?
  - Both repulsive and attractive forces depending on the configurations
    - Like poles repel each other while the unlike poles attract



Magnetism

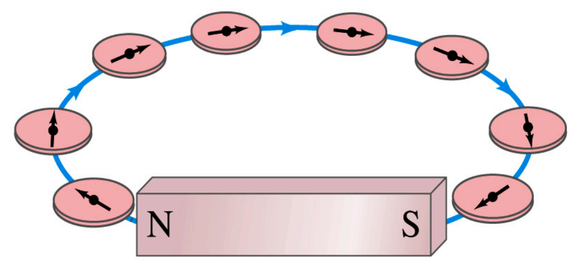
- So the magnetic poles are the same as the electric charge?
  - No. Why not?
  - While the electric charges (positive and negative) can be isolated, the magnetic poles cannot be isolated.
  - So what happens when a magnet is cut?
    - If a magnet is cut, two magnets are made.
    - The more they get cut, the more magnets are made

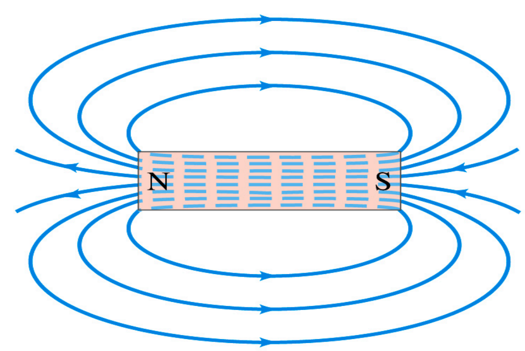


  - Single pole magnets are called the monopole but it has not been seen yet
- Ferromagnetic materials: Materials that show strong magnetic effects
  - Iron, cobalt, nickel, gadolinium and certain alloys
- Other materials show very weak magnetic effects

Magnetic Field

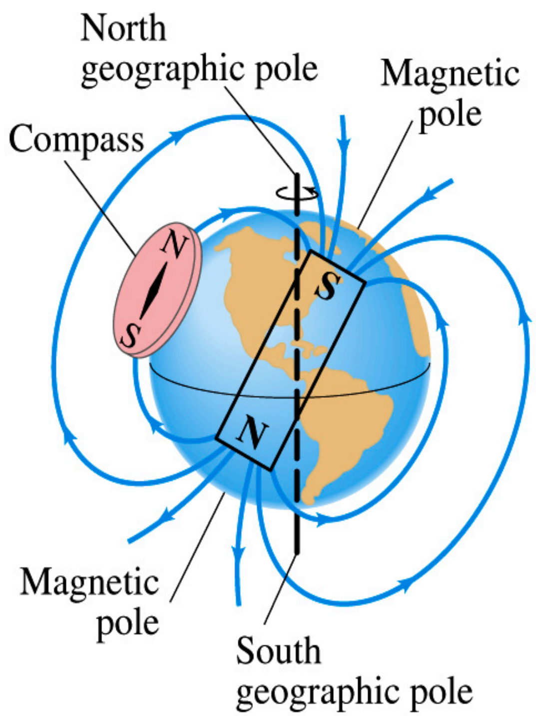
- Just like the electric field that surrounds electric charge, a magnetic field surrounds a magnet
- What does this mean?
  - Magnetic force is also a field force
  - The force one magnet exerts onto another can be viewed as the interaction between the magnet and the magnetic field produced by the other magnet
  - What kind of quantity is the magnetic field? Vector or Scalar? **Vector**
- So one can draw magnetic field lines, too.
  - The direction of the magnetic field is tangential to the field line at any point
  - The direction of the field is the direction the north pole of a compass would point to; out of N and into S
  - The number of lines per unit area is proportional to the strength of the magnetic field
  - Magnetic field lines continue inside the magnet
  - Since magnets always have both the poles, magnetic field lines form closed loops unlike electric field lines





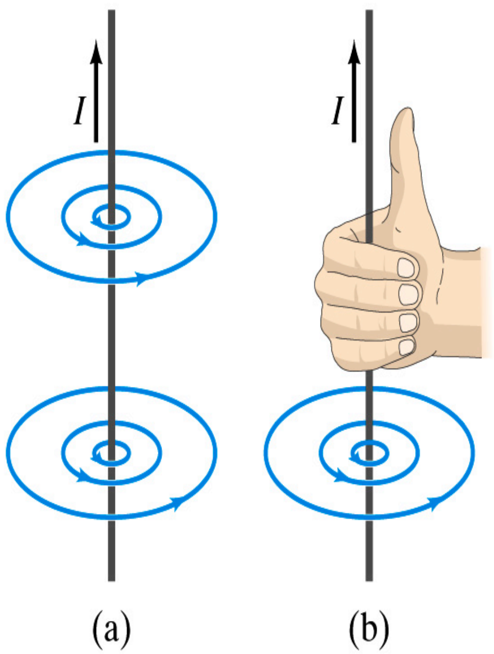
Earth's Magnetic Field

- What magnetic pole does the geographic North pole has to have?
  - Magnetic South pole. What? How do you know that?
  - Since the magnetic North pole points to the geographic North, the geographic north must have magnetic south pole
    - The pole in the North is still called geomagnetic North pole just because it is in the North
  - Similarly, South pole has magnetic North pole
- The Earth's magnetic poles do not coincide with the geographic poles ➔ magnetic declination
  - Geomagnetic North pole is in Northern Canada, some 900km off the true North pole
- Earth's magnetic field line is not tangent to the earth's surface at all points
  - The angle the Earth's field makes to the horizontal line is called the angle dip



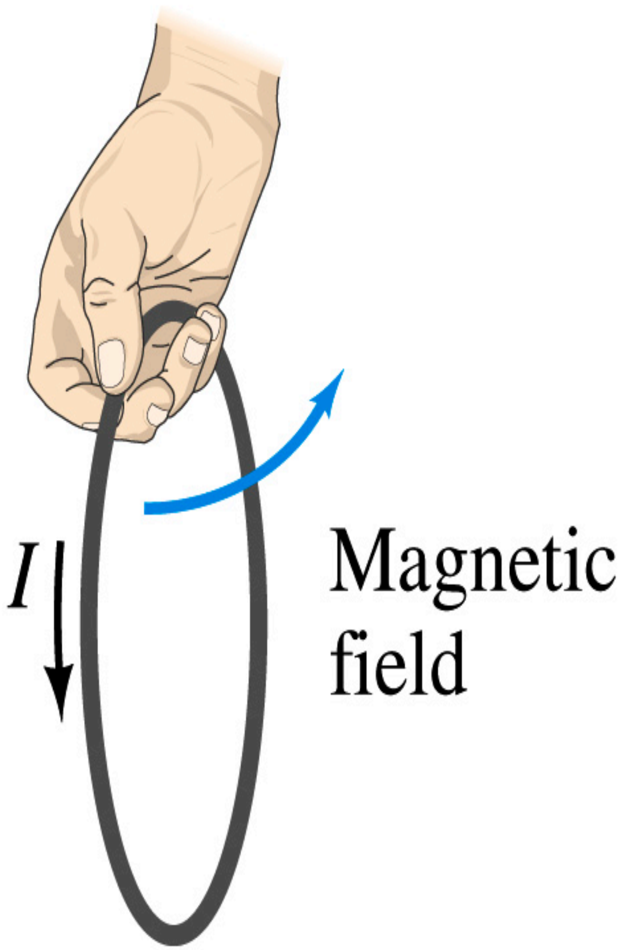
Electric Current and Magnetism

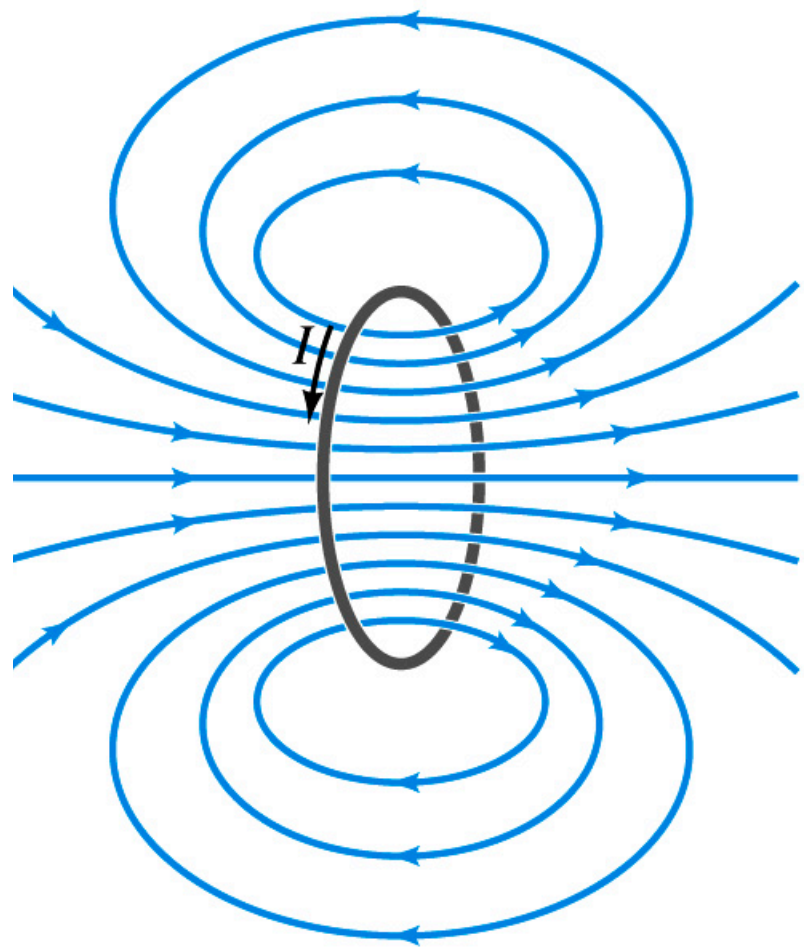
- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
  - Electric current produces a magnetic field
    - The first indication that electricity and magnetism are of the same origin
  - What about a stationary electric charge and magnet?
    - They don't affect each other.
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the "right-hand" rule
  - The field lines follow right-hand fingers wrapped around the wire when the thumb points to the direction of the electric current



Directions in a Circular Wire?

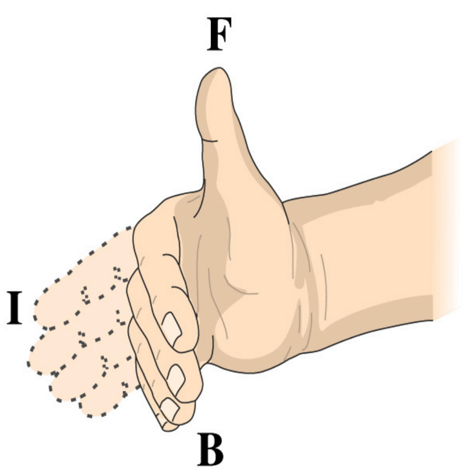
- OK, then what is the direction of the magnetic field generated by the current flowing through a circular loop?



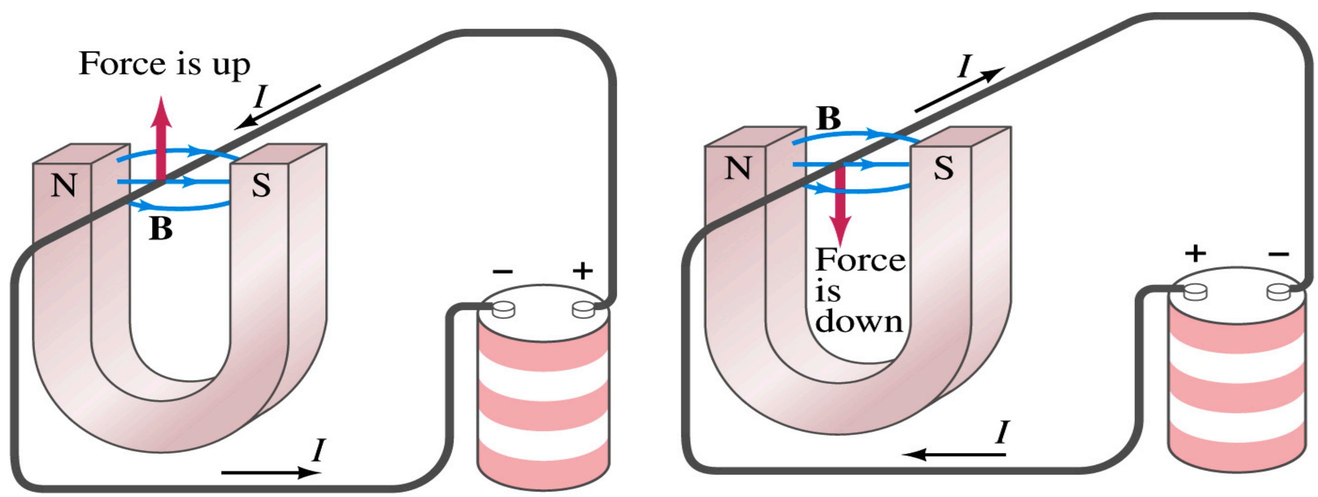


Magnetic Forces on Electric Current

- Since the electric current exerts force on a magnet, the magnet should also exert force on the electric current
  - Which law justifies this?
    - Newton's 3rd law
  - This was also discovered by Oersted
- Direction of the force is always
  - perpendicular to the direction of the current
  - perpendicular to the direction of the magnetic field, **B**
- Experimentally the direction of the force is given by another right-hand rule ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to the direction of the force



Right-hand rule



Magnetic Forces on Electric Current

- OK, we are set for the direction but what about the magnitude?
- It is found that the magnitude of the force is directly proportional
  - To the current in the wire
  - To the length of the wire in the magnetic field (if the field is uniform)
  - To the strength of the magnetic field
- The force also depends on the angle θ between the directions of the current and the magnetic field
  - When the wire is perpendicular to the field, the force is the strongest
  - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in the wire w/ length ℓ in a uniform field $\mathcal{B}$ is $\boxed{F \propto IlB\sin\theta}$