PHYS 1444 – Section 001
Lecture #15

Thursday, June 27, 2019

Dr. Jaehoon Yu

- Chapter 27: Magnetism and Magnetic Field
  - Charged Particle Path in a Magnetic Field
  - Cyclotron Frequency
  - Torque on a Current Loop
  - Magnetic Dipole Moment
- Chapter 28:Sources of Magnetic Field
  - Sources of Magnetic Field
  - Magnetic Field Due to Straight Wire
  - Magnetic Materials
  - Hysteresis

Today's homework is homework #10, due 11pm, Monday, July 1!!

Announcements

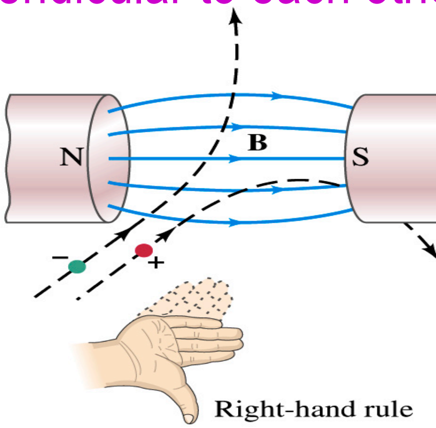
- Quiz #4
  - Beginning of the class coming Monday, July 1
  - Covers CH27.3 – what we learn today
  - BYOF: You may bring one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Reading Assignments: CH27.6 – 8 and CH28.6 – 10

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since a wire carrying electric current (moving charge) experiences force in a magnetic field, a freely moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by a given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for a charge q to travel a distance ℓ in the magnetic field **B**
    - Then, the length vector ℓ becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\boxed{\vec{F} = q\vec{v} \times \vec{B}}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity v in a field B is
    - $F = qvB\sin\theta$
    - What is the angle θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the B field and the velocity vector are perpendicular to each other.
    - $F_{max} = qvB \rightarrow B = \frac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field



Example 27 – 5

Magnetic force on a proton. A proton having a speed of $5x10^{6}$m/s in a magnetic field feels a force of F=$8.0x10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6 \times 10^{-19} C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

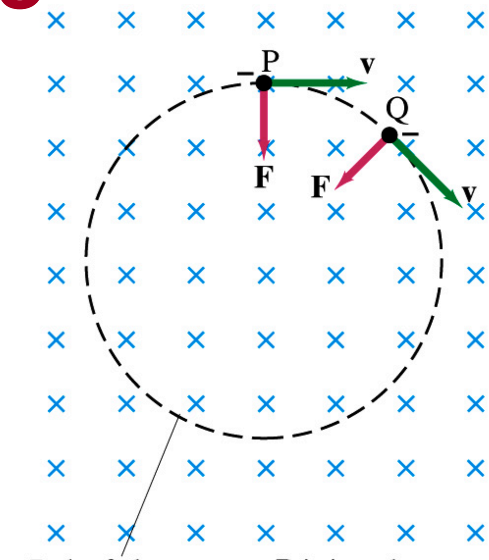
Since the particle feels force toward West, the field should be pointing to? North

Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14} N}{1.6 \times 10^{-19} C \cdot 5.0 \times 10^{6} m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field



- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward
  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity (the speed) is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^7m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\dfrac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is

$$F = evB$$

Since the magnetic force provides the centripetal force, we can establish an equation with the two forces

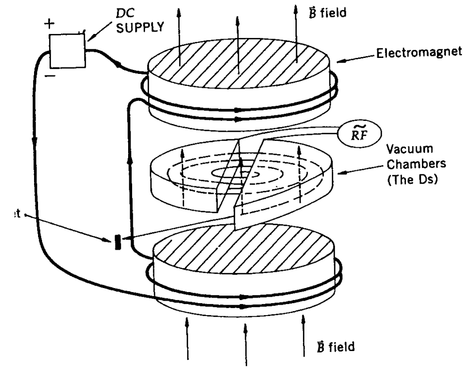
$$F = evB = m\frac{v^2}{r}$$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}kg\right)\cdot\left(2.0\times10^{7}\ m/s\right)}{\left(1.6\times10^{-19}C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ a constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



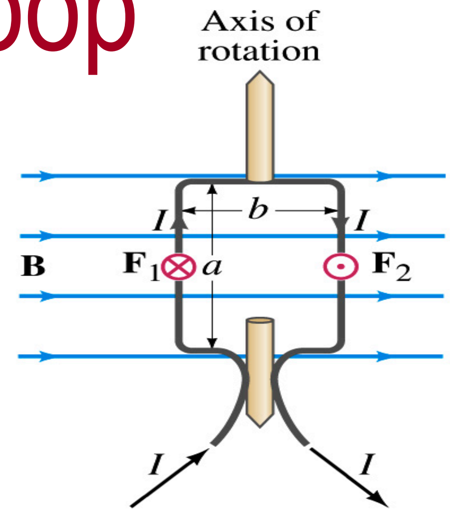
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$\boxed{f = \frac{1}{T} = \frac{qB}{2\pi m}}$$

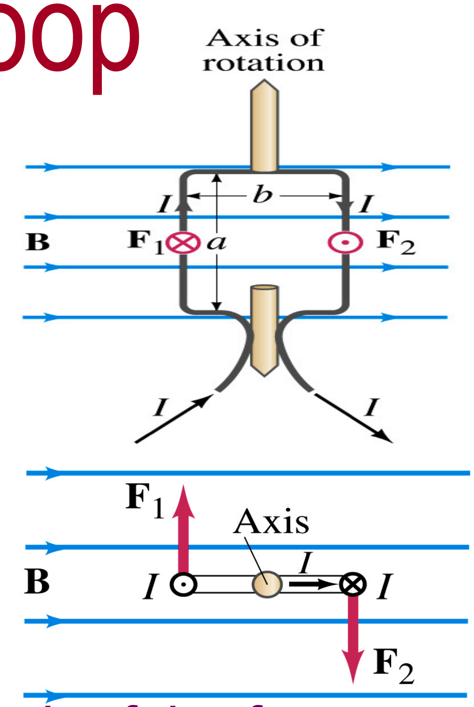
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with an electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used in?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque in the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not turn unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

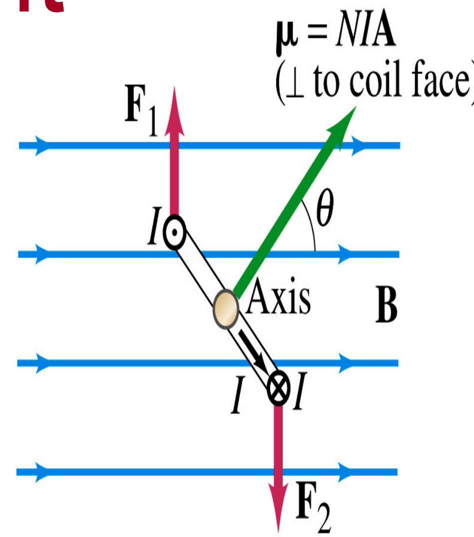
$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

    - Where $A = ab$ is the area of the coil loop
  - What is the total net torque if the coil consists of N loops of wire?

$$\boxed{\tau = NIAB}$$

  - If the coil makes an angle θ w/ the field $\boxed{\tau = NIAB\sin\theta}$

Magnetic Dipole Moment



- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity NIA is called the **magnetic dipole moment of the coil**
  - It is a vector quantity $\boxed{\vec{\mu} = NI\vec{A}}$
    - Its direction is the same as that of the area vector **A** and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finer cups around the loop in the same direction of the current
  - The tendency of an object to interact with an external magnetic field
  - Using the definition of magnetic moment, the torque can be rewritten in vector form

$$\boxed{\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}}$$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta \, d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at θ=π/2, then C=0
  - Thus the potential energy is $\boxed{U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}}$
    - Very similar to the electric dipole

Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius 0.529×10^{-10}m.

What provides the centripetal force? The Coulomb force

So we can obtain the speed of the electron from $F=\dfrac{e^2}{4\pi\varepsilon_0 r^2}=\dfrac{m_e v^2}{r}$

Solving for v

$$v=\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\sqrt{\frac{\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}}=2.19\times10^6\,m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current $I=\dfrac{e}{T}=\dfrac{ev}{2\pi r}$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu=IA=\frac{ev}{2\pi r}\pi r^2=\frac{evr}{2}=\frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 m_e r}}=\frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m_e}}$$

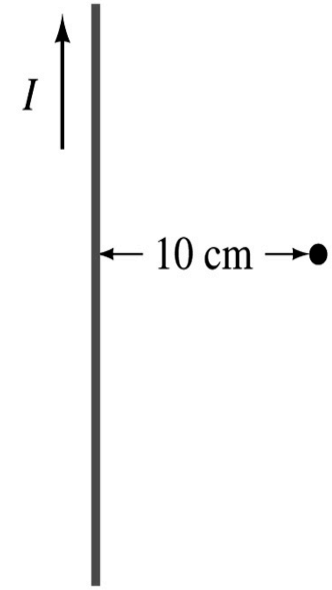
Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on the electric current and the moving charge
- We will now learn about the dynamics of magnetism
  - How do we determine magnetic field strengths in certain situations?
  - How do two wires with electric current interact?
  - What is the general approach to finding the connection between current and magnetic field?

Magnetic Field due to a Straight Wire

- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes $$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$
  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Example 28 – 1



Calculation of B near a wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10cm due East of this wire?

Using the formula for the magnetic field near a straight wire

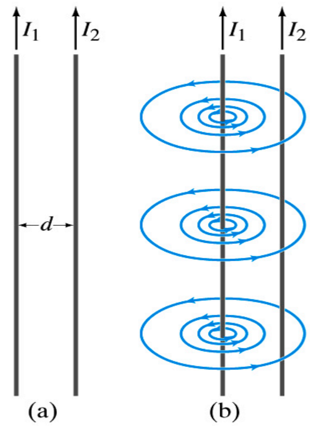
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ T \cdot m/A\right) \cdot (25A)}{(2\pi) \cdot (0.01m)} = 5.0 \times 10^{-5}\ T$$

Force Between Two Parallel Wires

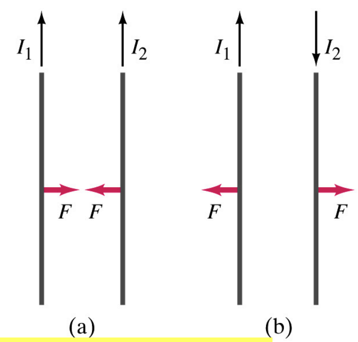
- We have learned that a wire carrying the electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force onto each other. Repel or attract?
  - Depending on the direction of the currents



- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance d, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is $$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

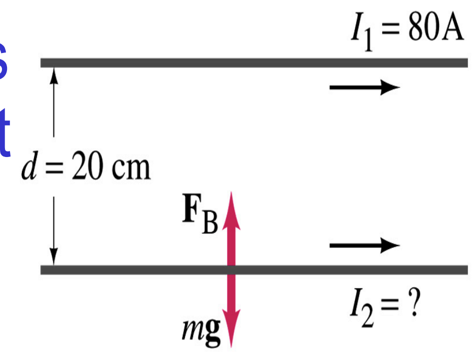
- The force F by a magnetic field B_1 on a wire of length l, carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi}\frac{I_1}{d}$
  - This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in opposite direction
- So the force per unit length is $\boxed{\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}}$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Down to the center of the Earth

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l}\frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi\left(9.8\,m/s^2\right)\cdot\left(0.12\times10^{-3}\,kg\right)\cdot\left(0.20m\right)}{\left(4\pi\times10^{-7}\;T\cdot m/A\right)\cdot\left(80A\right)} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- The unit of the current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

  – So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2x10^{-7}$N/m.

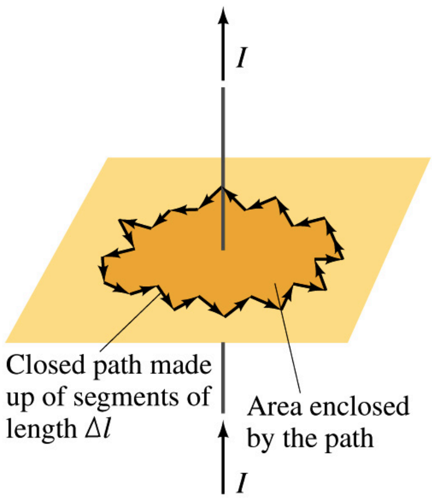
- Coulomb is then defined as exactly 1C=1A s.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.

Ampére's Law

- What is the relationship between the magnetic field strength and the current? $\boxed{B = \frac{\mu_0 I}{2\pi r}}$
  - Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shapes of the wire?
  - French scientist André Marie Ampére proposed such a relationship soon after Oersted's discovery

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path in small segments each of Δl long.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $\boxed{\sum B_{\parallel} \Delta l = \mu_0 I_{encl}}$
  - In the limit $\Delta l \to 0$, this relation becomes
  - $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}}$

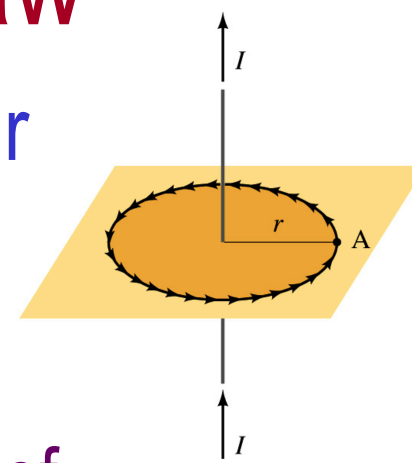


Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - This is a verification of Ampere's Law
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B\,dl = B\oint dl = 2\pi r B$$

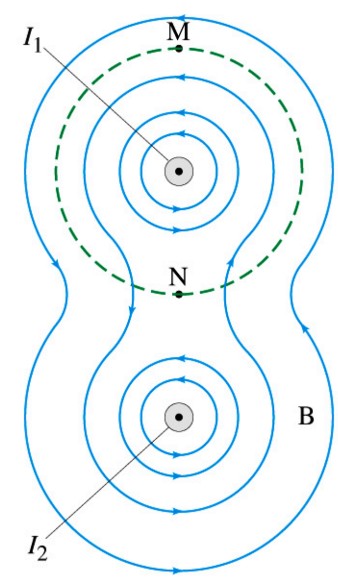
Solving for B

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments verified that it works for other cases too
  - The importance of this formula, however, is that it provides means to relate magnetic field to current

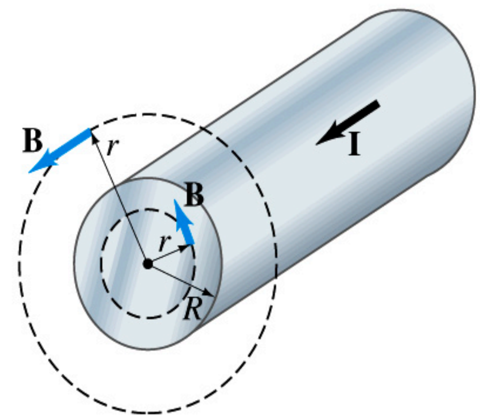
Verification of Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl}.
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

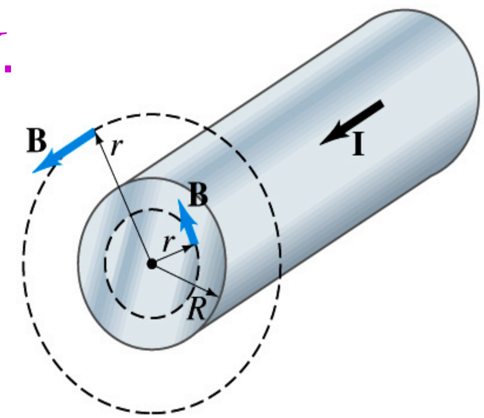
$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Example 28 – 6 cont'd

For r<R, the current inside the closed path is less than I. How much is it?



$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$

So using Ampere's law

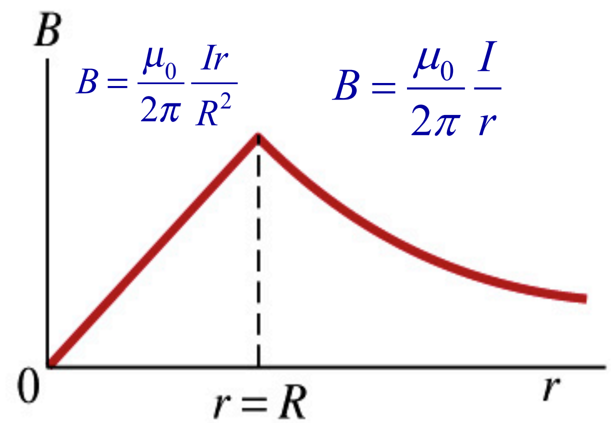
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

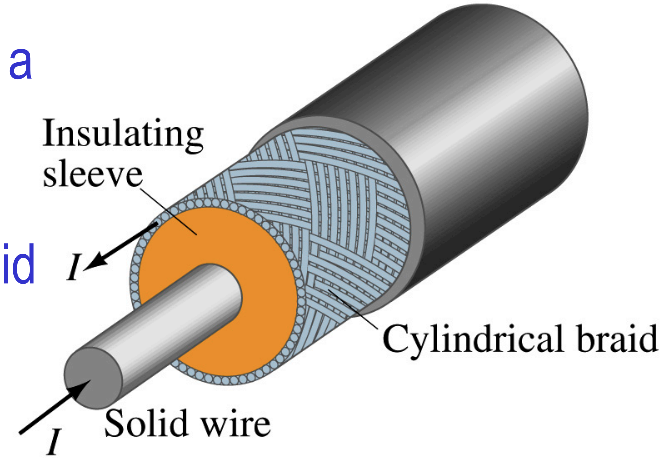
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.