PHYS 1444 – Section 001
Lecture #17

Tuesday, July 2, 2019

Dr. Jaehoon Yu

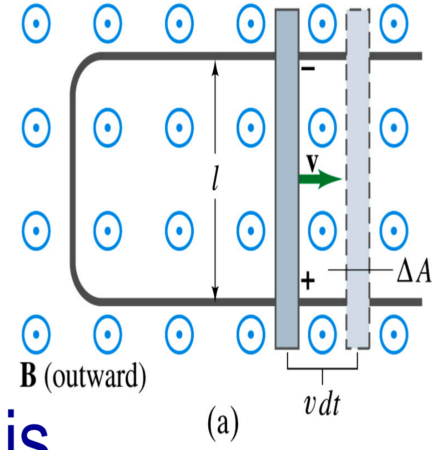
- Chapter 29:EM Induction & Faraday's Law
  - Generation of Electricity & Transformer
  - Mutual and Self Inductance
  - Energy Stored in Magnetic Field
- Chapter 30: Inductance
  - Mutual and Self Inductance
  - Energy Stored in Magnetic Field
- Chapter 31: Maxwell's Equations
  - Production of EM Waves
  - Light as EM Waves

Announcements

- Reading Assignments: CH29.5, 29.8, 30.10 and CH30.11
- Final Exam
  - In the class for 120min tomorrow Wednesday, July 3
  - Covers CH21.1 – What we finish today + Maxwell's equations (?)
  - BYOF: You may prepare one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
- Be sure to bring your planetarium extra credit and submit at the beginning of the exam this Wednesday, July 3
  - Tape one side of the ticket stub on a sheet of paper with your name on
- Term 2 results
  - Class average: 61.1/100
  - Equivalent to: 60.5/100, previous results: 56.4/100 and 65.8/100
  - Top score: 85/101
- Quiz 4 results
  - Class average: 42.1/80
  - Equivalent to: 52.6/100, previous results: 44.3/100, 52.2/100 and 68.2/100
  - Top score: 76

EMF Induced on a Moving Conductor

- Another way of inducing emf is using a U shaped conductor with a movable rod resting on it.
- As the rod moves at a speed v, it travels $v$dt in time dt, changing the area of the loop by dA=lvdt.



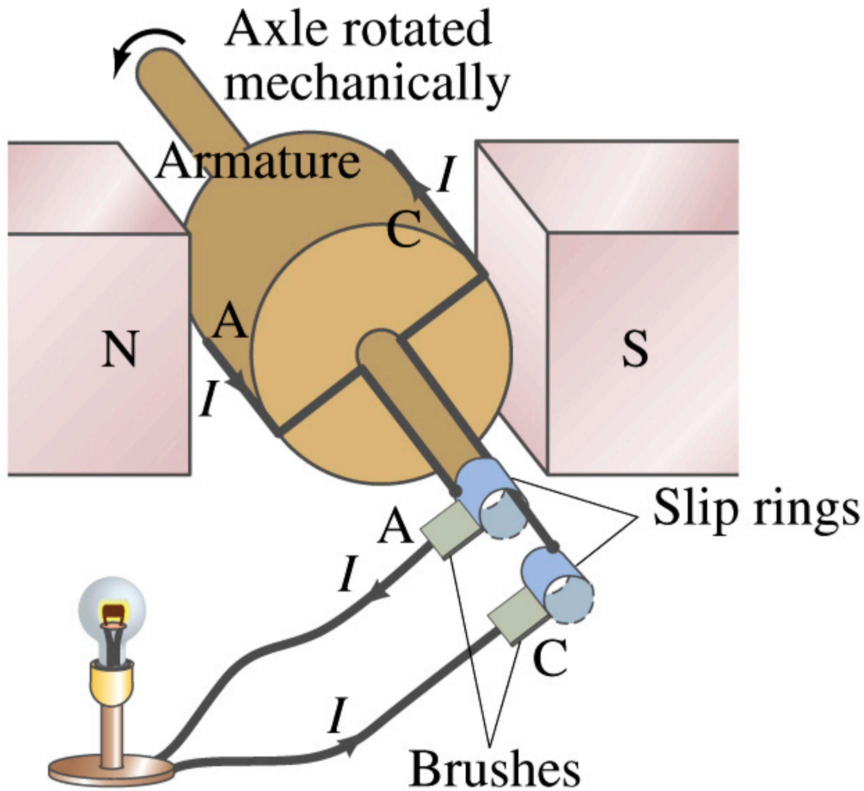
- Using Faraday's law, the induced emf for this loop is

$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{BdA}{dt} = \frac{Blvdt}{dt} = Blv$$

  – This equation is valid as long as B, l and v are perpendicular to each other. What do we do if not?
    - Use the scalar product of vector quantities
- An emf induced on a conductor moving in a magnetic field is called a **motional emf**

Electric Generators

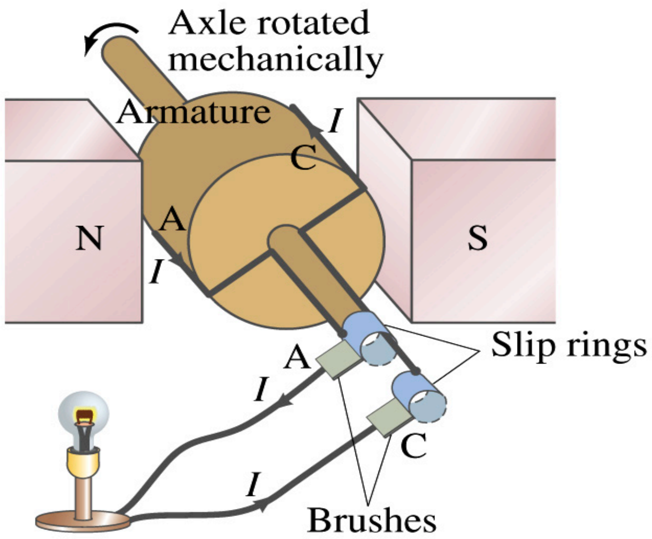
- What does a generator do?
  - Transforms mechanical energy into the electrical energy
  - What does this look like?
    - An inverse of an electric motor which transforms electrical energy to mechanical energy
  - An electric generator is also called a dynamo



- Whose law does the generator based on?
  - Faraday's law of induction

How does an Electric Generator work?

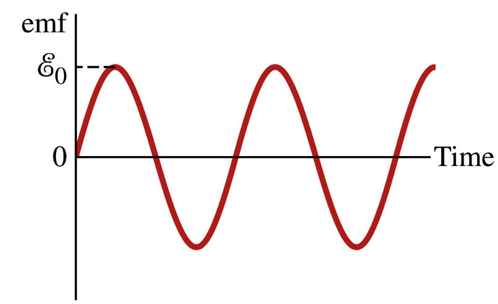
- An electric generator consists of
  - Many coils of wires wound on an armature that can rotate by mechanical means in a magnetic field
- An emf is induced in the rotating coil
- Electric current is the output of a generator



- Which direction does the output current flow when the armature rotates counterclockwise?
  - The conventional current flows outward on wire A toward the brush
  - After half the revolution the wire A will be where the wire C is and the current flow on A is reversed
- Thus the current produced is alternating its direction

How does an Electric Generator work?

- Let's assume the loop is rotating in a uniform B field w/ a constant angular velocity ω. The induced emf is
- $$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{A} = -\frac{d}{dt}\left[BA\cos\theta\right]$$
- What is the variable that changes above?
  - The angle θ. What is $d\theta/dt$?
    - The angular speed ω.
  - So $\theta=\theta_0+\omega t$
  - If we choose $\theta_0=0$, we obtain
  - $$\varepsilon = -BA\frac{d}{dt}\left[\cos\varpi t\right] = BA\varpi\sin\varpi t$$
  - If the coil contains N loops: $\varepsilon = -N\frac{d\Phi_B}{dt} = NBA\varpi\sin\varpi t = \varepsilon_0\sin\varpi t$
  - What is the shape of the output?
    - Sinusoidal w/ the amplitude $\varepsilon_0=NBA\omega$



- USA frequency is 60Hz. Europe is at 50Hz
  - Most the U.S. power is generated at steam plants

Example 29 – 9

An AC generator. The armature of a 60-Hz AC generator rotates in a 0.15-T magnetic field. If the area of the coil is $2.0\times10^{-2}m^2$, how many loops must the coil contain if the peak output is to be ε_0=170V?

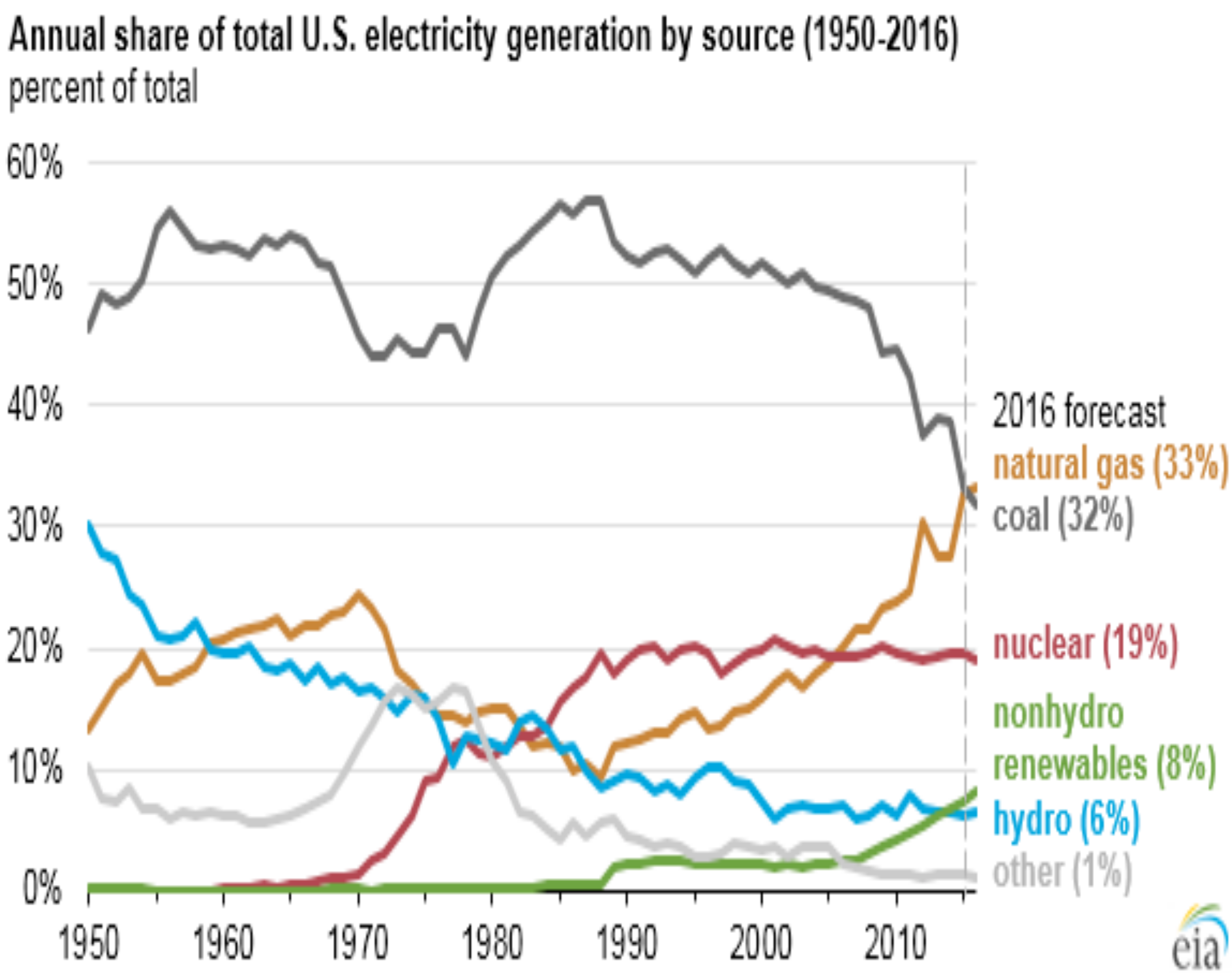
The maximum emf of a generator is $\varepsilon_0 = NBA\varpi$

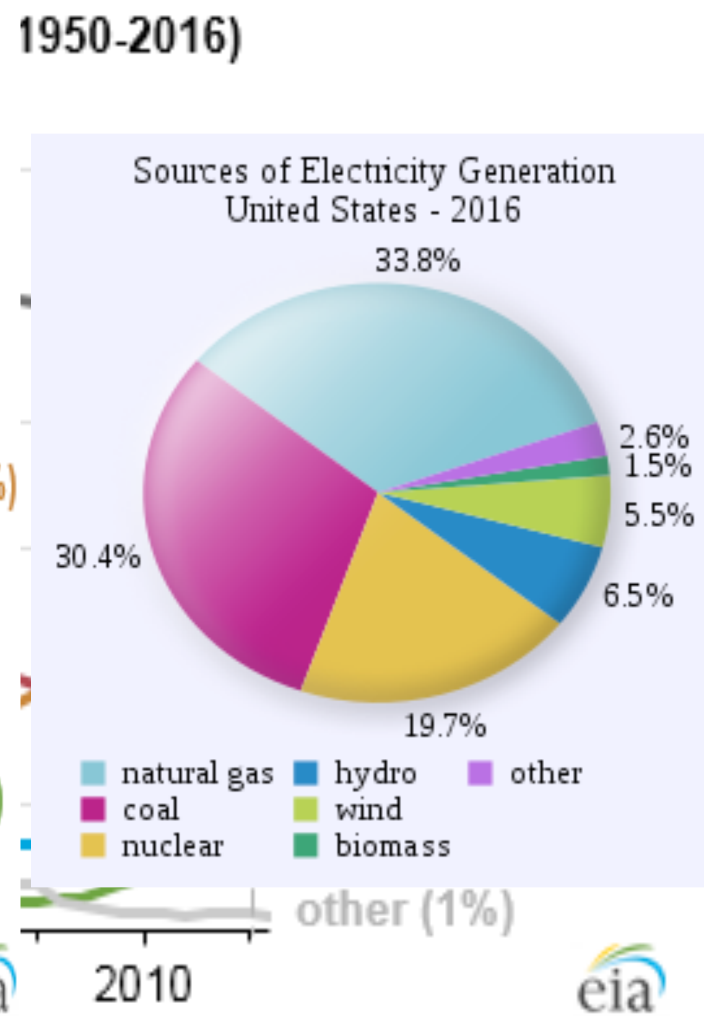
Solving for N $N = \dfrac{\varepsilon_0}{BA\varpi}$

Since $\varpi = 2\pi f$ We obtain

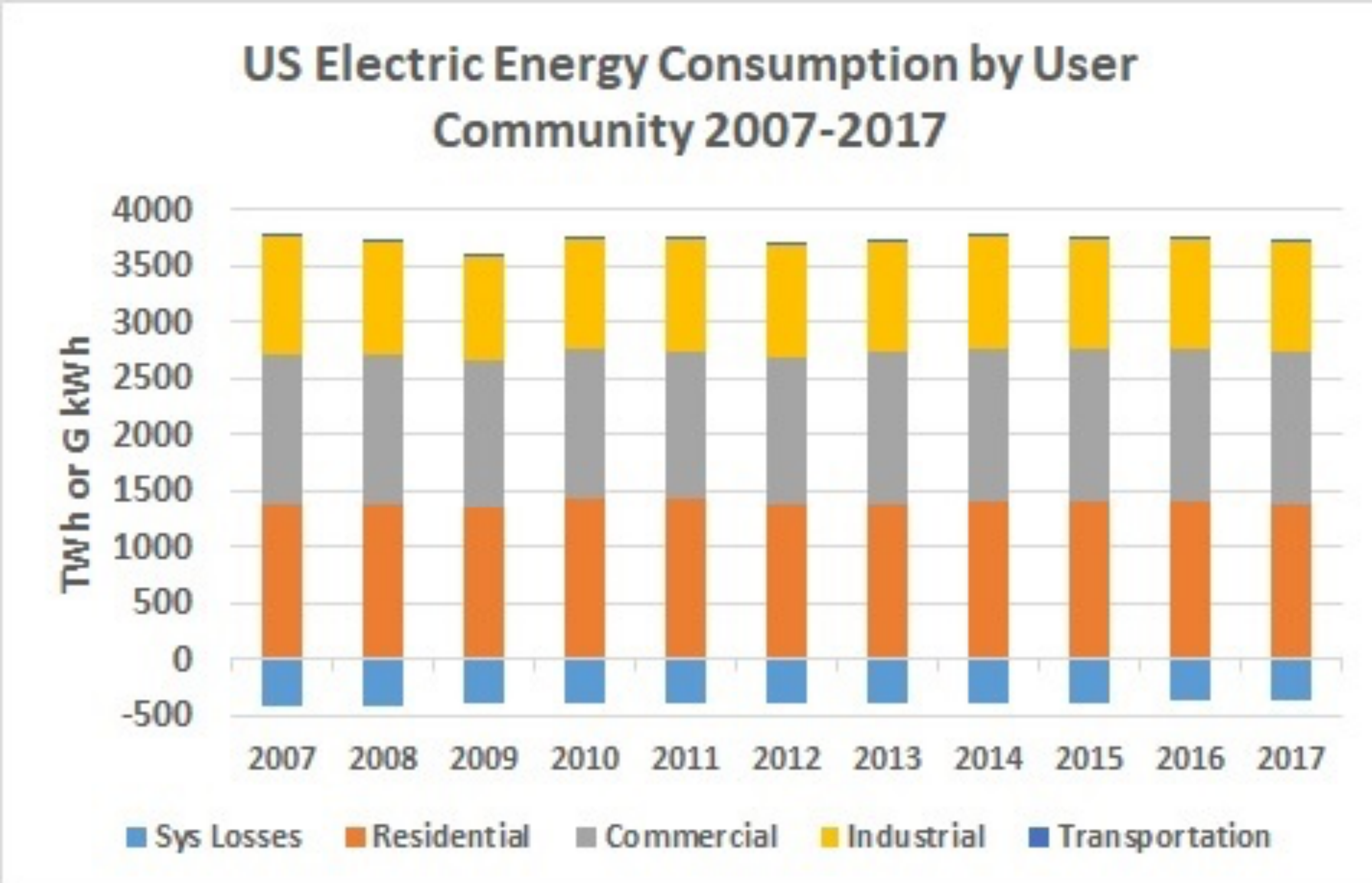
$$N = \frac{\varepsilon_0}{2\pi BAf} = \frac{170V}{2\pi \cdot (0.15T) \cdot \left(2.0\times10^{-2}m^2\right) \cdot \left(60s^{-1}\right)} = 150turns$$

US Electricity Sources





US Electric E Consumption by Users



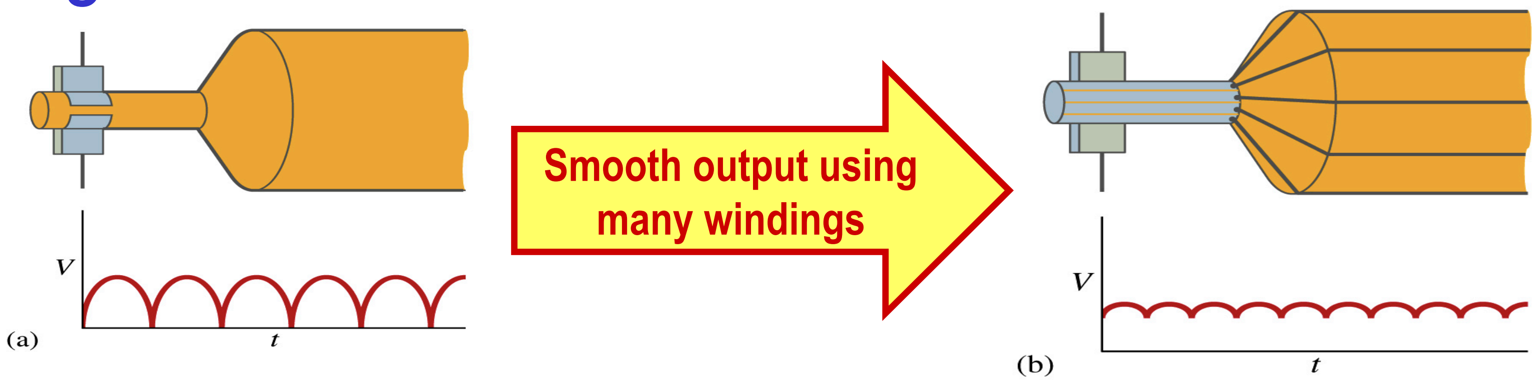
US Energy Information Administration http://www.eia.gov/electricity/

The World Energy Consumption

- In 2016, total worldwide energy consumption was 567 EJ (567×10^{18} J=157 PWh)➔ expected >1000EJ by 2050
  - Equivalent to an average energy consumption rate of 18 terawatts (1.8×10^{13} W)
  - US uses 39.1 PWh (1.38kWh/person, as of 2014)
- The potential for renewable energy
  - solar energy 1600 EJ (444,000 TWh)
  - wind power 600 EJ (167,000 TWh)
  - geothermal energy 500 EJ (139,000 TWh),
  - biomass 250 EJ (70,000 TWh)
  - hydropower 50 EJ (14,000 TWh) an
  - ocean energy 1 EJ (280 TWh)
  - Read this paper if you want to learn more

A DC Generator

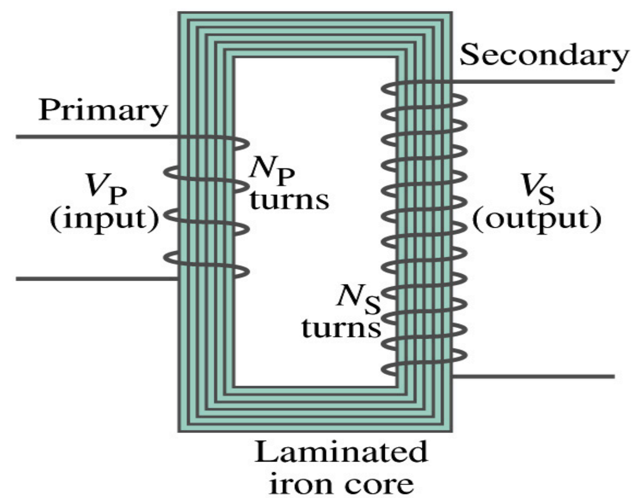
- A DC generator is almost the same as an AC generator except the slip rings are replaced by split-ring commutators



- Output can be smoothed out by placing a capacitor in parallel to the output
  - More commonly done using many armature windings

Transformer

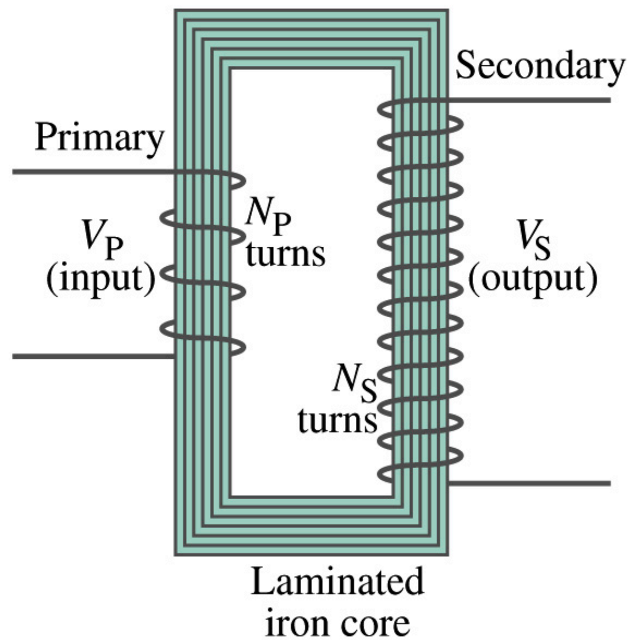
- What is a transformer?
  - A device for increasing or decreasing an AC voltage
  - A few examples?
    - TV sets to provide the high voltage to picture tubes, portable electronic device converters, transformers on the pole, etc
- A transformer consists of two coils of wires known as the primary and the secondary
  - The two coils can be interwoven or linked by a laminated soft iron core to reduce losses due to Eddy current
- Transformers are designed so that all magnetic flux produced by the primary coil pass through the secondary



How does a transformer work?

- When an AC voltage is applied to the primary, the changing B it produces will induce voltage of the same frequency in the secondary wire
- So how would we make the voltage different?
  - By varying the number of loops in each coil
  - From Faraday's law, the induced emf in the secondary is
  - $V_S = N_S \dfrac{d\Phi_B}{dt}$
  - The input primary voltage is
  - $V_P = N_P \dfrac{d\Phi_B}{dt}$
  - Since $d\Phi_B/dt$ is the same, we obtain
  - $\boxed{\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P}}$

Transformer Equation



Transformer Equation

- The transformer equation does not work for DC current
  - Since there is no change of magnetic flux!!
- If $N_S>N_P$, the output voltage is greater than the input so it is called a step-up transformer while $N_S<N_P$ is called step-down transformer
- Now, it looks like energy conservation is violated since we can get more emf from smaller ones, right?
  - Wrong! Wrong! Wrong! Energy is always conserved!
  - A well designed transformer can be more than 99% efficient
  - The power output is the same as the input:
  - $V_P I_P = V_S I_S$
  - $\boxed{\frac{I_S}{I_P} = \frac{V_P}{V_S} = \frac{N_P}{N_S}}$

The output current for a step-up transformer will be lower than the input, while it is larger for a step-down x-former than the input.

Example for A Transformer

Portable radio transformer. A transformer for home use of a portable radio reduces 120-V AC to 9.0V AC. The secondary contains 30 turns, and the radio draws 400mA. Calculate (a) the number of turns in the primary (b) the current in the primary and (c) the power transformed.

(a) What kind of a transformer is this? A step-down x-former

Since $\frac{V_P}{V_S} = \frac{N_P}{N_S}$ We obtain $N_P = N_S \frac{V_P}{V_S} = 30 \frac{120V}{9V} = 400 turns$

(b) Also from the transformer equation $\frac{I_S}{I_P} = \frac{V_P}{V_S}$ We obtain

$$I_P = I_S \frac{V_S}{V_P} = 0.4A \frac{9V}{120V} = 0.03A$$

(c) Thus the power transformed is

$$P = I_S V_S = (0.4A) \cdot (9V) = 3.6W$$

How about the input power? The same assuming 100% efficiency.

Example 29 – 13: Power Transmission

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$.

(a) If 120kW is sent at 240V, the total current is $I=\frac{P}{V}=\frac{120\times 10^3}{240}=500A.$

Thus the power loss due to transmission line is

$$P=I^2R=(500A)^2\cdot(0.4\Omega)=100kW$$

(b) If 120kW is sent at 24,000V, the total current is $I=\frac{P}{V}=\frac{120\times 10^3}{24\times 10^3}=5.0A.$

Thus the power loss due to transmission line is

$$P=I^2R=(5A)^2\cdot(0.4\Omega)=10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

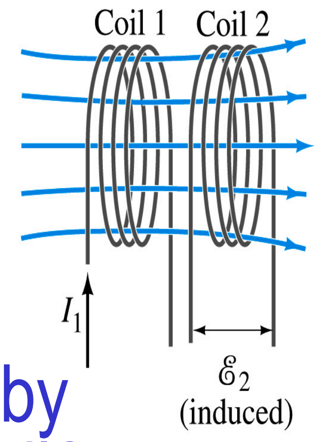
Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $\boxed{M_{21} = N_2\Phi_{21}/I_1}$.
- The emf induced in coil2 due to the changing current in coil1 is

$$\boxed{\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}}$$



Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

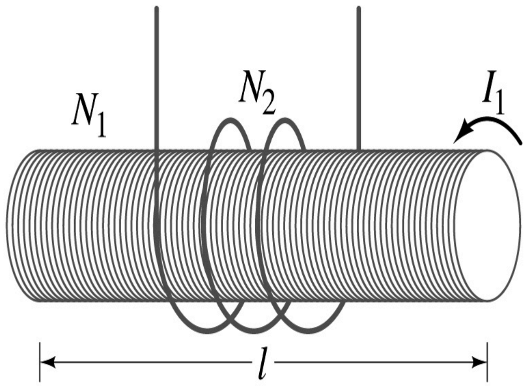
    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain
  - SI unit for mutual inductance is henry (H)

$$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$

$$1H = 1V \cdot s/A = 1\Omega \cdot s$$

Example 30 – 1



Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.

First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \frac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
- We define self-inductance, $\mathcal{L}$: $\boxed{L = \frac{N\Phi_B}{I}}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B=\mu_0 nI=\mu_0 NI/l$

The flux is, therefore, $\Phi_B=BA=\mu_0 NIA/l$

Using the formula for self inductance: $L=\dfrac{N\Phi_B}{I}=\dfrac{N\cdot\mu_0 NIA/l}{I}=\dfrac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}m^2\right)}{5.0\times10^{-2}m}=7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B=\mu NI/l$

$$L=\frac{\mu N^2 A}{l}=\frac{4000\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}m^2\right)}{5.0\times10^{-2}m}=0.030H=30mH$$

Energy Stored in the Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\dfrac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LIdI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LIdI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $\boxed{U = \frac{1}{2}LI^2}$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect

    $$L = \mu_0 N^2 A / l$$

  - The magnetic field in a solenoid is $B = \mu_0 NI / l$
  - Thus the energy stored in an inductor is

    $$U = \frac{1}{2} L I^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} Al$$

  - Thus the energy density is

    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$

  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What is this?

Volume V

$$\boxed{U = \frac{1}{2} \frac{B^2}{\mu_0} Al}$$ E

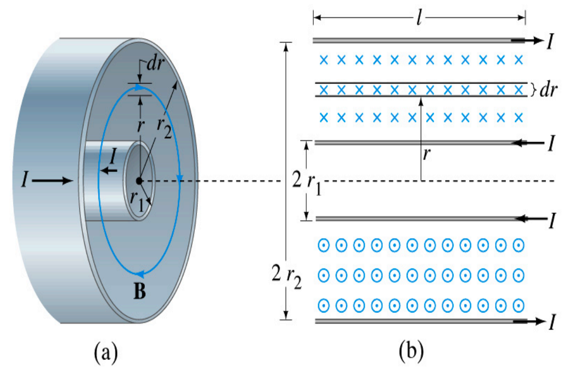
$$\boxed{u = \frac{1}{2} \frac{B^2}{\mu_0}}$$ E density

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0,

$$X_L = \varpi L = 0$$

So for DC power, the current is from Kirchhoff's rule

$$V - IR = 0$$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an AC power with f=60Hz, the reactance is

$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\boxed{\oint \vec{B}\cdot d\vec{A} = 0}$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt}}$$

Ampére's Law

A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes.
    - This changing magnetic field then in turn produces the electric field that changes.
    - This process continues.
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space