PHYS 1441 – Section 001
Lecture #5

Monday, June 15, 2020

Dr. Jaehoon Yu

- Chapter 22
  - Gauss' Law
  - Electric Flux
  - Gauss' Law with Multiple Charges
- Chapter 23
  - Electric Potential Energy
  - Electric Potential due to Point Charges

Today's homework is #3, due 11pm, Thursday, June 18!!

Announcements

- Reading assignment: CH23.9

Reminder: SP#2 – Angels & Demons

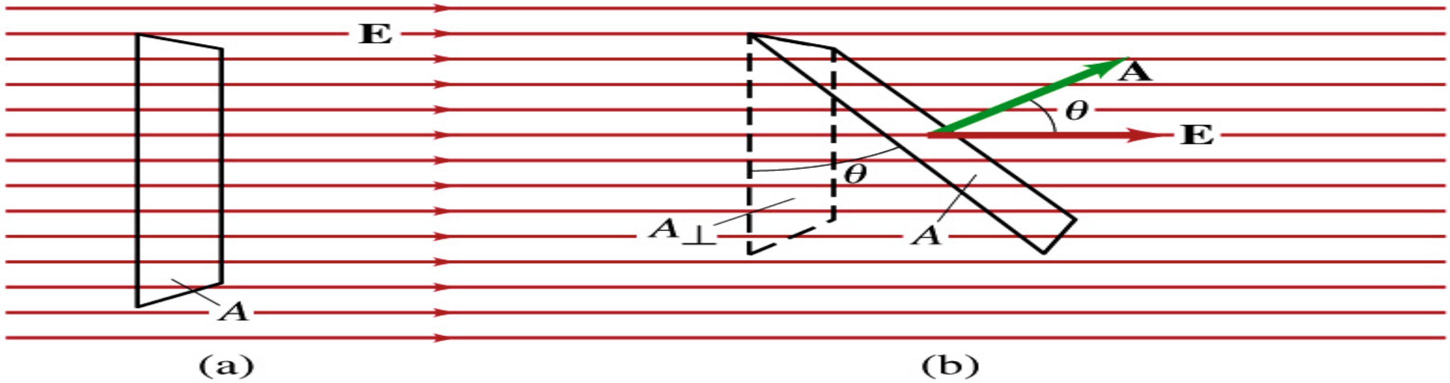
- Compute the total possible energy released from an annihilation of x-grams of anti-matter and the same quantity of matter, where x is the last two digits of your SS# or DL#. (20 points)
  - Use the famous Einstein's formula for mass-energy equivalence
- Compute the power output of this annihilation when the energy is released in x ns, where x is again the first two digits of your SS# or DL#. (10 points)
- Compute how many cups of gasoline (8MJ) this energy corresponds to. (5 points)
- Compute how many months of world electricity usage (3.6GJ/mo) this energy corresponds to. (5 points)
- Due by the beginning of the class Wednesday, June. 17
  - All pages must be in one PDF file with the name SP2-first-last-summer20.pdf in an email with the subject "Special Project 2, PHYS1444"

Gauss' Law

- Gauss' law states the relationship between the electric charge and the electric field.
  - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

Electric Flux

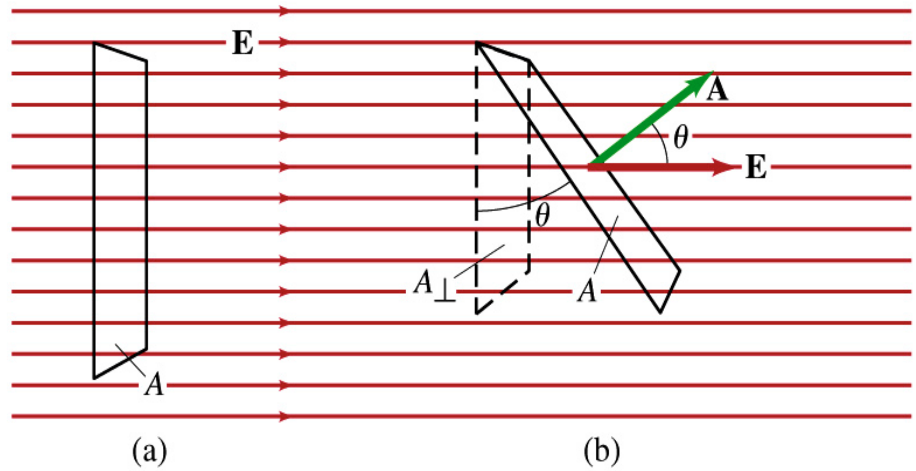
- Let's imagine a surface of area A through which a uniform electric field E passes



- The electric flux Φ_E is defined as
  - Φ_E=EA, if the field is perpendicular to the surface
  - Φ_E=EAcosθ, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

Example 22 – 1

- **Electric flux**. (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux if the angle is 30 degrees?



The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

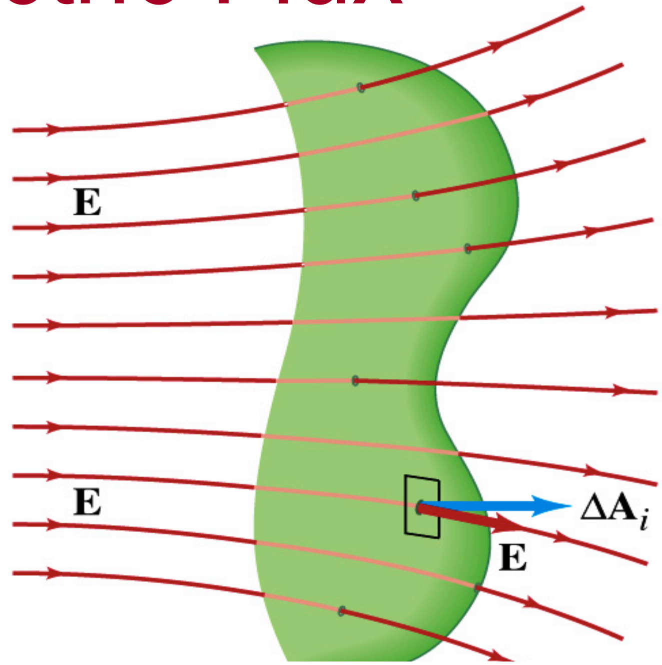
$$\Phi_E = EA\cos\theta = EA = (200N/C)\cdot(0.1\times 0.2m^2) = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees, we obtain

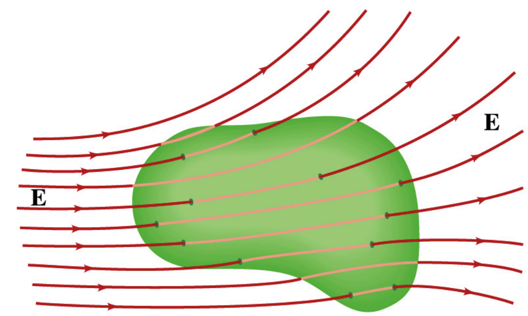
$$\Phi_E = EA\cos 30^\circ = (200N/C)\cdot(0.1\times 0.2m^2)\cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta\mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is approximately



$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta\vec{A}_i$$

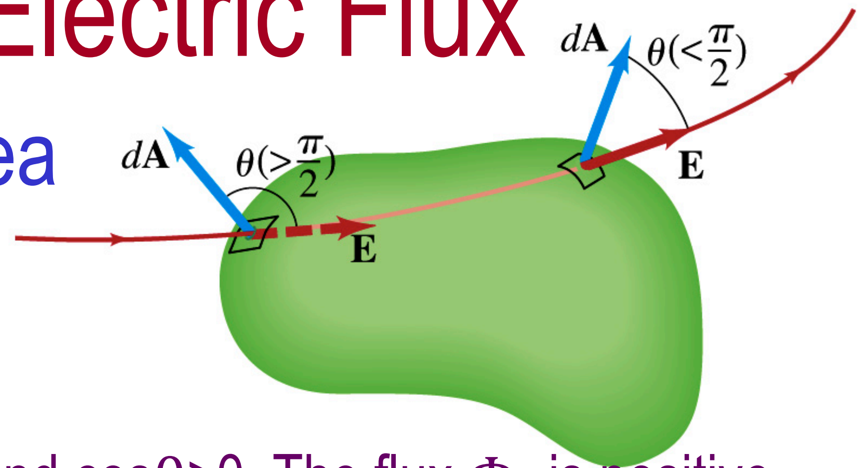


- In the limit where $\Delta\mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

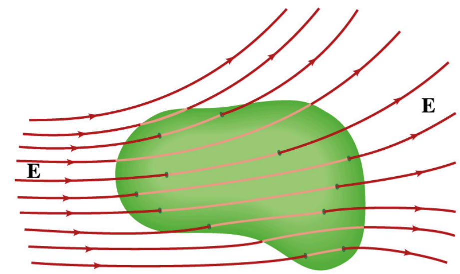
$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A}$$ enclosed surface

Generalization of the Electric Flux

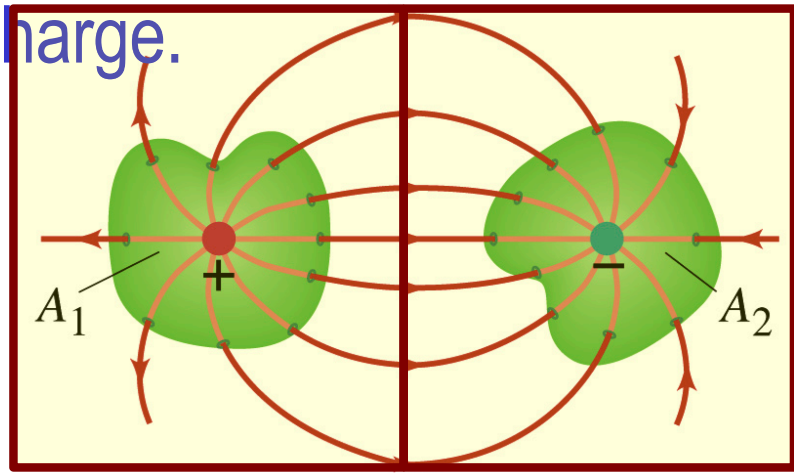


- We arbitrarily define that the area vector points outward from the enclosed volume.
  - For the line leaving the volume, $|\theta|<\pi/2$ and $\cos\theta>0$. The flux Φ_E is positive.
  - For the line coming into the volume, $|\theta|>\pi/2$ and $\cos\theta<0$. The flux Φ_E is negative.
  - If $\Phi_E>0$, there is net flux out of the volume.
  - If $\Phi_E<0$, there is flux into the volume.



- In the above figures, each field line that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0.$
- The flux is non-zero only if one or more lines start or end inside the surface.

Generalization of the Electric Flux



- The field line starts or ends only on a charge.
- Sign of the net flux on the surface A_1?
  - Net outward flux (positive flux)
- How about A_2?
  - Net inward flux (negative flux)
- What is the flux in the figure bottom right?
  - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.