PHYS 1441 – Section 001
Lecture #7

Wednesday, June 17, 2020

Dr. Jaehoon Yu

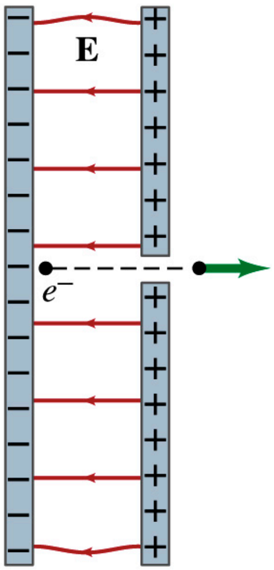
- Chapter 23
  - V due to Charge Distributions
  - Equi-potential Lines and Surfaces
  - Electric Potential Energy
- Chapter 24 Capacitance etc..
  - Capacitors
  - Capacitors in Series or Parallel

Today's homework is #4, due 11pm, Monday, June 22!!

Announcements

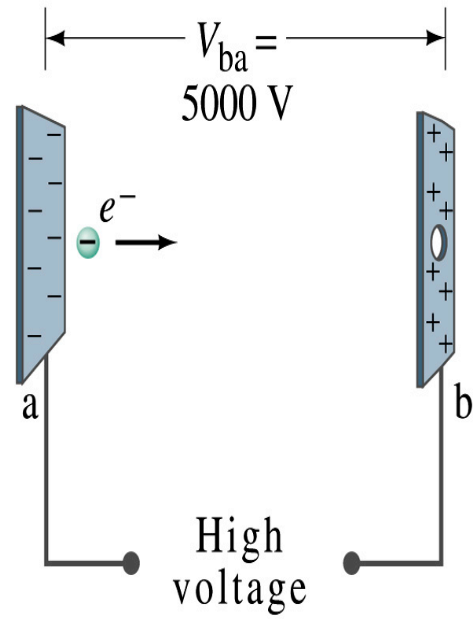
- Online Quiz #2 on Quest
  - Beginning of class tomorrow, Thursday, June 18
  - Covers: CH22.1 through what we finish today
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions or setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than 10am Monday morning
    - Once submitted, you cannot change, unless I ask you to delete some part of the sheet!
- Online comprehensive Mid-term Exam on Quest
  - In class, Tuesday, June 23
  - Covers CH21.1 to what we finish Monday, June 22 + A1 - A8, math
  - BYOF

Special Project #3



- **Particle Accelerator**. A charged particle of mass **M** with charge **-Q** is accelerated in the uniform field **E** between two parallel charged plates whose separation is **D** as shown in the figure on the right. The charged particle is accelerated from an initial speed $\mathbf{v_0}$ near the negative plate and passes through a tiny hole in the positive plate.
  - Derive the formula for the electric field E to accelerate the charged particle to a fraction *f* of the speed of light *c*. Express E in terms of **M, Q, D,** *f*, **c** and $\mathbf{v_0}$.
  - (a) Using the Coulomb force and the kinematic equations. (8 points)
  - (b) Using the work-kinetic energy theorem. (8 points)
  - (c) Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up and write down the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Must be handwritten and not copied from anyone else!
  - Follow the SP naming convention: SP3-first-last-summer20.pdf which includes all pages in one file ➔ Be sure to write your name onto all pages of the project report!
- Due beginning of the class Wednesday, June 24

Example 23 – 2



Electrons in TV tube: Suppose an electron in the picture tube of a television set is accelerated from rest through a potential difference V_{ba}=+5000V. (a) What is the change in potential energy of the electron? (b) What is the speed of the electron (m=9.1x10^{-31}kg) as a result of this acceleration? (c) Repeat for a proton (m=1.67x10^{-27}kg) that accelerates through a potential difference of V_{ba}=-5000V.

- What formula should you use? $U = qV$
- (a) What is the charge of an electron?
  - $e = -1.6\times10^{-19}C$
- So what is the change of its potential energy?

$$\Delta U = qV_{ba} = eV_{ba} = \left(-1.6\times10^{-19}C\right)\left(+5000V\right) = -8.0\times10^{-16}J$$

Example 23 – 2

- (b) Speed of the electron?
  - The entire potential energy of the electron turns to its kinetic energy. Thus the equation is

$$\Delta K = \frac{1}{2} m_e v_e^2 - 0 = W = -\Delta U = -eV_{ba} =$$

$$= -\left(-1.6\times10^{-19}C\right)5000V = 8.0\times10^{-16}J$$

$$v_e = \sqrt{\frac{2\times eV_{ba}}{m_e}} = \sqrt{\frac{2\times8.0\times10^{-16}}{9.1\times10^{-31}}} = 4.2\times10^{7}m/s$$

- (C) Speed of the proton?

$$\Delta K = \frac{1}{2} m_p v_p^2 - 0 = W = -\Delta U = -\left\{(-e)(-V_{ba})\right\} = -eV_{ba} = 8.0\times10^{-16}J$$

$$v_p = \sqrt{\frac{2\times eV_{ba}}{m_p}} = \sqrt{\frac{2\times8.0\times10^{-16}}{1.67\times10^{-27}}} = 9.8\times10^{5}m/s$$

Electric Potential and Electric Field

- The effect of a charge distribution can be described in terms of electric field or electric potential.
  - What kind of quantities are the electric field and the electric potential?
    - Electric Field: Vector
    - Electric Potential: Scalar
  - Since electric potential is a scalar quantity, it is often easier to handle.
- Well other than the above, how are these two quantities related?

Electric Potential and Electric Field

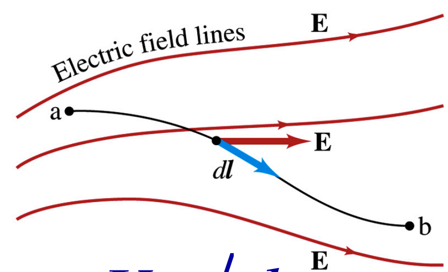
- **Potential energy** change is expressed in terms of a conservative force (point ***a*** at a higher potential)

$$U_b - U_a = -\vec{F}\cdot\vec{D} = -W_C$$

- For the electrical case, we are more interested in the **potential** difference:

$$V_{ba} = V_b - V_a = \frac{U_b - U_a}{q} = -\frac{\vec{F}}{q}\cdot\vec{D} = -\vec{E}\cdot\vec{D} = -ED\cos\theta$$

  – This formula can be used to determine V_{ba} when the electric field is given.



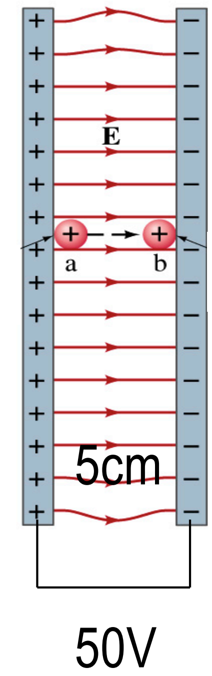
- When the field is uniform

$$V_b - V_a = -\vec{E}\cdot\vec{D} = -ED\cos\theta = -Ed \quad \text{so} \quad E = -V_{ba}/d$$

What does "-" sign mean? The direction of E is along that of decreasing potential.

Unit of the electric field in terms of potential? V/m Can you derive this from N/C?

Example



Uniform electric field obtained from voltage: Two parallel plates are charged to a voltage of 50V. If the separation between the plates is 5.0cm, calculate the magnitude of the electric field between them, ignoring any fringe effect.

What is the relationship between electric field and the potential for a uniform field?

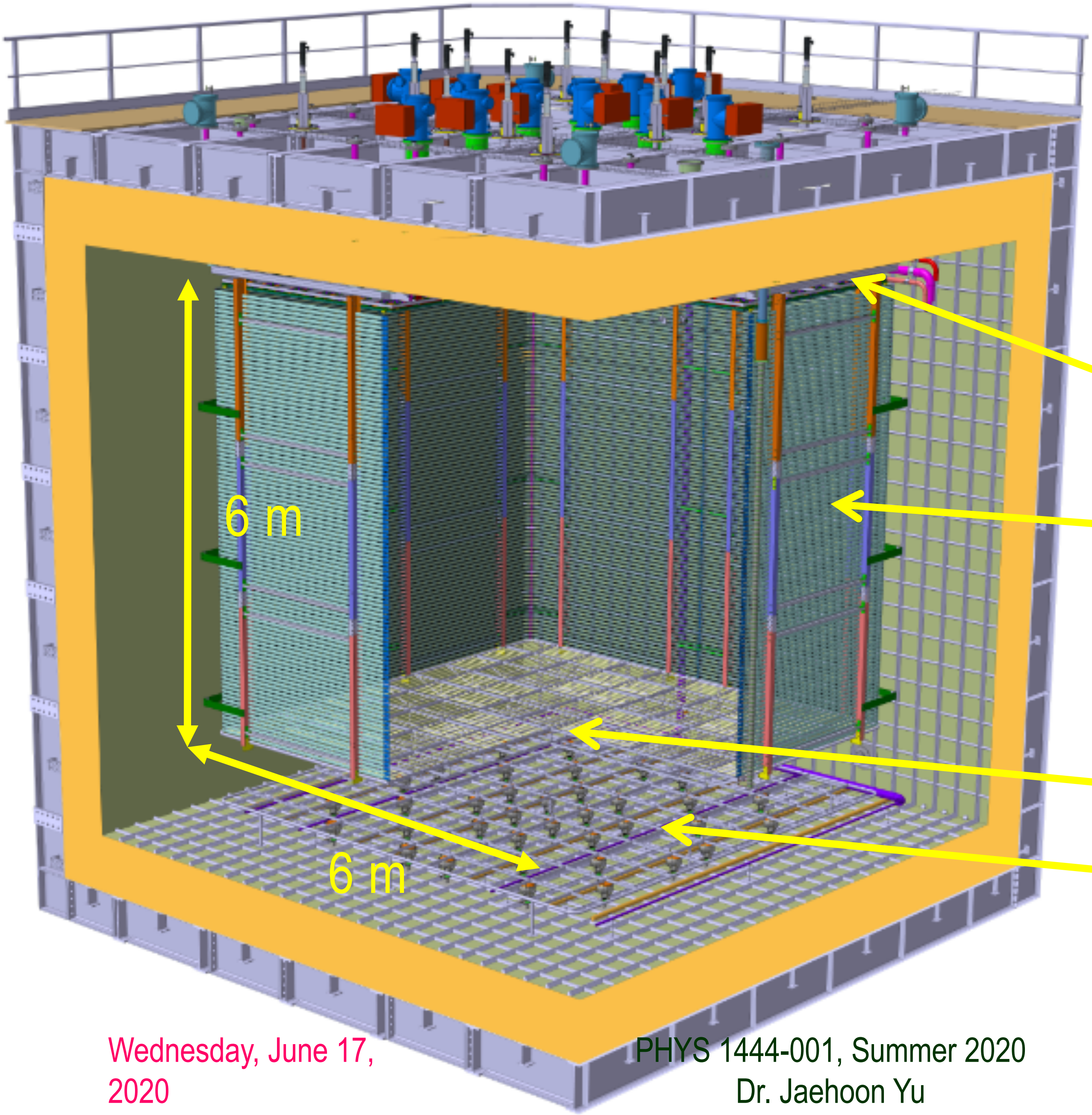
$$V = Ed$$

Solving for E

$$E = \frac{V}{d} = \frac{50V}{5.0cm} = \frac{50V}{5\times10^{-2}m} = 1000V/m$$

Which direction is the field? Direction of decreasing potential!

E Field in ProtoDUNE DP



- What is the electric field between the cathode and the ground grid?
- What do you need to know?
  - $V_{cathode}$= - 300kV
  - d= 70cm

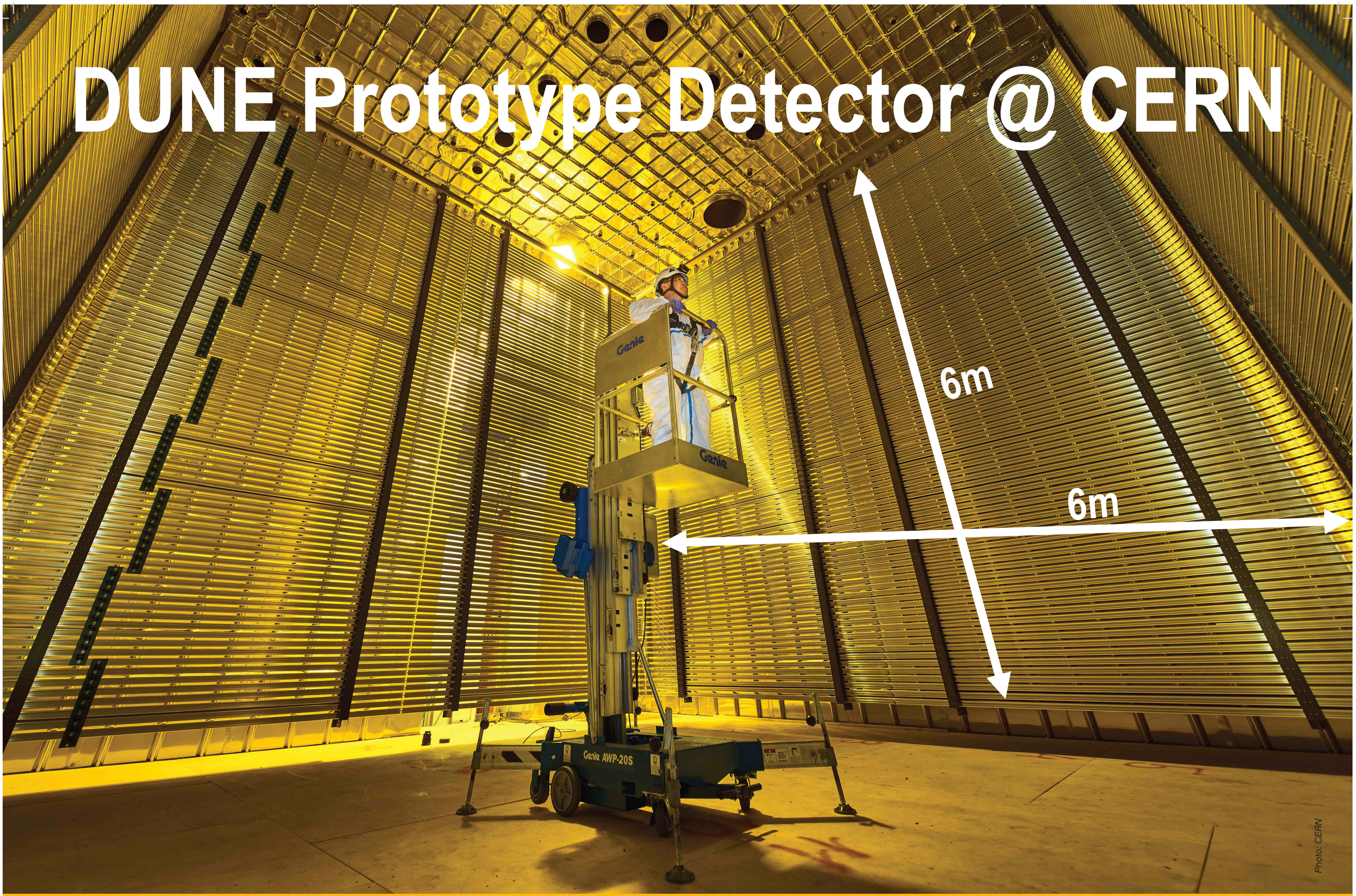
Charge Readout Planes

Field Cage (common structural elements with SP)

Cathode

Ground Grid

DUNE Prototype Detector @ CERN



Prototype detector for the Deep Underground Neutrino Experiment

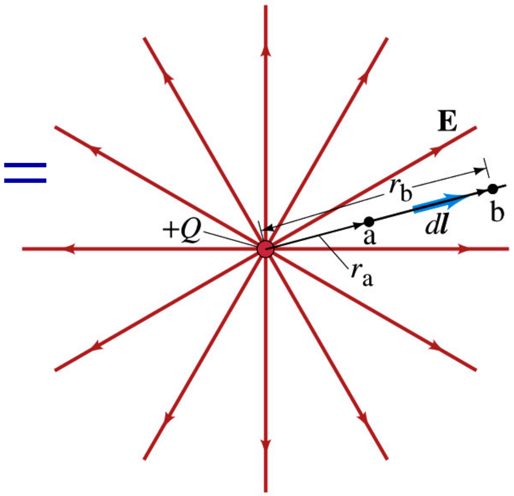
Electric Potential due to Point Charges

- What is the electric field by a single point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b in the radial direction away from the charge Q is (poll 9)

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\hat{r}}{r^2}\cdot\hat{r}\,dr =$$

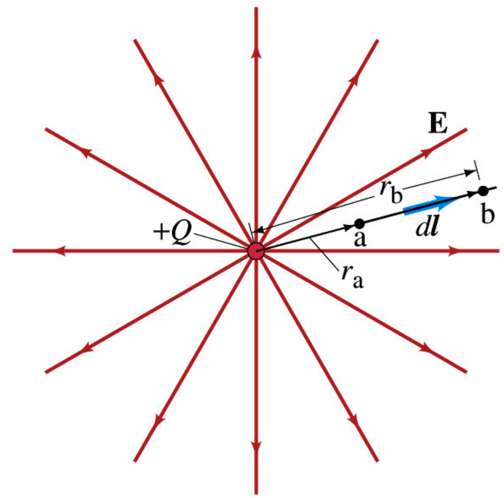


$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential by a single point charge can be thought of **the potential difference by a single point charge between r and infinity**

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?
  - Electric potential
    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential by each of the source charges to obtain the total potential from multiple charges, since potential is a scalar quantity

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

  - Electric field
    - Electric force per unit charge
    - Inversely proportional to the **square** of the distance
    - Need vector sums to obtain the total field from multiple source charges

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

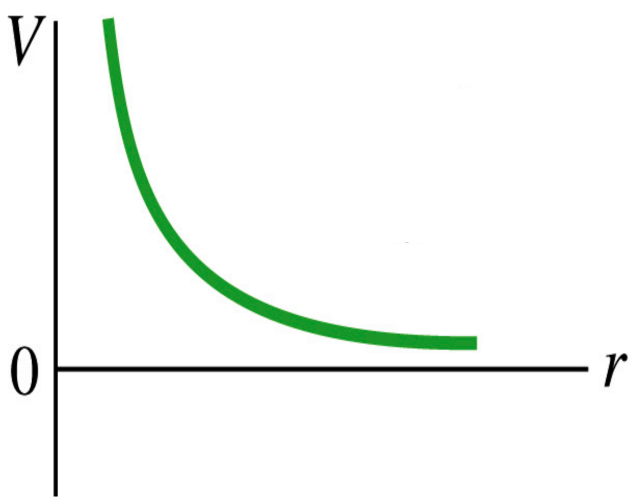
- Potential due to a positive charge is a large positive near the charge and decreases towards 0 at the large distance.
- Potential due to a negative charge is a large negative near the charge and increases towards 0 at a large distance.

Shape of the Electric Potential

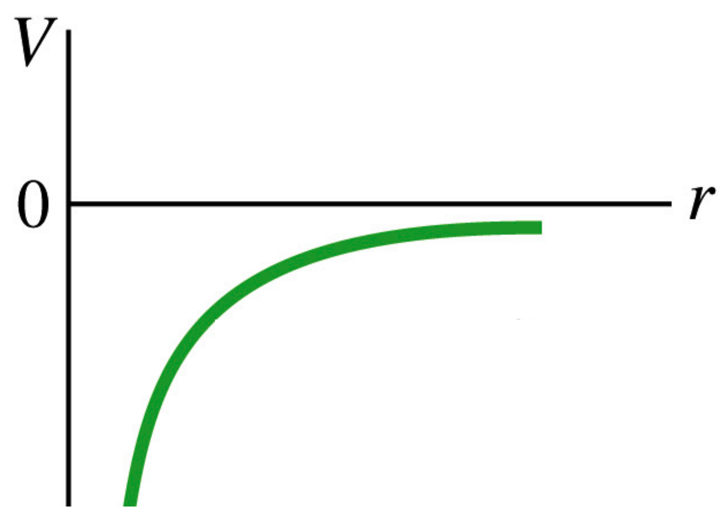
- So, how does the electric potential look like as a function of distance?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

If Q is Positive



If Q is Negative



Uniformly charged sphere would have the potential the same as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 23 – 6

Work to bring two positive charges close together: What is minimum work required by an external force to bring the charge q=+3.00μC from a great distance away (r=∞) to a point 0.500m from a charge Q=+20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(3.00\times10^{-6}C\right)\left(20.00\times10^{-6}C\right)}{0.500m} = -1.08J$$

Electric force does negative work. In other words, the external force must work +1.08J to bring the charge 3.00μC from infinity to 0.500m to the charge 20.0μC.

Electric Potential by Charge Distributions

- Let's consider the case of n individual point charges in a given space and V=0 at r=∞.
- Then the potential V_{ia} due to the charge Q_i at point *a*, at a distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

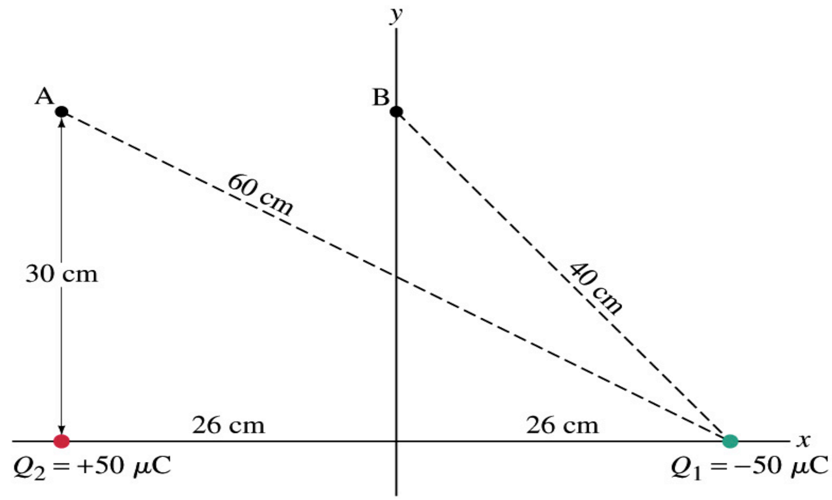
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Example

- **Potential due to two charges**: Calculate the electric potential (a) at point A in the figure due to the two charges shown, and (b) at point B.



- Potential is a scalar quantity, so one adds the potential by each of the source charge, as if they are numbers.

(a) potential at A is $V_A = V_{1A} + V_{2A} = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{iA}} =$

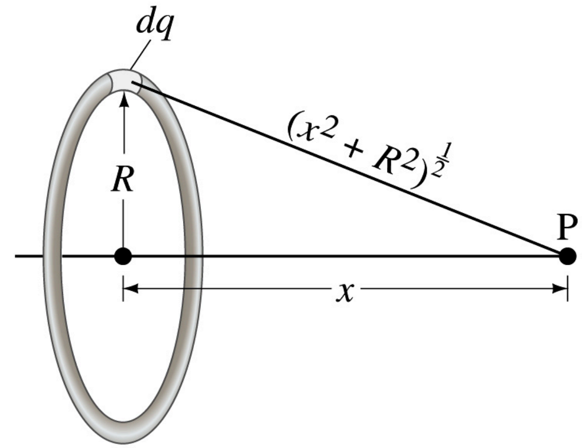
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{r_{2A}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{r_{1A}} + \frac{Q_2}{r_{2A}}\right)$$

$$= 9.0\times10^{9}\left(\frac{-50\times10^{-6}}{0.60} + \frac{50\times10^{-6}}{0.30}\right) = 7.5\times10^{5}V$$

(b) How about potential at B?

Example 23 – 8

- **Potential due to a ring of charge**: A thin circular ring of radius R carries a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center.



- Each point on the ring is at the same distance from the point P. What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0 r}\int dq =$$

$$\frac{1}{4\pi\varepsilon_0\sqrt{x^2+R^2}}\int dq = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

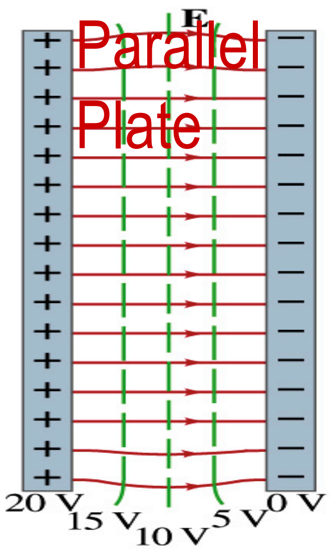
What's this?

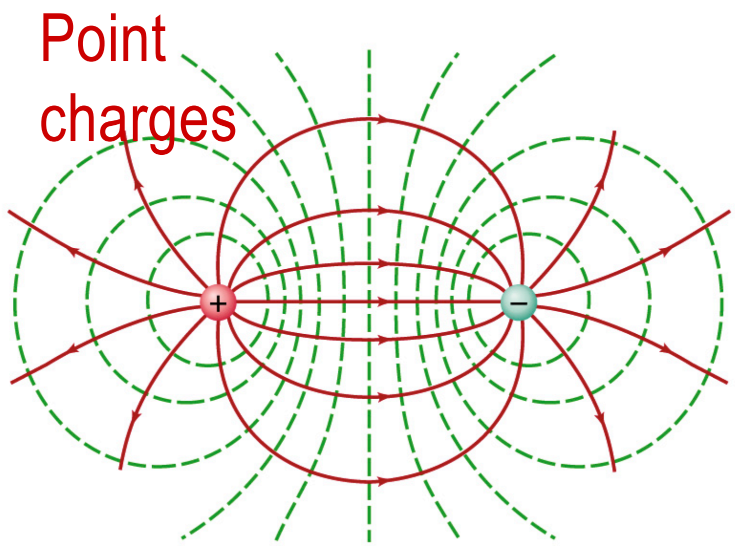
Equi-potential Surfaces

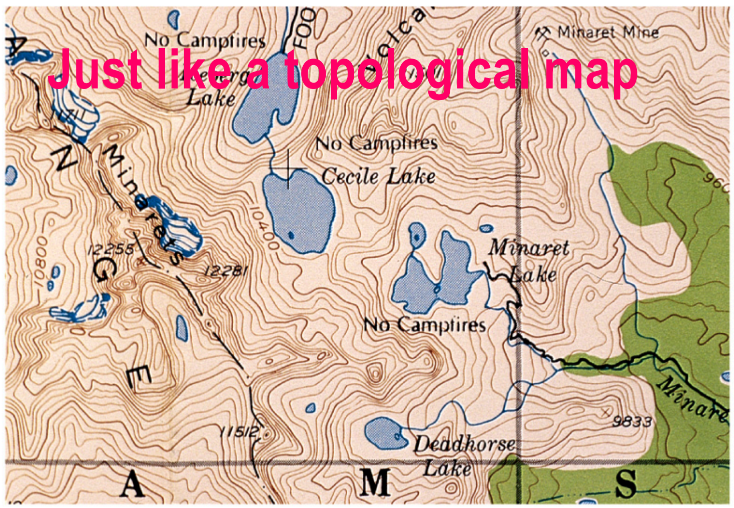
- Electric potential can be graphically shown using the equipotential lines in 2-D or the equipotential surfaces in 3-D
- Any two points on the equipotential surfaces (lines) are at the same potential
- What does this mean in terms of the potential difference?
  - The potential difference between any two points on an equipotential surface is 0.
- How about the potential energy difference?
  - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  - No work is necessary to move a charge between these two points.

Equi-potential Surfaces

- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- Since there can be no electric field within a conductor in a static case, the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.

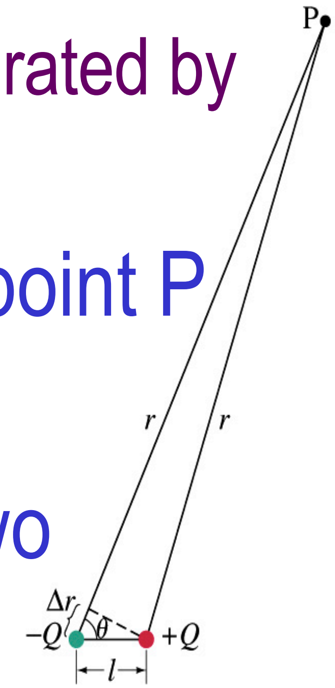






Electric Potential due to Electric Dipoles

- What is an electric dipole?
  - Two equal point charge Q of opposite signs separated by a distance ℓ and behaves like one entity: p=Qℓ
- For the electric potential due to a dipole at a point P
  - We take V=0 at r=∞
- The simple sum of the potential at P by the two charges is



$$V = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{(-Q)}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

- Since Δr=ℓcosθ and if r>>ℓ, r>>Δr, thus r+Δr ~r and

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

V by a dipole at a distance r from the dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

E Determined from V

- Potential difference between two points under an electric field is $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$
- So in a differential form, we can write

$$dV = -\vec{E} \cdot d\vec{l} = -E_l dl$$

  – What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between the two points separated by a distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.
- Thus we can write the field component E_ℓ as

$$E_l = -\frac{dV}{dl}$$

Physical Meaning?

The component of the electric field in any direction is equal to the negative rate of change of the electric potential as a function of distance in that direction!!

E Determined from V

- The quantity dV/dℓ is called the **gradient of V** in a particular direction
  - If no direction is specified, the term gradient refers to the direction in which **V changes most rapidly** and this would be the direction of the field vector **E** at that point.
  - So if **E** and dℓ are parallel to each other, $E = -\frac{dV}{dl}$
- If E is written as a function x, y and z, the ℓ refers to x, y and z $E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$
- $\frac{\partial V}{\partial x}$ is the "partial derivative" of V with respect to x, while y and z are held constant
- In vector form, $\vec{E} = -gradV = -\vec{\nabla} V = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)V$

$\vec{\nabla} = -\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$ is called ***del*** or the ***gradient operator*** and is a ***vector operator***.

Electrostatic Potential Energy

- Consider a case in which a point charge q is moved between points *a* and *b* where the electrostatic potential due to other charges in the system is V_a and V_b
- The change in electrostatic potential energy of q in the field by other charges is

$$\Delta U = U_b - U_a = q\left(V_b - V_a\right) = qV_{ba}$$

- Now what is the electrostatic potential energy of a system of charges?
  - Let's choose V=0 at r=∞
  - If there are no other charges around, single point charge Q_1 in isolation has no potential energy and is under no electric force

Electrostatic Potential Energy; Two charges

- If the second point charge Q_2 is brought close to Q_1 at a distance r_{12}, the potential due to Q_1 at the position of Q_2 is

$$V = \frac{Q_1}{4\pi\varepsilon_0} \frac{1}{r_{12}}$$

- The potential energy of the two charges relative to V=0 at r= ∞ is

$$U = Q_2 V = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{r_{12}}$$

  - This is the work that needs to be done by an external force to bring Q_2 from infinity to the distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy; Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - Work needed to bring Q_1 to a certain location without the presence of any charge is 0.
  - Work needed to bring Q_2 to a distance to Q_1 is $U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$
  - Work need to bring Q_3 to certain distances to Q_1 and Q_2 is

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_{23}}$$

- So the total electrostatic potential energy of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right) \quad \left[V = 0 \text{ at } r = \infty\right]$$

  - What about a four charge system or N charge system?

Electrostatic Potential Energy: electron Volt

- What is the unit of the electrostatic potential energy? (poll 8)
  - Joules
- Joules is a very large unit in dealing with electrons, atoms, molecules or any atomic scale problems
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the magnitude of the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot 1V = 1.6\times10^{-19}J$
- Note that eV, however, is **NOT a standard SI unit**. You must convert the energy to Joules for computations.
- What is the speed of an electron with kinetic energy 5000eV?