PHYS 1441 – Section 001
Lecture #14

Tuesday, June 30, 2020

Dr. Jaehoon Yu

CH 27: Magnetism and Magnetic Field

- Magnetic Force on Electric Current
- Magnetic Force on a Moving Charge
- Charged Particle Path in a Magnetic Field
- Cyclotron Frequency
- Torque on a Current Loop
- Magnetic Dipole Moment

CH 28: Sources of Magnetic Field

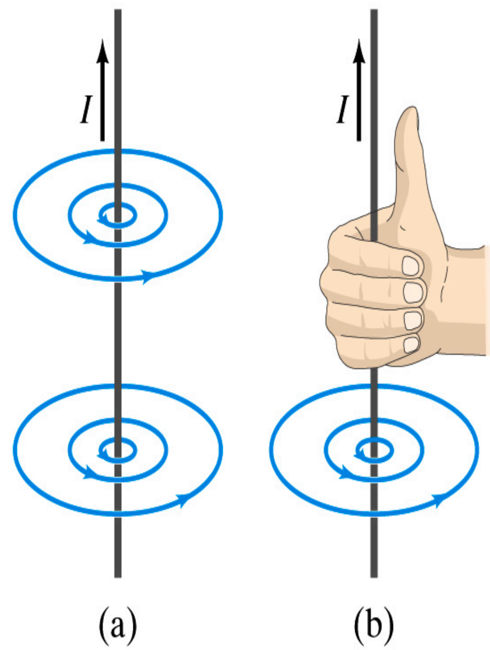
- Sources of Magnetic Field

Announcements

- Reading Assignments: CH27.6 – 8 and CH28.6 – 10
- 2nd Non-comprehensive term exam
  - Beginning of the class tomorrow, Wednesday, July 1 (roll call starts at 10:20am!)
  - Covers CH25.1 – end of CH27.4
  - BYOF: You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, figures, pictures, arrows, or setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than 10am!
    - Once submitted, you cannot change, unless I ask you to delete some part of the sheet!
- Special seminar on COVID–19 extra credit Monday, July 6
  - Dr. Linda Lee, a frontline doctor
- Quiz 3 results
  - Class average: 42.1/70 ➔ equivalent to 60.1/100
    - Previous quizzes: 61/100 and 53.8/100
  - Top score: 70/70

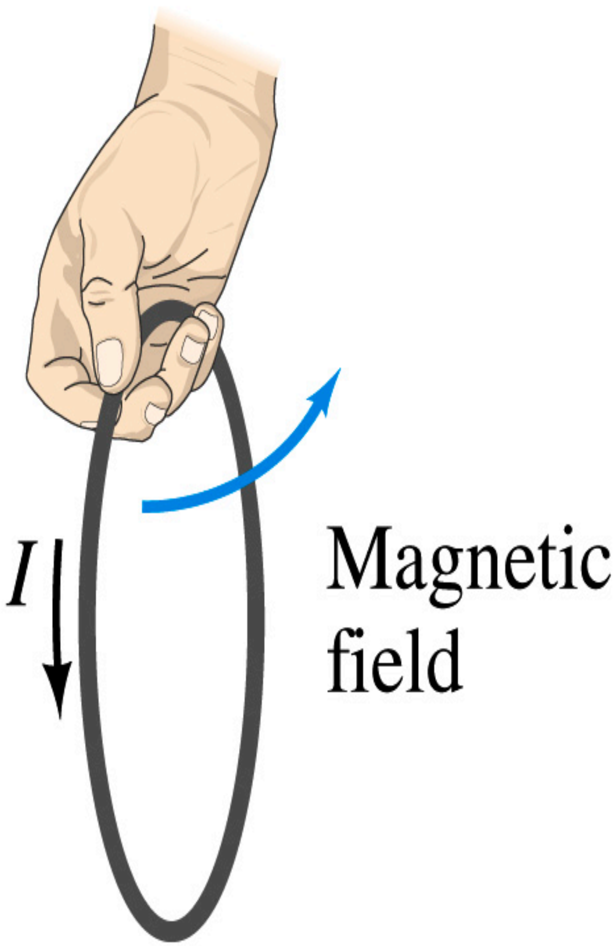
Electric Current and Magnetism

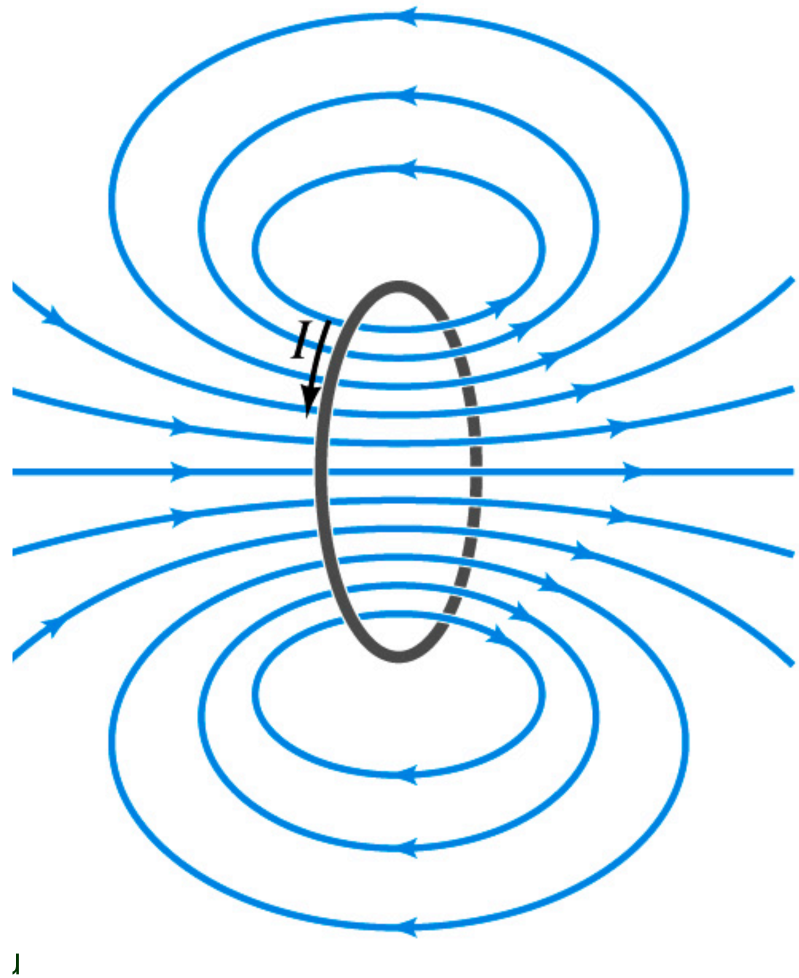
- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
  - Electric current produces a magnetic field (poll 1)
    - The first indication that electricity and magnetism are of the same origin
  - What about a stationary electric charge and magnet?
    - They don't affect each other.
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the "right-hand" rule
  - The field lines follow right-hand fingers wrapped around the wire when the thumb points to the direction of the electric current



Directions in a Circular Wire?

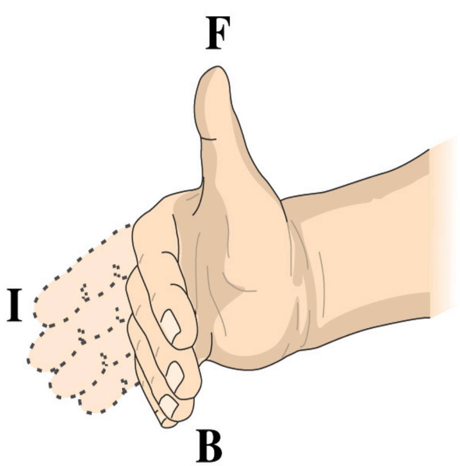
- OK, then what is the direction of the magnetic field generated by the current flowing through a circular loop?



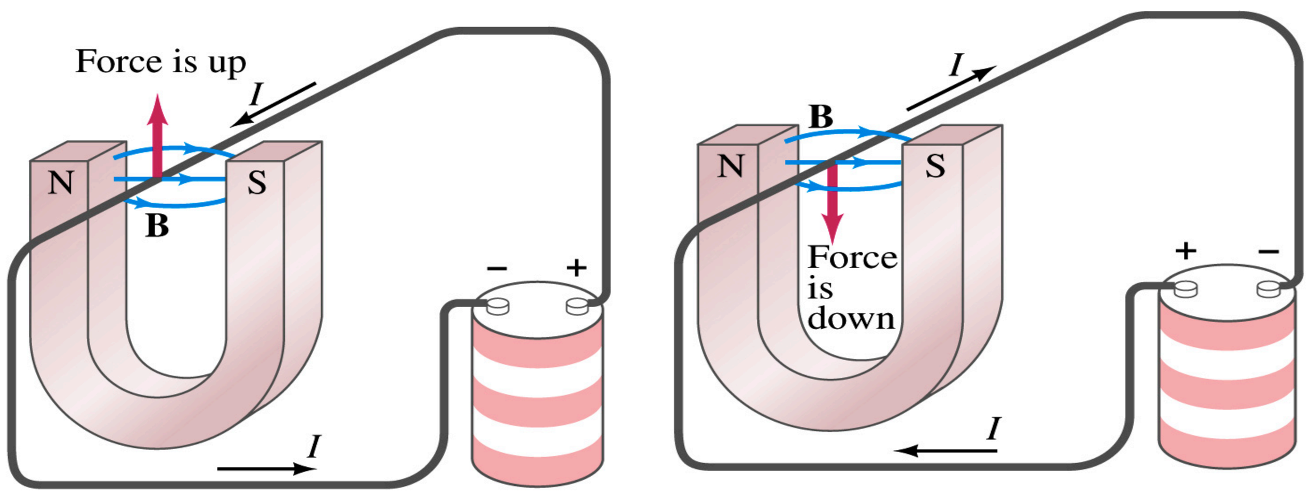


Magnetic Forces on Electric Current

- Since the electric current exerts force on a magnet, the magnet should also exert force on the electric current
  - Which law justifies this? (poll 13)
    - Newton's 3rd law
  - This was also discovered by Oersted
- Direction of the force is always
  - perpendicular to the direction of the current
  - perpendicular to the direction of the magnetic field, **B**
- Experimentally the direction of the force is given by another right-hand rule ➔ When the fingers of the right-hand points to the direction of the current and the finger tips bent to the direction of magnetic field B, the direction of thumb points to the direction of the force (poll 15)



Right-hand rule



Magnetic Forces on Electric Current

- OK, we are set for the direction but what about the magnitude?
- It is found that the magnitude of the force is directly proportional (poll 16)
  - To the current in the wire
  - To the length of the wire in the magnetic field (if the field is uniform)
  - To the strength of the magnetic field
- The force also depends on the angle θ between the directions of the current and the magnetic field
  - When the wire is perpendicular to the field, the force is the strongest
  - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in the wire w/ length l in a uniform field $\mathcal{B}$ is

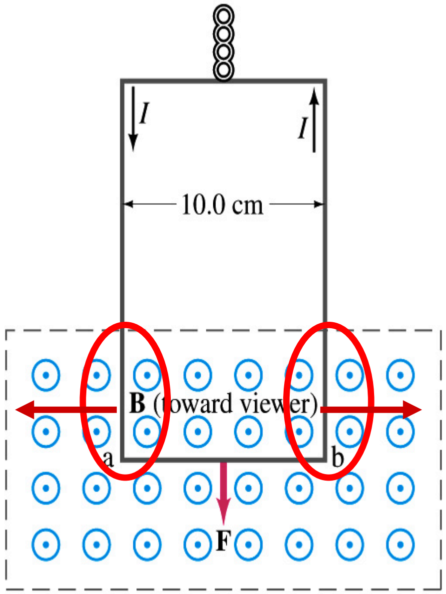
$$F \propto IlB\sin\theta$$

Magnetic Forces on Electric Current

- Magnetic field strength B can be defined using the proportionality relationship w/ the constant 1. $F = IlB\sin\theta$ (poll 17)
- if θ=90°, $F_{max} = IlB$ and if θ=0° $F_{min} = 0$
- So the magnitude of the magnetic field B can be defined as
  - $B = F_{max}/Il$ where F_{max} is the magnitude of the force on a straight length l of the wire carrying the current I when the wire is perpendicular to **B**
- The relationship between F, B and I can be written in a vector formula: $\vec{F} = I\vec{l} \times \vec{B}$
  - l is the vector whose magnitude is the length of the wire in B and its direction is along the wire in the direction of the **conventional current**
  - This formula works only if **B** is uniform.
- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length dl is $d\vec{F} = Id\vec{l} \times \vec{B}$

Example 27 – 2

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field **B** is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field **B** is very nearly uniform along the horizontal portion of wire ab (length l=10.0cm) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of F=3.48×10^{-2}N when the wire carries a current I=0.245A. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is $\vec{F} = I\vec{l} \times \vec{B}$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

SI unit of B field is Tesla or T

Solving for B $B = \frac{F}{Il} = \frac{3.48\times10^{-2}\,N}{0.245A\cdot0.10m} = 1.42T$

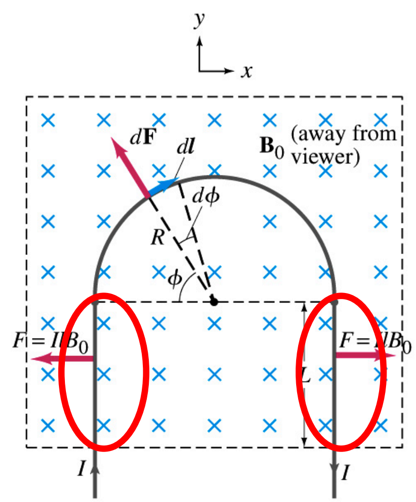
1 T = 10^4 gauss

Something is not right! What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.

Example 27 – 3

Magnetic force on a semi-circular wire. A rigid wire, carrying the current I, consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field $\mathbf{B_0}$. The straight portions each has length ℓ within the field. Determine the net force on the wire due to the magnetic field $\mathbf{B_0}$.



As in the previous example, the forces on the straight sections of the wire is equal and in opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle? $d\vec{F} = Id\vec{l} \times \vec{B}$

We divide the semicircle into infinitesimal straight sections. $dl = Rd\phi$

What is the net x component of the force exerting on the circular section? 0 **Why?**

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = d\left(F\sin\phi\right) = IRB_0 d\phi$

Integrating over φ=0 - π

$$F = \int_0^\pi d\left(F\sin\phi\right) = IB_0 R\int_0^\pi \sin\phi d\phi = -IB_0 R\left[\cos\phi\right]_0^\pi = 2RIB_0$$

Which direction?

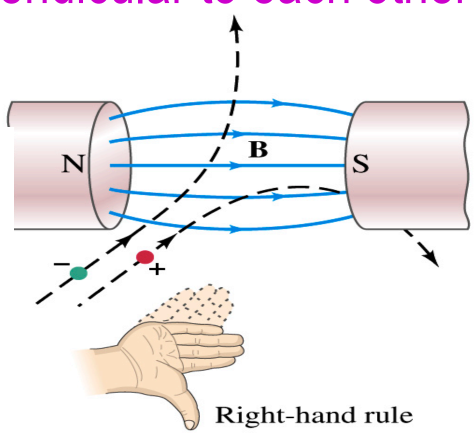
Vertically upward direction. The wire will be pulled deeper into the field.

Magnetic Forces on a Moving Charge

- Will moving charge in a magnetic field experience force?
  - Yes
  - Why?
  - Since a wire carrying electric current (moving charge) experiences force in a magnetic field, a freely moving charge must feel the same kind of force…☺
- OK, then how much force would it experience?
  - Let's consider N moving particles with charge q each, and they pass by a given point in a time interval t.
    - What is the current? $I = Nq/t$
  - Let t be the time for a charge q to travel a distance ℓ in the magnetic field **B**
    - Then, the length vector ℓ becomes $\vec{l} = \vec{v}t$
    - Where **v** is the velocity of the particle
- Thus the force on N particles by the field is $\vec{F} = I\vec{l} \times \vec{B} = Nq\vec{v} \times \vec{B}$
- The force on one particle with charge q, $\vec{F} = q\vec{v} \times \vec{B}$

Magnetic Forces on a Moving Charge

- This can be an alternative way of defining the magnetic field.
  - How?
  - The magnitude of the force on a particle with charge q moving with a velocity v in a field B is
    - $F = qvB\sin\theta$
    - What is the angle θ?
      - The angle between the magnetic field and the direction of particle's movement
    - When is the force maximum?
      - When the B field and the velocity vector are perpendicular to each other.
    - $F_{max} = qvB$ ➔ $B = \frac{F_{max}}{qv}$
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field (p 15)



Example 27 – 5

Magnetic force on a proton. A proton having a speed of $5x10^6$m/s in a magnetic field feels a force of F=$8.0x10^{-14}$N toward West when it moves vertically upward. When moving horizontally in a northerly direction, it feels zero force. What is the magnitude and the direction of the magnetic field in this region?

What is the charge of a proton? $q_p = +e = 1.6 \times 10^{-19} C$

What does the fact that the proton does not feel any force in a northerly direction tell you about the magnetic field?

The field is along the north-south direction. Why?

Because the particle does not feel any magnetic force when it is moving along the direction of the field.

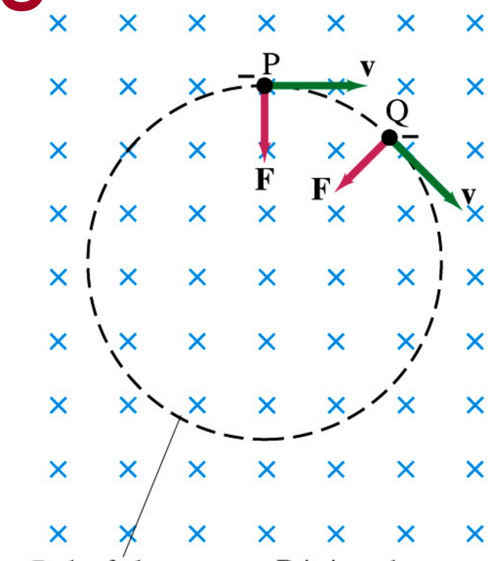
Since the particle feels force toward West, the field should be pointing to? North

Using the formula for the magnitude of the field B, we obtain

$$B = \frac{F}{qv} = \frac{8.0 \times 10^{-14} N}{1.6 \times 10^{-19} C \cdot 5.0 \times 10^{6} m/s} = 0.10T$$

We can use magnetic field to measure the momentum of a particle. How?

Charged Particle's Path in Magnetic Field



- What shape do you think is the path of a charged particle on a plane perpendicular to a uniform magnetic field?
  - Circle!! Why?
  - An electron moving to right at the point P in the figure will be pulled downward
  - At a later time, the force is still perpendicular to the velocity
  - Since the force is always perpendicular to the velocity, the magnitude of the velocity (the speed) is constant
  - The direction of the force follows the right-hand-rule and is perpendicular to the direction of the magnetic field
  - Thus, the electron moves on a circular path with a centripetal force F.

Example 27 – 7

Electron's path in a uniform magnetic field. An electron travels at the speed of 2.0×10^7m/s in a plane perpendicular to a 0.010-T magnetic field. What is the radius of the electron's path?

What is formula for the centripetal force? $F = ma = m\frac{v^2}{r}$

Since the magnetic field is perpendicular to the motion of the electron, the magnitude of the magnetic force is $F = evB$

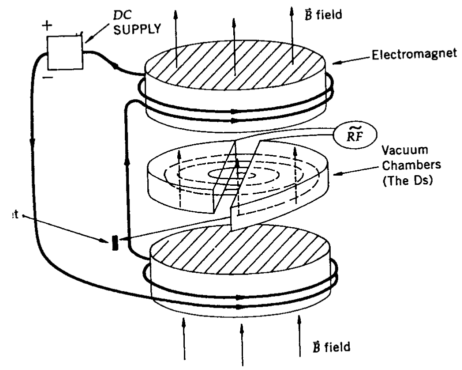
Since the magnetic force provides the centripetal force, we can establish an equation with the two forces $F = evB = m\frac{v^2}{r}$

Solving for r

$$r = \frac{mv}{eB} = \frac{\left(9.1\times10^{-31}\,kg\right)\cdot\left(2.0\times10^{7}\,m/s\right)}{\left(1.6\times10^{-19}\,C\right)\cdot\left(0.010T\right)} = 1.1\times10^{-2}\,m$$

Cyclotron Frequency

- The time required for a particle of charge q moving w/ a constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.