PHYS 1441 – Section 001
Lecture #19

Wednesday, July 8, 2020

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CH 29:EM Induction & Faraday's Law
  – Electric Field due to Magnetic Flux
- Chapter 30: Inductance
  – Mutual and Self Inductance
  – Energy Stored in Magnetic Field
- Chapter 31: Maxwell's Equations
  – Expansion of Ampere's Law
  – Gauss' Law for Magnetism
  – Production of EM Waves

Announcements

- Reading Assignments: CH9.8, CH30.7 – 30.11 and CH31.4
- Course feedback survey ➔ Deadline today!
- Online Final Exam in class tomorrow, Thursday, July 9 (roll call at 10:20am)
  - Covers CH21.1 – CH31.4 plus the math refresher
  - BYOF You may bring a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
  - No derivations, word definitions, figures, pictures, arrows, or setups or solutions of any problems!
  - No additional formulae or values of constants will be provided!
  - Must send me the photos of front and back of the formula sheet, including the blank, no later than 10am tomorrow!
    - Once submitted, you cannot change, unless I ask you to delete some part of the sheet!
- Quiz 4 results
  - Class average: 34.2/60 equivalent to 57/100
    - Previous results: 61/100, 54/100 and 60/100
  - Top score: 60/60

Power Transmission – Why HV?

Transmission lines. An average of 120kW of electric power is sent to a small town from a power plant 10km away. The transmission lines have a total resistance of 0.4Ω. Calculate the power loss if the power is transmitted at (a) 240V and (b) 24,000V.

We cannot use $P=V^2/R$ since we do not know the voltage along the transmission line. We, however, can use $P=I^2R$.

(a) If 120kW is sent at 240V, the total current is $I=\frac{P}{V}=\frac{120\times10^3}{240}=500A.$

Thus the power loss due to transmission line is

$$P=I^2R=(500A)^2\cdot(0.4\Omega)=100kW$$

(b) If 120kW is sent at 24,000V, the total current is $I=\frac{P}{V}=\frac{120\times10^3}{24\times10^3}=5.0A.$

Thus the power loss due to transmission line is

$$P=I^2R=(5A)^2\cdot(0.4\Omega)=10W$$

The higher the transmission voltage, the smaller the current, causing less loss of energy. This is why power is transmitted w/ HV, as high as 170kV.

Electric Field due to Magnetic Flux Change

- When the electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- Recall the relationship between the electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- Induced emf in a circuit is equal to the work done per unit charge by the electric field
- $\varepsilon = \int_a^b \vec{E} \cdot d\vec{l}$
- So we obtain

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

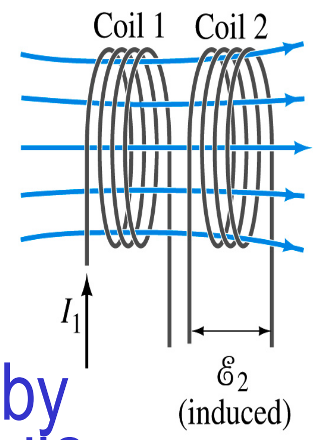
Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit (Poll 13)
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - and induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21}=M_{21}I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined as $\boxed{M_{21}=N_2\Phi_{21}/I_1}$.
- The emf induced in coil2 due to the changing current in coil1 is

$$\boxed{\varepsilon_2=-N_2\frac{d\Phi_{21}}{dt}=-\frac{d\left(N_2\Phi_{21}\right)}{dt}=-M_{21}\frac{dI_1}{dt}}$$



Mutual Inductance

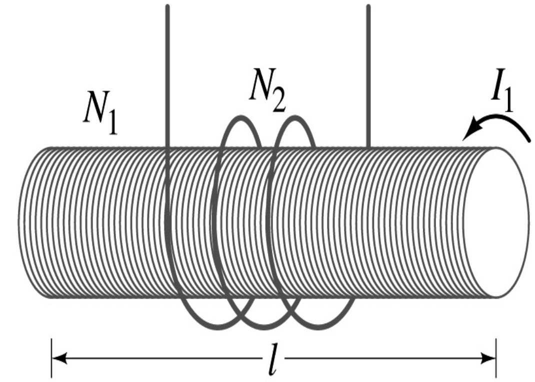
- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - In most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $\varepsilon_1 = -M\frac{dI_2}{dt}$ and $\varepsilon_2 = -M\frac{dI_1}{dt}$
  - SI unit for mutual inductance is henry (H) $1H = 1V \cdot s/A = 1\Omega \cdot s$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assuming all the flux from coil 1 (the solenoid) passes through coil 2, calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \dfrac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is $M_{21} = \dfrac{N_2 \Phi_{21}}{I_1} = \dfrac{N_2}{I_1} \dfrac{\mu_0 N_1 I_1}{l} A = \dfrac{\mu_0 N_1 N_2}{l} A$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this? (Poll 13, 14)
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through an N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
- We define self-inductance, $\mathcal{L}$: $\boxed{L = \frac{N\Phi_B}{I}}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V\cdot s/A = 1\Omega\cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in the opposite direction to cancel the magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls the changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

$$X_L = \varpi L$$

Example 30 – 3

Solenoid inductance. (a) Determine the formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B=\mu_0 nI=\mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L=\frac{N\Phi_B}{I}=\frac{N\cdot\mu_0 NIA/l}{I}=\frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L=\frac{\mu_0 N^2 A}{l}=\frac{\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B=\mu NI/l$

$$L=\frac{\mu N^2 A}{l}=\frac{4000\left(4\pi\times10^{-7}\,T\cdot m/A\right)100^2\left(0.30\times10^{-4}\,m^2\right)}{5.0\times10^{-2}\,m}=0.030H=30mH$$

Energy Stored in the Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\dfrac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LIdI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LIdI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in the Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $\boxed{U = \frac{1}{2}LI^2}$

    Energy Stored in a magnetic field inside an inductor
  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect

    $$L = \mu_0 N^2 A/l$$
  - The magnetic field in a solenoid is $B = \mu_0 NI/l$
  - Thus the energy stored in an inductor is

    $$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$

    What is this?

    $$\boxed{U = \frac{1}{2}\frac{B^2}{\mu_0}Al}$$ E
  - Thus the energy density is

    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$

    Volume V

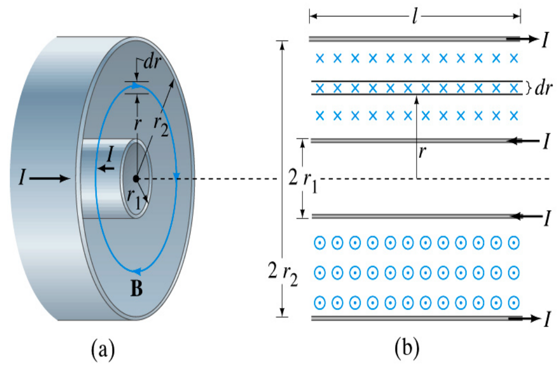
    $$\boxed{u = \frac{1}{2}\frac{B^2}{\mu_0}}$$ E density
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.

Example 30 – 9

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V DC is applied to it; (b) 120 V AC (rms) at 60.0Hz is applied.

Is there a reactance for DC? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for DC power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

For an AC power with f=60Hz, the reactance is

$$X_L = \varpi L = 2\pi fL = 2\pi \cdot \left(60.0s^{-1}\right) \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is

$$I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$$

Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced somewhat by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verifications that the electromagnetic waves can travel through the empty space
  - What do you think this accomplishment did?
    - Open a new world of communication
    - It also yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of human intellect

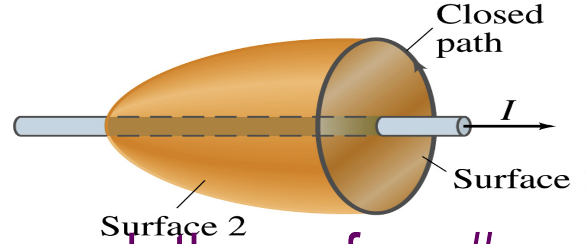
Ampere's Law

- Do you remember the mathematical expression of Oersted discovery of a magnetic field produced by an electric current, given by Ampere?

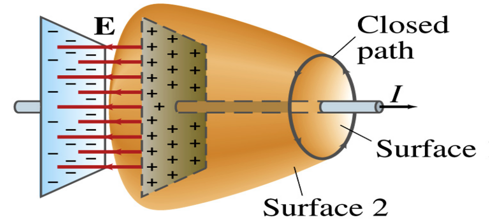
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

- We've learned that a varying magnetic field produces an electric field
- Then can the reverse phenomena, that a changing electric field producing a magnetic field, possible?
  - If this is the case, it would demonstrate a beautiful symmetry in nature between electricity and magnetism

Expanding Ampere's Law



- Let's consider a wire carrying current I
  - The current that is enclosed in the loop passes through the surface # 1 in the figure
  - We could imagine a different surface # 2 that shares the same enclosed path but cuts through the wire in a different location. What is the current that passes through the surface?
    - Still I.
  - So the Ampere's law still works



- We could then consider a capacitor being charged up or being discharged.
  - The current I enclosed in the loop passes through the surface #1
  - However the surface #2 that shares the same closed loop do not have any current passing through it.
    - There is magnetic field present since there is current ➔ In other words there is a changing electric field in between the plates
    - Maxwell resolved this by adding an additional term to Ampere's law involving the changing electric field

Modifying Ampere's Law

- To determine what the extra term should be, we first have to figure out what the electric field between the two plates is
  - The charge Q on the capacitor with capacitance C is Q=CV
    - Where V is the potential difference between the plates
  - Since V=Ed
    - Where E is the uniform field between the plates, and d is the separation of the plates
  - And for parallel plate capacitor $C=\varepsilon_0 A/d$
  - We obtain

$$Q = CV = \left(\varepsilon_0 \frac{A}{d}\right) Ed = \varepsilon_0 AE$$

Modifying Ampere's Law

– If the charge on the plate changes with time, we can write

$$\frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

– Using the relationship between the current and charge we obtain

$$I = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE}{dt} = \varepsilon_0 \frac{d\left(AE\right)}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Where Φ_E=EA is the electric flux through the surface between the plates

– So in order to make Ampere's law work for the surface 2 in the figure, we must write it in the following form

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

Extra term by Maxwell

– This equation represents the general form of Ampere's law

- This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Example 31 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area A=100cm^2. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instance the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0\left(1 - e^{-t/RC}\right)$

For the initial current (t=0), we differentiate the charge with respect to time.

$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC}e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega\cdot} = 35A$$

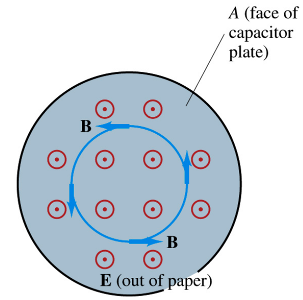
The electric field is $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}$

Change of the electric field is $\frac{dE}{dt} = \frac{dQ/dt}{A\varepsilon_0} = \frac{35A}{\left(8.85\times10^{-12}\,C^2/N\cdot m^2\right)\cdot\left(1.0\times10^{-2}\,m^2\right)} = 4.0\times10^{14}\,V/m\cdot s$

Example 31 – 1

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry



Whose law can we use to determine B?

Extended Ampere's Law w/ I_{encl}=0!

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0\varepsilon_0 \pi r^2 \frac{dE}{dt}$$

Solving for B

$$\boxed{B = \mu_0\varepsilon_0 \frac{r}{2}\frac{dE}{dt}}$$

For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface.

$$\Phi_E = EA = E\pi r_{plate}^2$$

So from Ampere's law, we obtain $$B\cdot(2\pi r) = \mu_0\varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0\varepsilon_0 \pi r_{plate}^2 \frac{dE}{dt}$$

Solving for B

$$\boxed{B = \frac{\mu_0\varepsilon_0 r_{plate}^2}{2r}\frac{dE}{dt}}$$

For $r>r_{plate}$

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent to an electric current
  - He called this term as the displacement current, I_D
  - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{encl} + I_D \right)}$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is a symmetry between electricity and magnetism, there must be an equivalent law in magnetism as the Gauss' Law in electricity
- For a magnetic field B, the magnetic flux Φ_B through the surface is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

  – Where the integration is over the area of either an open or a closed surface
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was the Gauss' law in the electric case?
  – The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

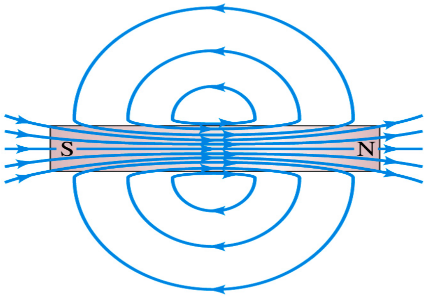
- Why is result of the integral zero?
  – There is no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does the Gauss' law in magnetism mean physically?

$$\boxed{\oint \vec{B} \cdot d\vec{A} = 0}$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopole does not exist, there is no starting or stopping point of the flux lines
    - Electricity do have the source and the sink
  - Magnetic field lines must be continuous
  - Even for bar magnets, the field lines exist both insides and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

A magnetic field is produced by an electric current or by a changing electric field

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in an empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces the magnetic field that also changes.
    - This changing magnetic field then in turn produces the electric field that changes.
    - This process continues.
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through the space