PHYS 1444 – Section 003
Lecture #12

Tuesday October 9, 2012

Dr. Andrew Brandt

- Chapter 25
  - Power
  - Alternating Current
  - Microscopic Current
- Chapter 26
  - EMF and Terminal Voltage
  - Resistors in Series and Parallel
  - Energy loss in Resistors

Electric Power

- How do we find out the power of an electric device?
  - What is definition of the power?
    - The rate at which work is done or the energy is transferred
- What energy is transferred when an infinitesimal charge dq moves through a potential difference V?
  - dU=Vdq
  - If dt is the time required for an amount of charge dq to move through the potential difference V, the power P is
  - $P = dU/dt = V\,dq/dt$

What is this?

  - Thus, we obtain $\boxed{P = IV}$.

In terms of resistance $\boxed{P = I^2R = \frac{V^2}{R}}$

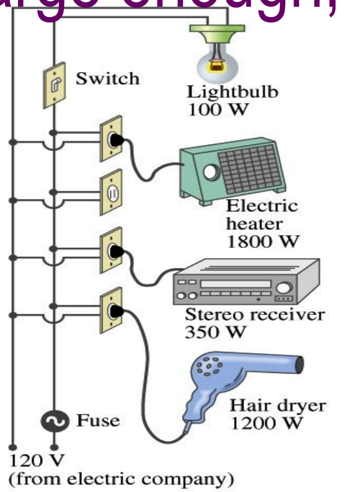
  - What is the unit?

Watts = J/s

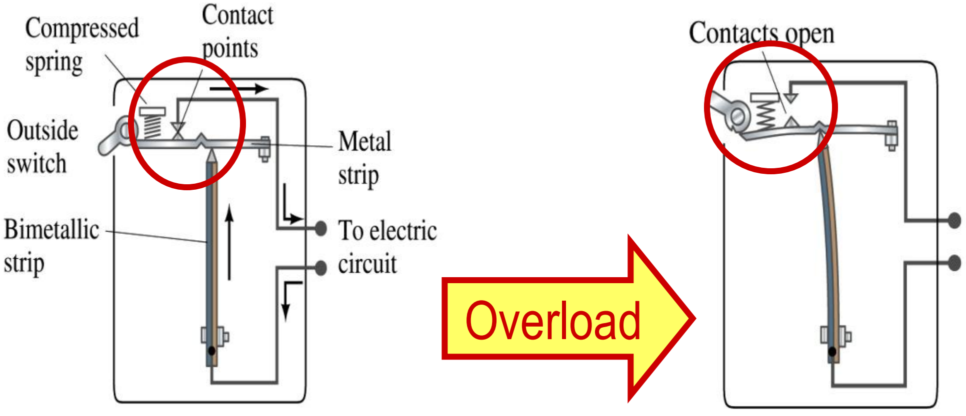
  - What kind of quantity is the electrical power?
    - Scalar
  - P=IV can apply to any device, while the formulae involving resistance only applies to Ohmic resistors.

Power in Household Circuits

- Household devices usually have small resistance
  - But since they draw current, if they become large enough, wires can heat up (overload) and cause a fire
    - Why is using thicker wires safer?
      - Thicker wires has less resistance, lower heat



- How do we prevent this?
  - Put in a switch that disconnects the circuit when overloaded
    - Fuse or circuit breakers
    - They open up the circuit when the current exceeds a certain value



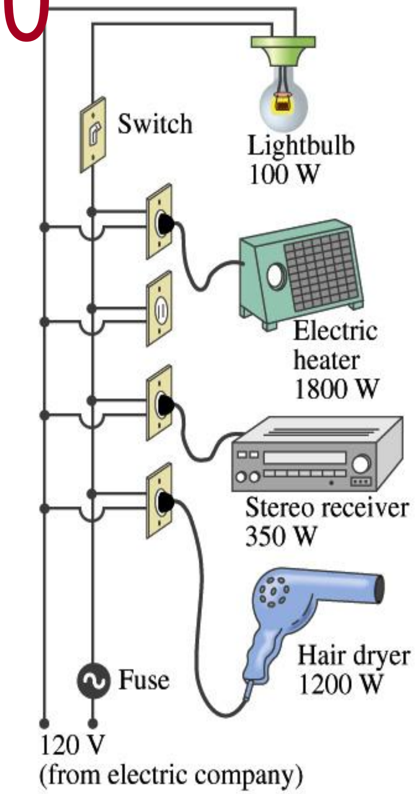
(b) Circuit breaker

(c) Circuit breaker

Example 25 – 10

Will a 30A fuse blow?

Determine the total current drawn by all the devices in the circuit in the figure.



The total current is the sum of current drawn by the individual devices.

$$P = IV$$

Solve for I

$$I = P/V$$

Bulb $I_B = 100W/120V = 0.8A$

Heater $I_H = 1800W/120V = 15.0A$

Stereo $I_S = 135W/120V = 2.9A$

Dryer $I_D = 1200W/120V = 10.0A$

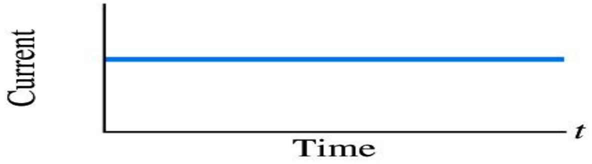
Total current

$$I_T = I_B + I_H + I_S + I_D = 0.8A + 15.0A + 2.9A + 10.0A = 28.7A$$

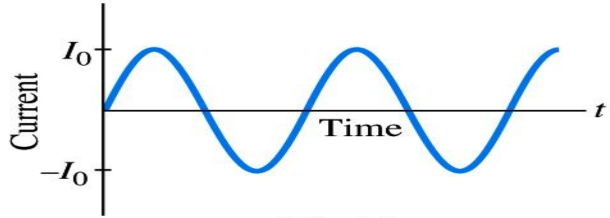
What is the total power? $P_T = P_B + P_H + P_S + P_D = 100W + 1800W + 350W + 1200W = 3450W$

Alternating Current

- Does the direction of the flow of current change when a battery is connected to a circuit?
  - No. Why?
    - Because its source of potential difference is constant.
  - This kind of current is called the Direct Current (DC
    - How would DC look as a function of time?
      - A horizontal line
- Electric generators at electric power plant produce alternating current (AC)
  - AC reverses direction many times a second
  - AC is sinusoidal as a function of time
- Most currents supplied to homes and business are AC.



(a) DC



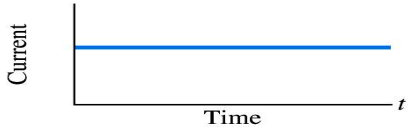
(b) AC

Alternating Current

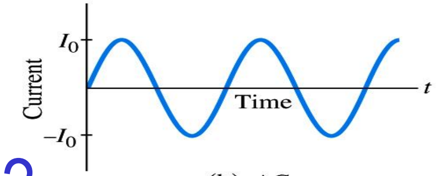
- The voltage produced by an AC electric generator is sinusoidal
  - This is why the current is sinusoidal
- Voltage produced can be written as

$$V = V_0 \sin 2\pi ft = V_0 \sin \omega t$$

- What are the maximum and minimum voltages?
  - ➢ V_0 and $-V_0$
  - ➢ The potential oscillates between $+V_0$ and $-V_0$, the peak voltages or amplitude
  - ➢ What is f?
    - The frequency, the number of complete oscillations made per second. What is the unit of f? What is the normal size of f in the US?
      - f = 60 Hz in the US and Canada.
      - Many European countries have f = 50Hz.
  - ➢ $\omega = 2\pi f$



(a) DC



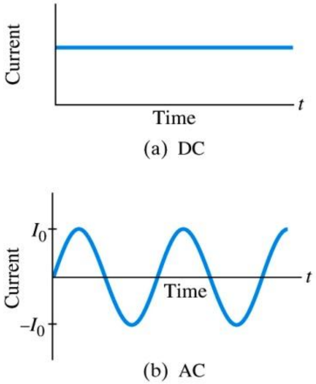
(b) AC

Alternating Current

- Since V=IR, if a voltage V exists across a resistance R, the current I is

What is this?

$$I = \frac{V}{R} = \frac{V_0}{R}\sin 2\pi ft = I_0 \sin \omega t$$



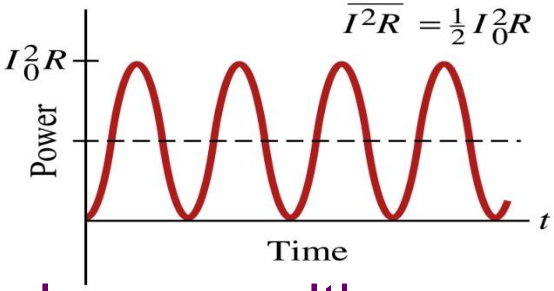
(a) DC

(b) AC

- What are the maximum and minimum currents?
  - I_0 and $-I_0$
  - The current oscillates between $+I_0$ and $-I_0$, the peak currents or amplitude. The current is positive when electron flows in one direction and negative when they flow in the opposite direction.
  - What is the average current?
    - Zero. So there is no power and no heat produced in a heater?
      - Wrong! The electrons actually flow back and forth, so power is delivered.

Power Delivered by Alternating Current

- AC power delivered to a resistance is:



$$\boxed{P = I^2R = I_0^2R\sin^2\omega t}$$

  - Since the current is squared, the power is always positive
- The average power delivered is $\boxed{\bar{P} = \frac{1}{2} I_0^2 R}$
- Since the power is also P=V^2/R, we can obtain

$$\boxed{P = V_0^2/R \ \sin^2\omega t}$$

Average power

$$\boxed{\bar{P} = \frac{1}{2}\left(\frac{V_0^2}{R}\right)}$$

- The average of the square of current and voltage are important in calculating power:

$$\boxed{\overline{I^2} = \frac{1}{2} I_0^2} \qquad \boxed{\overline{V^2} = \frac{1}{2} V_0^2}$$

Power Delivered by Alternating Current

- The square root of each of these are called root-mean-square, or rms:

$$\boxed{I_{rms} = \sqrt{\overline{I^2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0} \qquad \boxed{V_{rms} = \sqrt{\overline{V^2}} = \frac{V_0}{\sqrt{2}} = 0.707 V_0}$$

- rms values are sometimes called effective values
  - These are useful quantities since they can substitute current and voltage directly in power equations, as if they were DC values

$$\boxed{\overline{P} = \frac{1}{2} I_0^2 R = I_{rms}^2 R} \qquad \boxed{\overline{P} = \frac{1}{2}\frac{V_0^2}{R} = \frac{V_{rms}^2}{R}} \qquad \boxed{\overline{P} = I_{rms} V_{rms}}$$

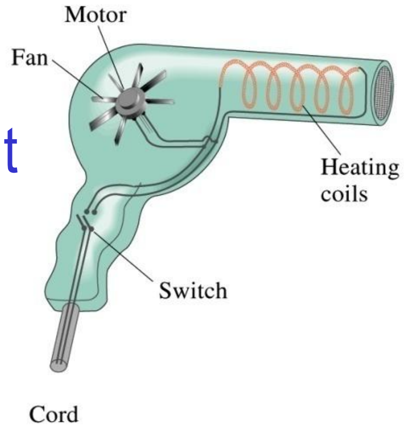
  - In other words, an AC of peak voltage V_0 or peak current I_0 produces as much power as DC voltage of V_{rms} or DC current I_{rms}.
  - So normally, rms values in AC are specified or measured.
    - US uses 115V rms voltage. What is the peak voltage?

$$V_0 = \sqrt{2} V_{rms} = \sqrt{2} \cdot 115V = 162.6V$$

    - Europe uses 240V

$$V_0 = \sqrt{2} V_{rms} = \sqrt{2} \cdot 240V = 340V$$

Example 25 – 11



Hair Dryer. (a) Calculate the resistance and the peak current in a 1000-W hair dryer connected to a 120-V AC line. (b) What happens if it is connected to a 240-V line in Britain?

The rms current is: $I_{rms} = \frac{\overline{P}}{V_{rms}} = \frac{1000W}{120V} = 8.33A$

The peak current is: $I_0 = \sqrt{2} I_{rms} = \sqrt{2} \cdot 8.33A = 11.8A$

Thus the resistance is: $R = \frac{\overline{P}}{I_{rms}^2} = \frac{1000W}{8.33A^{\;2}} = 14.4\Omega$

(b) If connected to 240V in Britain …

The average power provide by the AC in UK is

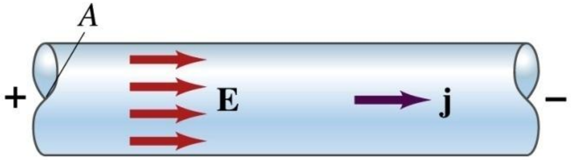
$$\overline{P} = \frac{V_{rms}^2}{R} = \frac{240V^{\;2}}{14.4\Omega} = 4000W$$

So? The heating coils in the dryer will melt!

Microscopic View of Electric Current

- When a potential difference is applied to the two ends of a wire of uniform cross-section, the direction of electric field is parallel to the walls of the wire

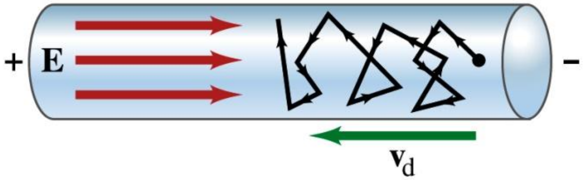
- Let's define a microscopic vector quantity, the current density, **j**, the electric current per unit cross-sectional area
  - j=I/A or I = jA if the current density is uniform
  - If not uniform $I = \int \vec{j} \cdot d\vec{A}$
  - The direction of **j** is the direction the positive charge would move when placed at that position, generally the same as E



- The current density exists at any point in space while the current I refers to a conductor as a whole

Microscopic View of Electric Current

- The direction of j is the direction of a positive charge. So in a conductor, since negatively charged electrons move, their direction is –**j**.
- Let's think about the current in a microscopic view again. When voltage is applied to the end of a wire:
  - Electric field is generated by the potential difference
  - Electrons feel force and get accelerated
  - Electrons soon reach a steady average speed (drift velocity, $\mathbf{v}_d$) due to collisions with atoms in the wire
  - The drift velocity is normally much smaller than electrons' average random speed.



Microscopic View of Electric Current

- How do we relate v_d to the macroscopic current I?
  - In a time interval Δt, the electrons travel $l = v_d \Delta t$ on average
  - If the wire's x-sectional area is A, in time Δt the electrons in a volume $V = lA = Av_d\Delta t$ will pass through the area A
  - If there are n free electrons (of charge –e) per unit volume, the total charge ΔQ that pass through A in time Δt is
  - $\Delta Q = \text{total number of particles, N} \times \text{charge per particle} = nV \quad -e \quad = - \quad nAv_d \Delta te$
  - The current I in the wire is $I = \dfrac{\Delta Q}{\Delta t} = -neAv_d$
  - The density in vector form is $\vec{j} = \dfrac{I}{A} = -ne\vec{v}_d$
  - For any type of charge:

$$\boxed{I = \sum_i n_i q_i v_{di} A} \qquad \boxed{\vec{j} = \sum_i n_i q_i \vec{v}_{di}}$$

Microscopic View of Electric Current

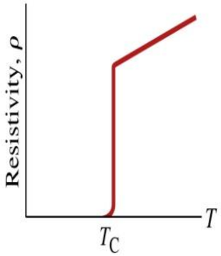
- The drift velocity of electrons in a wire is only about 0.05 mm/s. How does a light turned on immediately then?
  - While the electrons in a wire travel slowly, the electric field travels essentially at the speed of light. Then what is all the talk about electrons flowing through?
    - It is just like water. When you turn on a faucet, water flows right out of the faucet despite the fact that the water travels slowly.
    - Electricity is the same. Electrons fill the wire and when the switch is flipped on or a potential difference is applied, the electrons close to the positive terminal flow into the bulb.

Ohm’s Law in Microscopic View

- Ohm’s law can be written in microscopic quantities.
  - Resistance in terms of resistivity is $R = \rho \frac{l}{A}$
  - We can rewrite V and I as: I=jA, V=El.
  - For a uniform electric field in an ohmic conductor:
  - $V = I\ R$
  - $El = \ jA \left(\rho \frac{l}{A} \right) = \ j\rho l$
  - So
  - $\boxed{j = \frac{E}{\rho} = \sigma E}$
  - Since ρ or σ are properties of material independent of j or electric field E, the current density j is proportional E ➔ Microscopic statement of Ohm’s Law
  - In vector form, the density can be written as $\boxed{\vec{j} = \frac{\vec{E}}{\rho} = \sigma \vec{E}}$

Superconductivity

- At temperatures near absolute 0K, the resistivity of certain materials approaches 0.
  - This state is called the “superconducting” state.
  - Observed in 1911 by H. K. Onnes when he cooled mercury to 4.2K (-269^{o}C).
    - Resistance of mercury suddenly dropped to 0.
  - In general superconducting materials become superconducting below a transition temperature.
  - The highest temperature superconductor so far is 160K
    - First observation above the boiling temperature of liquid nitrogen is in 1987 at 90K observed from a compound of yttrium, barium, copper and oxygen.



- Since a much smaller amount of material can carry just as much current more efficiently, superconductivity can make electric cars more practical, computers faster, and capacitors store higher energy (not to mention LHC magnets)

Electric Hazards: Leakage Currents

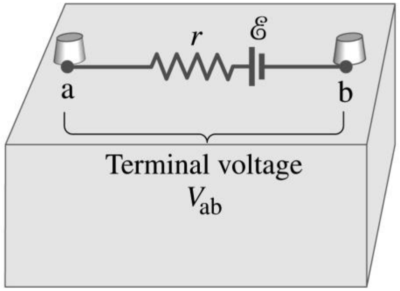
- How does one feel an electric shock?
  - Electric current stimulates nerves and muscles, and we feel a shock
  - The severity of the shock depends on the amount of current, how long it acts and through what part of the body it passes
  - Electric current heats tissues and can cause burns
- Currents above 70mA on a torso for a second or more is fatal, causing heart to function irregularly, “ventricular fibrillation”
- Dry skin has a resistance of 10^4 to 10^6 Ω.
- When wet, it could be $10^3\Omega$.
- A person in good contact with the ground who touches 120V DC line with wet hands can receive a fatal current

$$I=\frac{V}{R}=\frac{120V}{1000\Omega}=120mA$$

EMF and Terminal Voltage

- What do we need to have current in an electric circuit?
  - A device that provides a potential difference, such as battery or generator
    - typically it converts some type of energy into electric energy
    - These devices are called sources of electromotive force (*emf*)
      - This does NOT refer to a real "force".
- The potential difference between terminals of the source, when no current flows to an external circuit, is called the *emf* (ε) of the source.
- A battery itself has some **internal resistance** (r) due to the flow of charges in the electrolyte
  - Why do headlights dim when you start the car?
    - The starter needs a large amount of current but the battery cannot provide charge fast enough to supply current to both the starter and the headlights

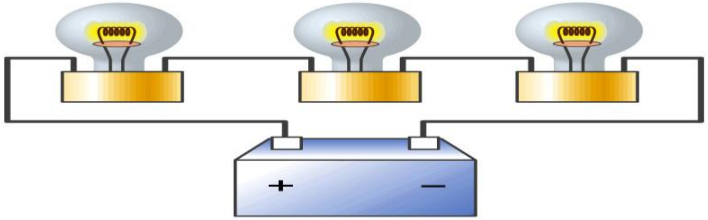
EMF and Terminal Voltage

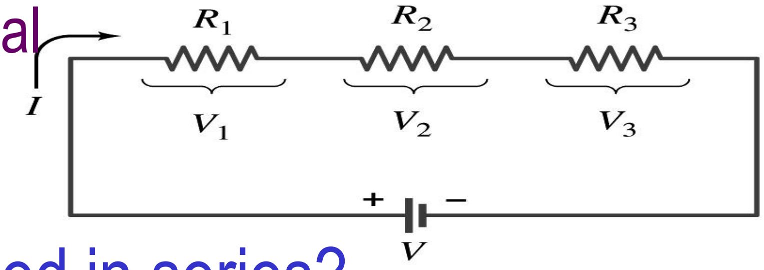


- Since the internal resistance is inside the battery, we cannot separate the two.
- So the terminal voltage difference is $V_{ab}=V_a-V_b$.
- When no current is drawn from the battery, the terminal voltage equals the *emf* which is determined by the chemical reaction; $V_{ab}=\varepsilon$.
- However when the current I flows from the battery, there is an internal drop in voltage which is equal to Ir. Thus the actual **delivered** terminal voltage is

$$\boxed{V_{ab}=\varepsilon-Ir}$$

Resistors in Series





- Resistors are in series when two or more of them are connected end to end
  - These resistors represent simple electrical devices in a circuit, such as light bulbs, heaters, dryers, etc.
- What is common in a circuit connected in series?
  - the current is the same through all the elements in series
- Potential difference across each element in the circuit is:

  $V_1=IR_1$, $V_2=IR_2$ and $V_3=IR_3$
- Since the total potential difference is V, we obtain

  $V=IR_{eq}=V_1+V_2+V_3=I(R_1+R_2+R_3)$

  Thus, $R_{eq}=R_1+R_2+R_3$

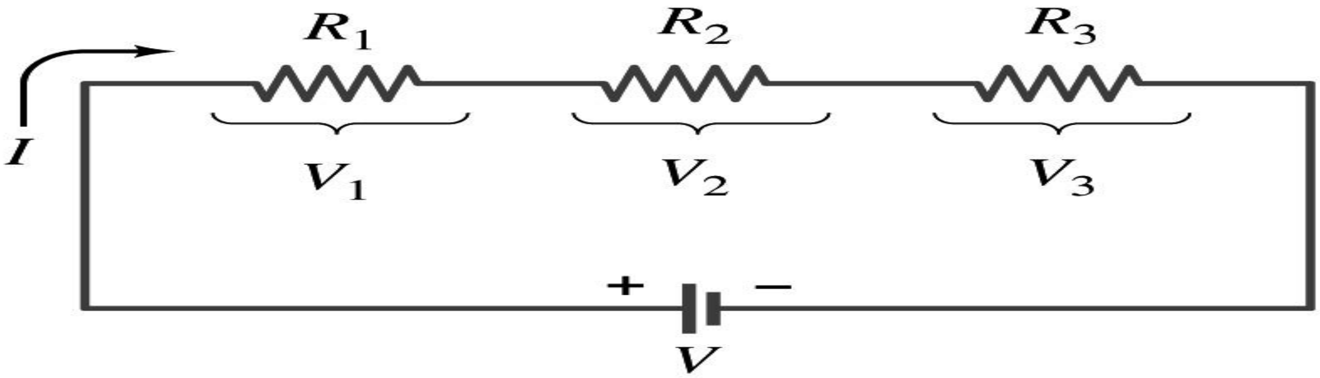
$$\boxed{R_{eq}=\sum_i R_i}$$

Resistors in series

When resistors are connected in series, the total resistance increases and the current through the circuit decreases compared to a single resistor.

Energy Losses in Resistors

- Why is it true that $V=V_1+V_2+V_3$?



- What is the potential energy loss when charge q passes through the resistor R_1, R_2 and R_3

  $\Delta U_1=qV_1$, $\Delta U_2=qV_2$, $\Delta U_3=qV_3$

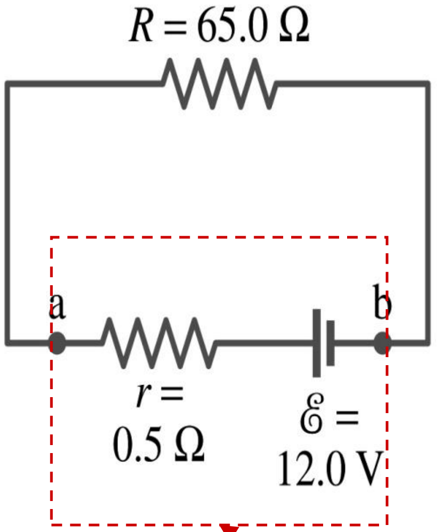
- Since the total energy loss should be the same as the energy provided to the system by the battery , we obtain

  $\Delta U=qV=\Delta U_1+\Delta U_2+\Delta U_3=q(V_1+V_2+V_3)$

  Thus, $V=V_1+V_2+V_3$

Example 26 – 1

Battery with internal resistance. A 65.0-Ω resistor is connected to the terminals of a battery whose emf is 12.0V and whose internal resistance is 0.5-Ω. Calculate (a) the current in the circuit, (b) the terminal voltage of the battery, V_{ab}, and (c) the power dissipated in the resistor R and in the battery's internal resistor.



What is this?

A battery or a source of ***emf.***

(a) Since $V_{ab} = \varepsilon - Ir$ We obtain $V_{ab} = IR = \varepsilon - Ir$

Solve for ***I***

$$I = \frac{\varepsilon}{R + r} = \frac{12.0V}{65.0\Omega + 0.5\Omega} = 0.183A$$

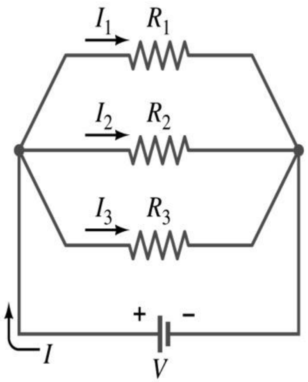
(b) The terminal voltage V_{ab} is $V_{ab} = \varepsilon - Ir = 12.0V - 0.183A \cdot 0.5\Omega = 11.9V$

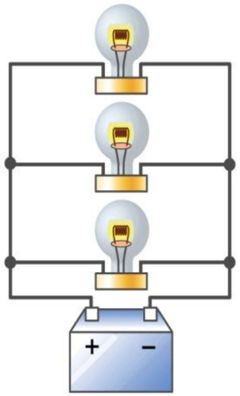
(c) The power dissipated in R and r are

$$P = I^2R = 0.183A\ ^{2} \cdot 65.0\Omega = 2.18W$$

$$P = I^2r = 0.183A\ ^{2} \cdot 0.5\Omega = 0.02W$$

Resistors in Parallel





- Resisters are in parallel when two or more resistors are connected in separate branches
  - Most house and building wirings are arranged this way.
- What is common in a circuit connected in parallel?
  - The voltage is the same across all the resistors.
  - The total current that leaves the battery, is however, split.
- The current that passes through every element is

  $I_1=V/R_1$, $I_2=V/R_2$, $I_3=V/R_3$
- Since the total current is I, we obtain

  $I=V/R_{eq}=I_1+I_2+I_3=V(1/R_1+1/R_2+1/R_3)$

  Thus, $1/R_{eq}=1/R_1+1/R_2+1/R_3$

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Resisters in parallel

When resistors are connected in parallel, the total resistance decreases and the current through the circuit increases compared to a single resistor.

Resistor and Capacitor Arrangements

- Parallel Capacitor arrangements $$\boxed{C_{eq} = \sum_i C_i}$$
- Series Resistor arrangements $$\boxed{R_{eq} = \sum_i R_i}$$
- Series Capacitor arrangements $$\boxed{\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}}$$
- Parallel Resistor arrangements $$\boxed{\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}}$$