



PHYS 1444 – Section 003

Lecture #13

Tuesday October 16, 2012

Dr. Andrew Brandt

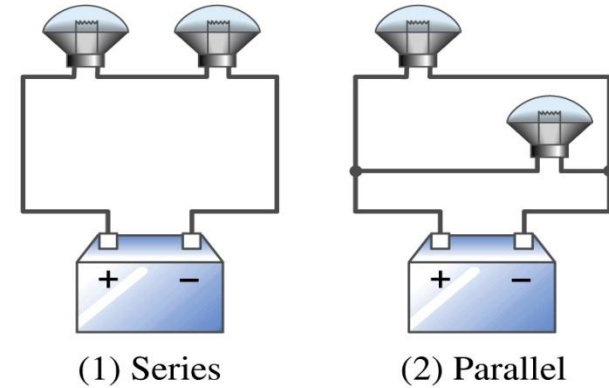
- Chapter 26
- Resistors
- Kirchoff's Rules
- Chapter 27 Magnetism

Grades Posted on website
HW on ch26 due Tues 23



Example 26 – 2

Series or parallel? (a) The light bulbs in the figure are identical and have identical resistance R . Which configuration produces more light? (b) Which way do you think the headlights of a car are wired?



(a) What are the equivalent resistances for the two cases?

Series $\rightarrow R_{eq} = 2R$ Parallel $\rightarrow \frac{1}{R_{eq}} = \frac{2}{R}$ So $\rightarrow R_{eq} = \frac{R}{2}$

The bulbs get brighter when the total power transformed is larger.

series $P_S = IV = \frac{V^2}{R_{eq}} = \frac{V^2}{2R}$ parallel $P_P = IV = \frac{V^2}{R_{eq}} = \frac{2V^2}{R} = 4P_S$

So parallel circuit provides brighter lighting.

(b) Car's headlights are in parallel to provide brighter light and also to prevent both lights going out at the same time when one burns out.

So what is bad about parallel circuits? Dr. A Uses more energy in a given time.



Example 26 – 5

Current in one branch. What is the current flowing through the 500-Ω resistor in the figure?

What do we need to find first? We need to find the total current.

To do that we need to compute the equivalent resistance.

R_{eq} of the small parallel branch is: $\frac{1}{R_P} = \frac{1}{500} + \frac{1}{700} = \frac{12}{3500}$ $R_P = \frac{3500}{12}$

R_{eq} of the circuit is: $R_{eq} = 400 + \frac{3500}{12} = 400 + 292 = 692\Omega$

Thus the total current in the circuit is $I = \frac{V}{R_{eq}} = \frac{12}{692} = 17mA$

The voltage drop across the parallel branch is $V_{bc} = IR_P = 17 \times 10^{-3} \cdot 292 = 4.96 V$

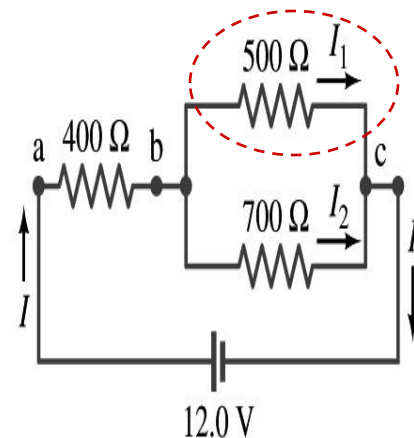
The current flowing across 500-Ω resistor is therefore

$$I_{500} = \frac{V_{bc}}{R} = \frac{4.96}{500} = 9.92 \times 10^{-3} = 9.92 mA$$

What is the current flowing in the 700-Ω resistor?

$$I_{700} = I - I_{500} = 17 - 9.92 = 7.08 mA$$

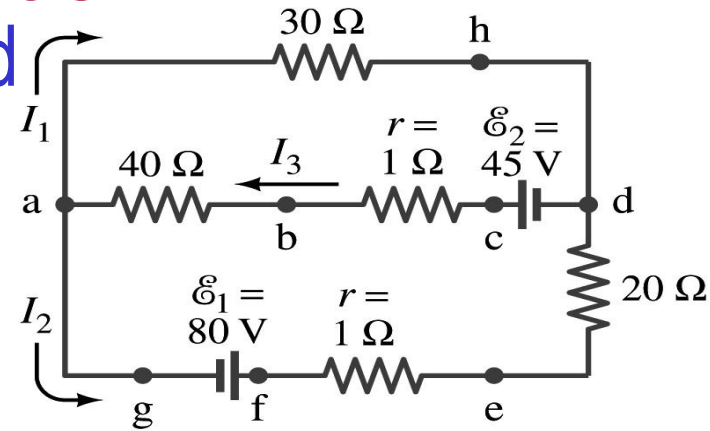
What is the current flowing in the 400-Ω resistor?





Kirchhoff's Rules

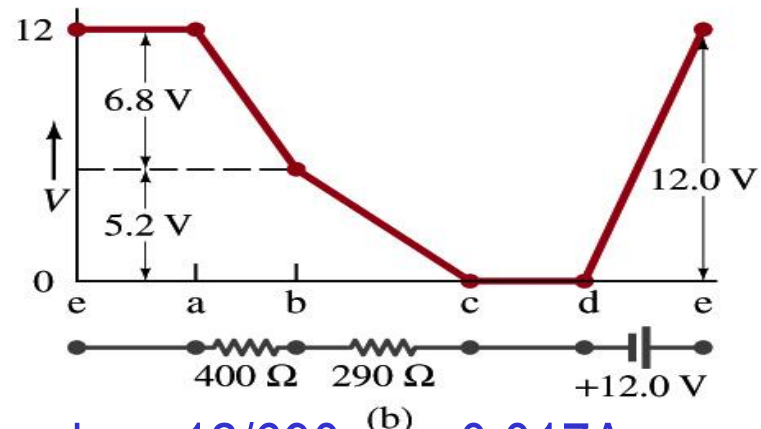
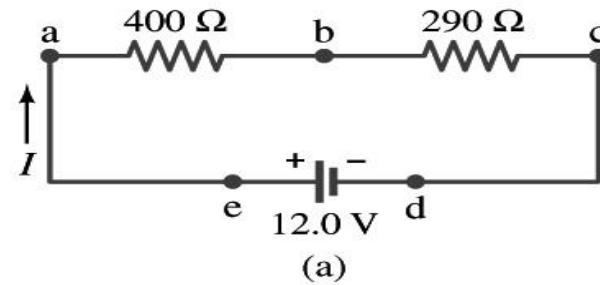
- Some circuits are very complicated to analyze using the simple rules for combining resistors
 - G. R. Kirchhoff devised two rules to deal with complicated circuits.
- Kirchhoff's rules are based on conservation of charge and energy
 - Kirchhoff's 1st rule: Junction rule, charge conservation.
 - At any junction point, the sum of all currents entering the junction must equal to the sum of all currents leaving the junction.
 - In other words, charge is conserved
 - At junction *a* in the figure, I_3 goes into the junction while I_1 and I_2 leaves: $I_3 = I_1 + I_2$





Kirchhoff's 2nd Rule

- Kirchhoff's 2nd rule: Loop rule, uses conservation of energy.
 - The sum of the changes in potential around any closed path of a circuit must be zero.



Kirchhoff's 2nd Rule

- The current in the circuit in the figure is $I = 12/690 = 0.017\text{A}$.
 - Point e is the highest potential point while point d is the lowest potential.
 - When the test charge starts at e and returns to e , the total potential change is 0.
 - Between point e and a , no potential change since there is no source of potential nor any resistance.
 - Between a and b , there is a 400Ω resistance, causing $IR = 0.017 \cdot 400 = 6.8\text{V}$ drop.
 - Between b and c , there is a 290Ω resistance, causing $IR = 0.017 \cdot 290 = 5.2\text{V}$ drop.
 - Since these are voltage drops, we use negative V sign for these, -6.8V and -5.2V .
 - No change between c and d while from d to e there is $+12\text{V}$ change.
 - Thus the total change of the voltage through the loop is: $-6.8\text{V} - 5.2\text{V} + 12\text{V} = 0\text{V}$.



Example 26 – 8

Use Kirchhoff's rules. Calculate the currents I_1 , I_2 and I_3 .

The directions of the current through the circuit is not known *a priori* but since the current tends to move away from the positive terminal of a battery, we arbitrarily choose the direction of the currents as shown.

We have three unknowns so we need three equations.

Using Kirchhoff's junction rule at point a , we obtain

$$I_3 = I_1 + I_2$$

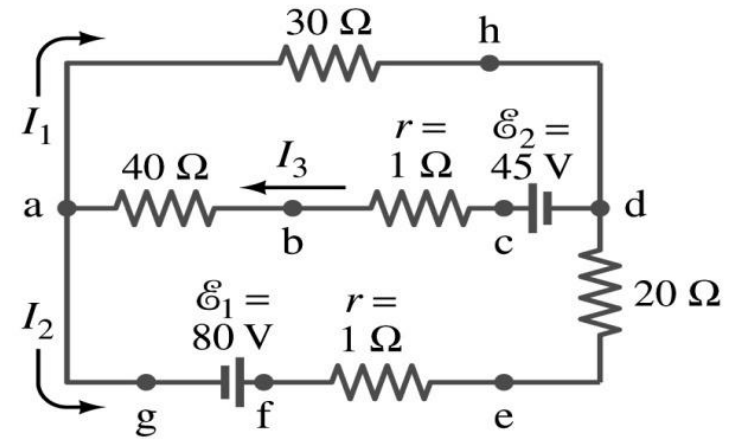
This is the same for junction d as well, so it gives no additional information.

Now apply K's second rule to the top loop **ahdcba**

$$V_{ah} = -I_1 30 \quad V_{hd} = 0 \quad V_{dc} = +45 \quad V_{cb} = -I_3 \quad V_{ba} = -40I_3$$

The total voltage change in loop *ahdcba* is.

$$V_{ahdcba} = -30I_1 + 45 - I_3 - 40I_3 = 45 - 30I_1 - 41I_3 = 0$$





Using Kirchhoff's Rules

1. Determine the flow of currents at the junctions.
 - It does not matter which direction you assign a current.
 - If the value of the current after completing the calculations are negative, it means you chose the wrong direction
2. Write down the current equation based on Kirchhoff's 1st rule at various junctions.
3. Choose closed loops in the circuit
4. Write down the potential in each interval of the junctions, keeping the sign properly.
5. Write down the potential equations for each loop.
6. Solve the equations for unknowns.

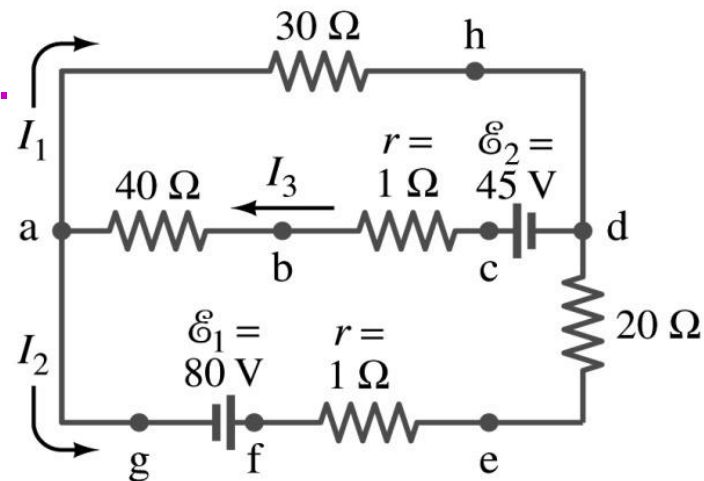


Example 26 – 8, cnt'd

Now the 2nd rule on the bottom loop **agfedcba**.

$$V_{ag} = 0 \quad V_{gf} = +80 \quad V_{fe} = -I_2 \quad V_{ed} = -I_2 20$$

$$V_{dc} = +45 \quad V_{cb} = -I_3 \quad V_{ba} = -40I_3$$



The total voltage change in loop **agfedcba** $V_{agfedcba} = -21I_2 + 125 - 41I_3 = 0$

So the three equations are

$$I_3 = I_1 + I_2$$

$$45 - 30I_1 - 41I_3 = 0$$

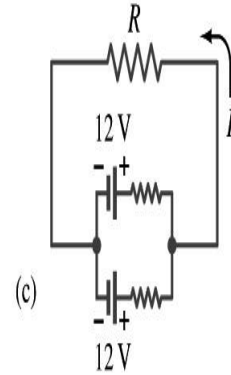
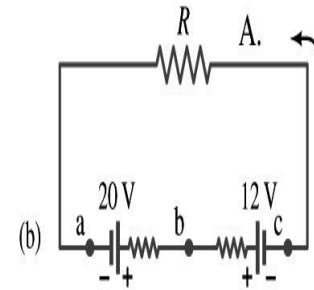
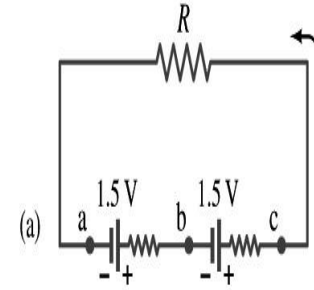
$$125 - 21I_2 - 41I_3 = 0$$

We can obtain the three current by solving these equations for I_1 , I_2 and I_3 .



EMFs in Series and Parallel: Charging a Battery

- When two or more sources of emfs, such as batteries, are connected in series
 - The total voltage is the algebraic sum of their voltages, if their direction is the same
 - $V_{ab} = 1.5 + 1.5 = 3.0V$ in figure (a).
 - If the batteries are arranged in an opposite direction, the total voltage is the difference between them
 - $V_{ac} = 20 - 12 = 8.0V$ in figure (b)
 - Connecting batteries in opposite direction might seem wasteful.
 - This, however, is the way a battery charger works.
 - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
 - Some battery are rechargeable since their chemical reactions are reversible but most the batteries can not reverse their chemical reactions



Parallel arrangements (c) are used only to increase currents.



RC Circuits

- Circuits containing both resistors and capacitors

- RC circuits are used commonly in everyday life

- Control windshield wiper
- Timing of traffic light from red to green
- Camera flashes and heart pacemakers

- What does an RC circuit look like?

- There should be a source of emf, capacitors and resistors

- What happens when the switch S is closed?

- Current immediately starts flowing through the circuit.

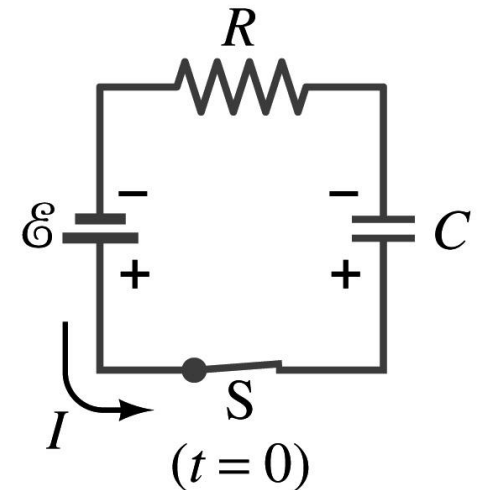
- Electrons flow out of negative terminal of the emf source, through the resistor R and accumulate on the upper plate of the capacitor

- The electrons from the bottom plate of the capacitor will flow into the positive terminal of the battery, leaving only positive charge on the bottom plate

- As the charge accumulates on the capacitor, the potential difference across it increases

- The current reduces gradually to zero -- at that point the voltage across the capacitor is the same as that of the emf

- The charge on the capacitor increases until it reaches to its maximum $C\varepsilon$.





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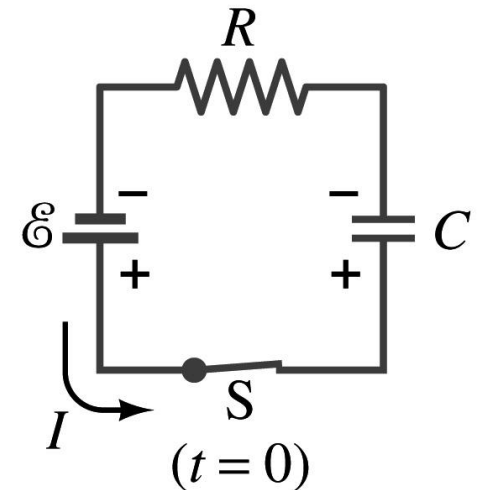
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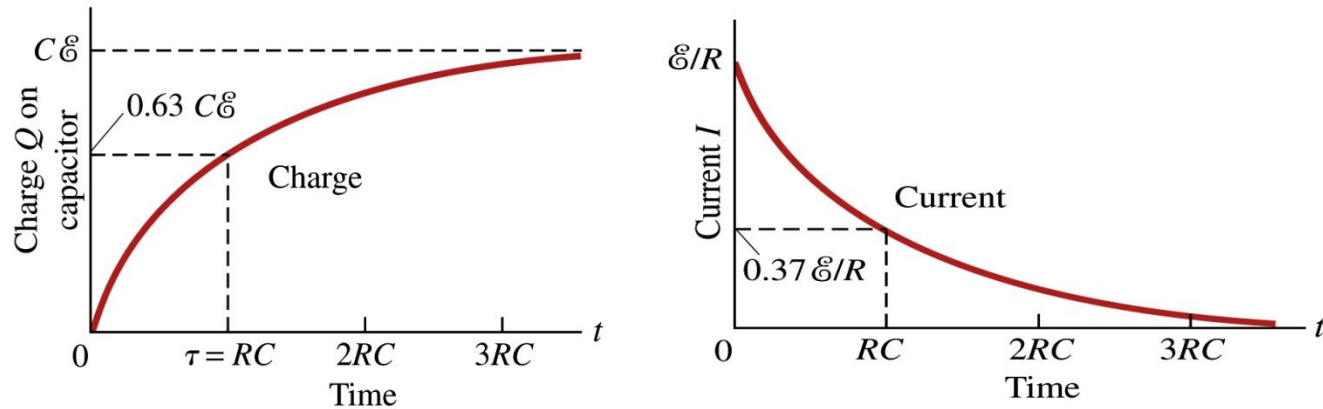
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RC Circuits

- What does all this look like graphically?
 - Charge on the capacitor and current as a function of time

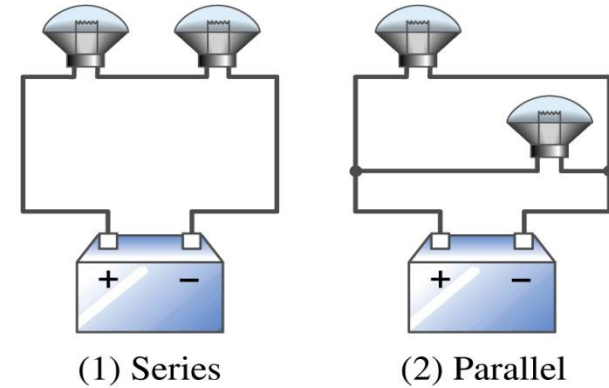


- From energy conservation (Kirchhoff's 2nd rule), the emf ϵ must be equal to the voltage drop across the capacitor and the resistor
 - $\epsilon = IR + Q/C$
 - \mathcal{R} includes all resistance in the circuit, including the internal resistance of the battery, I is the current in the circuit at any instant, and Q is the charge of the capacitor at that same instant



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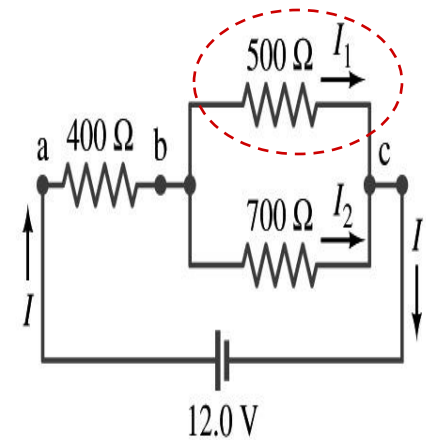
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What is the current flowing in the 400- Ω resistor? PHYS 1444-003 Dr. Andrew Brandt





Magnetism

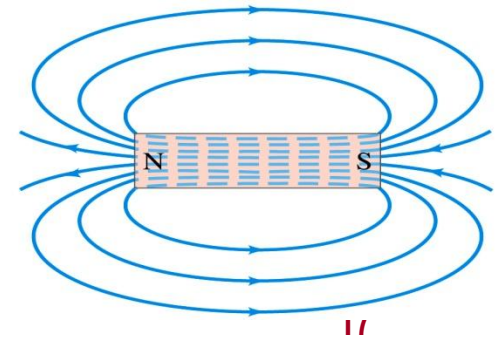
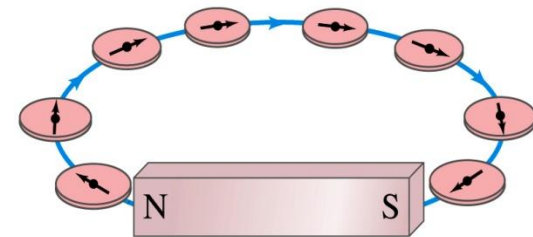
- So are magnet poles analogous to electric charge?
 - No. Why not?
 - While the electric charges (positive and negative) can be isolated, magnet poles cannot.
 - So what happens when a magnet is cut?
 - You get two magnets!
 - The more they get cut, the more magnets are made
 - Single-pole magnets are called “monopoles,” but to date none have been observed...
- Ferromagnetic materials: Materials that show strong magnetic effects
 - Iron, cobalt, nickel, gadolinium and certain alloys
- Other materials show very weak magnetic effects





Magnetic Field

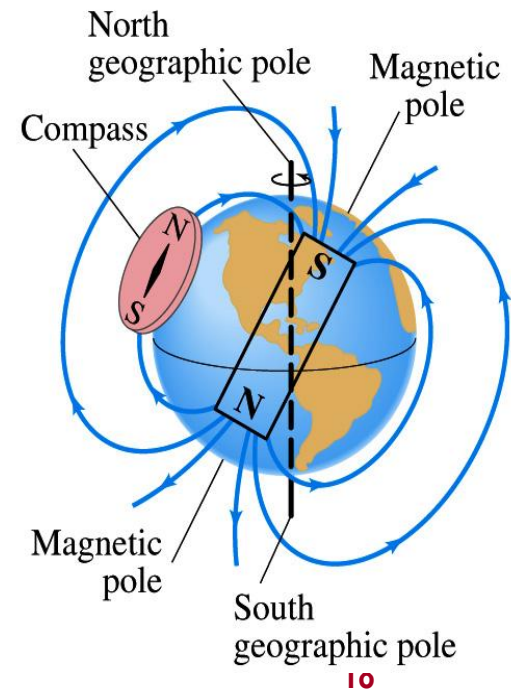
- Just like an electric field surrounds electric charge, a magnetic field surrounds a magnet
- What does this mean?
 - Magnetic force is also a field force
 - The force one magnet exerts on another can be viewed as the interaction between the magnet and the magnetic field produced by the other magnet
 - What kind of quantity is the magnetic field? Vector or Scalar? **Vector**
- So one can draw magnetic field lines, too.
 - The direction of the magnetic field is tangent to a line at any point
 - The direction of the field is the direction the north pole of a compass would point to
 - The number of lines per unit area is proportional to the strength of the magnetic field
 - Magnetic field lines continue inside the magnet
 - Since magnets always have both poles, magnetic field lines form closed loops, unlike electric field lines





Earth's Magnetic Field

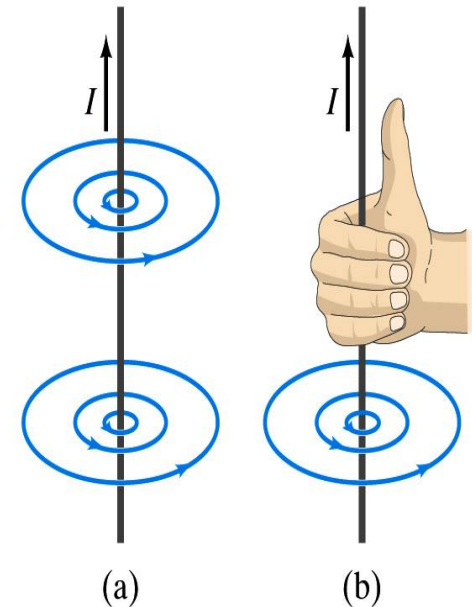
- Which way does a compass point?
- So the magnetic pole of the geographic North pole is ...
 - Yep South!
 - Since the magnetic north pole points to the geographic north, the geographic north must have magnetic south pole
 - The pole in the north is still called geomagnetic north pole just because it is in the north
 - Similarly, south pole has magnetic north pole
- To add confusion: the Earth's magnetic poles do not coincide with the geographic poles → magnetic declination
 - Geomagnetic north pole is in northern Canada, some 1300km off the true north pole
- Earth's magnetic field line is not tangent to the earth's surface at all points
 - The angle the Earth's field makes to the horizontal line is called the angle of dip





Electric Current and Magnetism

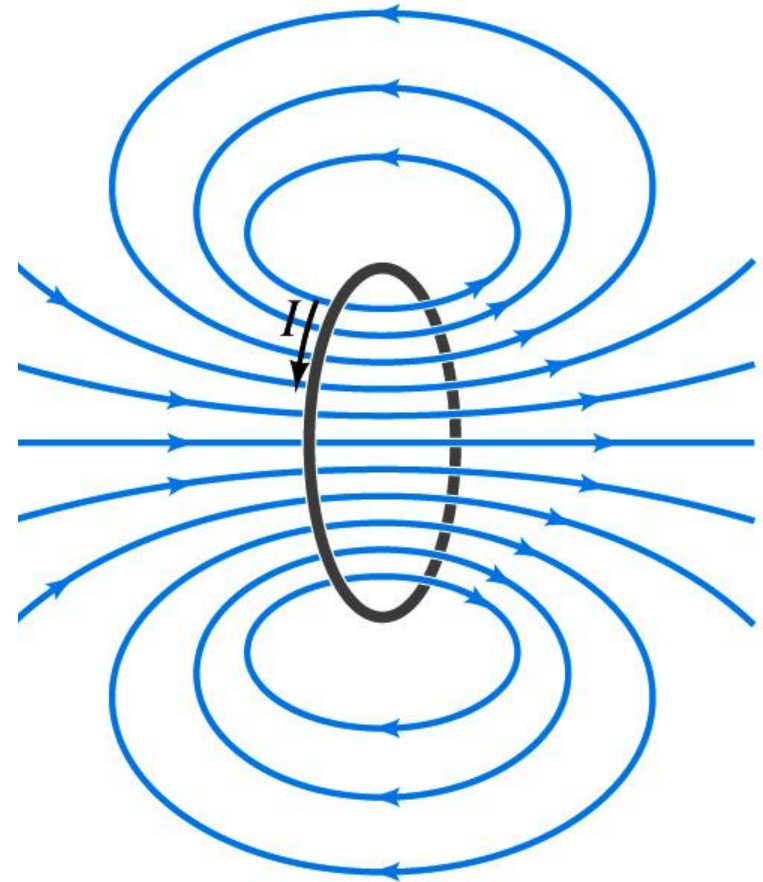
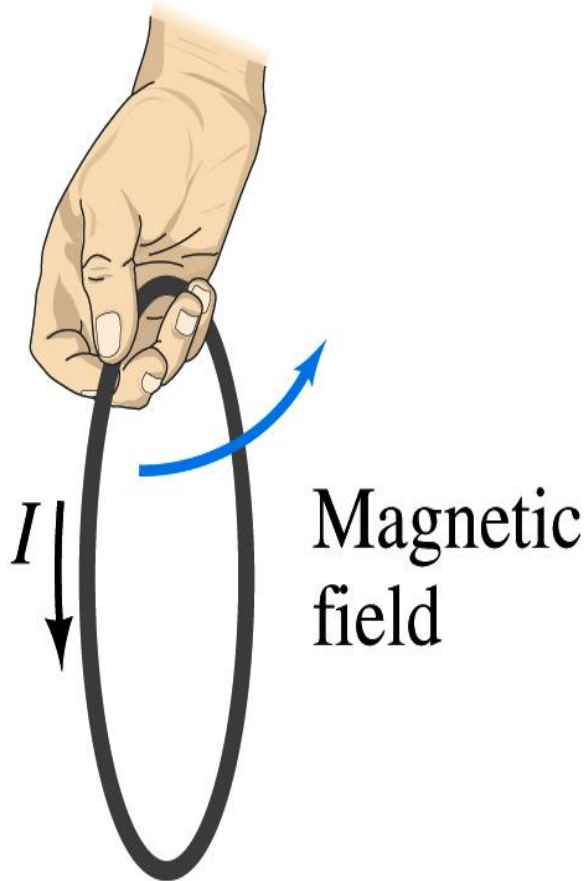
- In 1820, Oersted found that when a compass needle is placed near an electric wire, the needle deflects as soon as the wire is connected to a battery and the current flows
 - Electric current produces a magnetic field
 - The first indication that electricity and magnetism are linked
 - What about a stationary electric charge and magnet?
 - They don't affect each other
- The magnetic field lines produced by a current in a straight wire is in the form of circles following the “right-hand” rule
 - The field lines follow right-hand's fingers wrapped around the wire when the thumb points to the direction of the electric current





Directions in a Circular Wire?

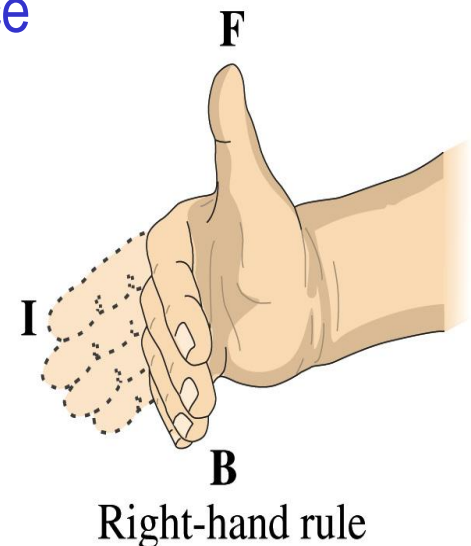
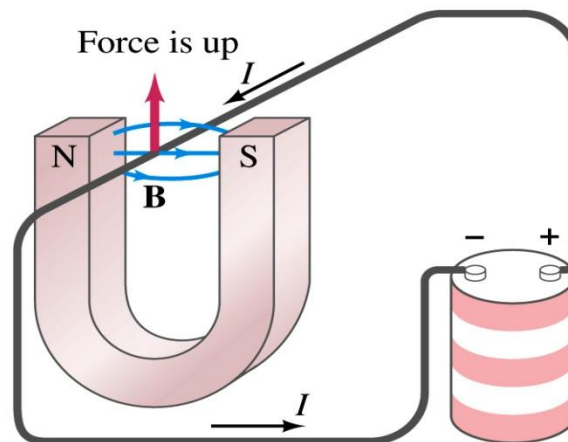
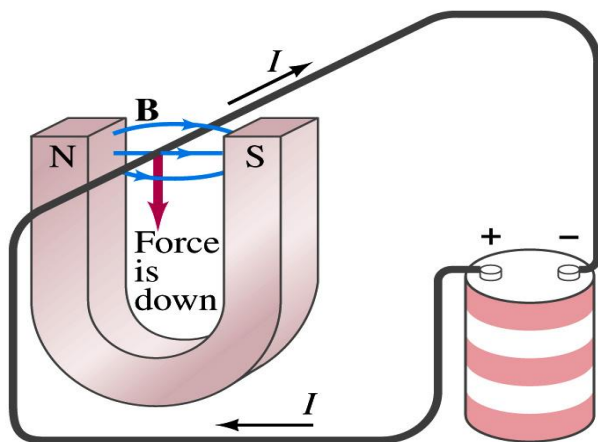
- OK, then what are the directions of the magnetic fields generated by the current flowing through circular loops?





Magnetic Forces on Electric Current

- Since electric current exerts force on a magnet, the magnet should also exert force on the electric current
 - How do we know this?
 - Newton's 3rd law (confirmed in this case by Oerste)
- Direction of the force is always
 - perpendicular to the direction of the current and also
 - perpendicular to the direction of the magnetic field, \mathbf{B}
 - **How is this possible?**
- Experimentally the direction of the force is given by another right-hand rule → When the fingers of the right-hand point in the direction of the current and the finger tips bend in the direction of magnetic field \mathbf{B} , the direction of thumb points to the direction of the force





Another Version of RHR

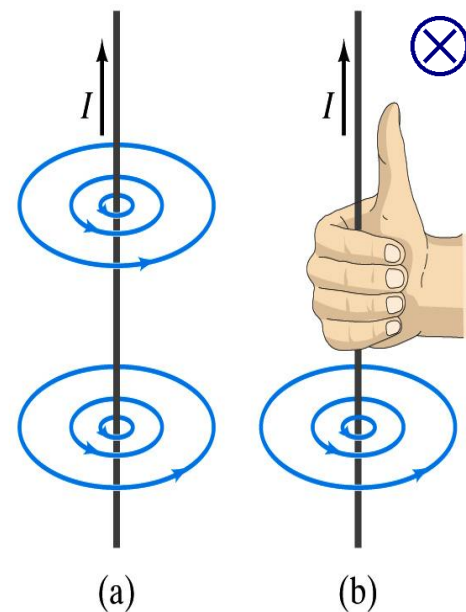
- Suppose we keep thumb in I direction, and fingers in B direction, then the palm will give direction of force!
- Example:

Simply evaluate the force on 2nd wire due to magnetic field in wire 1

The magnetic field at wire 2 from wire 1 is into the page, so the force on wire 2 is to the left.

What about the force on wire 1 due to wire 2?

What if I flipped the direction of the current on one wire?





Magnetic Forces on Electric Current

- The right-hand rule gives the direction, but what about magnitude?
- Observations show that the magnitude is directly proportional to
 - The current in the wire
 - The length of the wire in the magnetic field (if the field is uniform)
 - The strength of the magnetic field
- The force also depends on the angle θ between the directions of the current and the magnetic field
 - When the wire is perpendicular to the field, the force is the strongest
 - When the wire is parallel to the field, there is no force at all
- Thus the force on current I in a wire of length ℓ in a uniform field \mathcal{B} is

$$F \propto I\ell B \sin \theta$$



Magnetic Forces on Electric Current

- Magnetic field strength B can be defined using the previous proportionality relationship with the constant=1: $F = IlB \sin \theta$
- if $\theta=90^\circ$, $F_{\max} = IlB$ and if $\theta=0^\circ$ $F_{\min} = 0$
- So the magnitude of the magnetic field B can be defined as
 $B = F_{\max} / Il$ where F_{\max} is the magnitude of the force on a straight length l of wire carrying a current I when the wire is perpendicular to \mathbf{B}

- The relationship between F , B and I can be written in a vector formula: $\vec{F} = I \vec{l} \times \vec{B}$

where l is a vector with magnitude is the length of the wire and the direction is that of the conventional current

This formula works if B is uniform.

- If B is not uniform or l does not form the same angle with B everywhere, the infinitesimal force acting on a differential length $d\vec{l}$ is $d\vec{F} = Id\vec{l} \times \vec{B}$



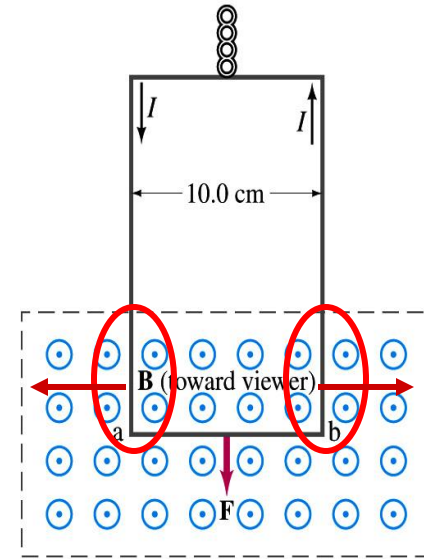
About the Magnetic Field, B

- The magnetic field is a vector quantity
- The SI unit for B is tesla (T)
 - What is the definition of 1 Tesla in terms of other known units?
 - $1\text{T}=1\text{N}/\text{A}\cdot\text{m}$
 - An older unit: 1 tesla is the same as a weber per meter-squared
 - $1\text{Wb}/\text{m}^2=1\text{T}$
- The cgs unit for B is gauss (G)
 - How many T is one G?
 - $1\text{G}=10^{-4}\text{T}$
 - For computation in SI units , one MUST convert Gauss to Tesla
- Magnetic field on the Earth's surface is about $0.5\text{G}=0.5\times 10^{-4}\text{T}$
- On a diagram, \odot for field out of page and \otimes for field in.



Example 27 – 1

Measuring a magnetic field. A rectangular loop of wire hangs vertically as shown in the figure. A magnetic field \mathbf{B} is directed horizontally perpendicular to the wire, and points out of the page. The magnetic field \mathbf{B} is very nearly uniform along the horizontal portion of wire ab (length $l=10.0\text{cm}$) which is near the center of a large magnet producing the field. The top portion of the wire loop is free of the field. The loop hangs from a balance which measures a downward force (in addition to the gravitational force) of $F=3.48 \times 10^{-2}\text{N}$ when the wire carries a current $I=0.245\text{A}$. What is the magnitude of the magnetic field B at the center of the magnet?



Magnetic force exerted on the wire due to the uniform field is

$$\vec{F} = I\vec{l} \times \vec{B}$$

Since $\vec{B} \perp \vec{l}$ Magnitude of the force is $F = IlB$

What about gravitational force?

Solving for B
$$B = \frac{F}{Il} = \frac{3.48 \times 10^{-2} \text{ N}}{0.245 \text{ A} \cdot 0.10 \text{ m}} = 1.42 \text{ T}$$

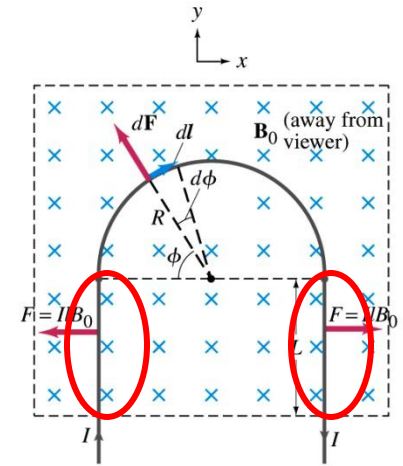
What happened to the forces on the loop on the side?

The two forces cancel out since they are in opposite direction with the same magnitude.



Example 27 – 2

Magnetic force on a semi-circular wire. A rigid wire, carrying a current I , consists of a semicircle of radius R and two straight portions as shown in the figure. The wire lies in a plane perpendicular to the uniform magnetic field \mathbf{B}_0 . The straight portions each have length ℓ within the field. Determine the net force on the wire due to the magnetic field \mathbf{B}_0 .



As in the previous example, the forces on the straight sections of the wire are equal and opposite direction. Thus they cancel.

What do we use to figure out the net force on the semicircle?

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

We divide the semicircle into infinitesimal straight sections.

$$dl = R d\phi$$

What is the net X component of the force acting on the circular section?

0 Why?

Because the forces on left and the right-hand sides of the semicircle balance.

Since $\vec{B}_0 \perp d\vec{l}$ Y-component of the force dF is $dF_y = dF \sin \phi = IB_0 R d\phi \sin \phi$

Integrate over $\phi=0 \rightarrow \pi$

$$F = \int_0^\pi dF \sin \phi = IB_0 R \int_0^\pi \sin \phi d\phi = -IB_0 R \cos \phi \Big|_0^\pi = 2RIB_0$$

Which direction? ¹⁶ Vertically upward direction. The wire will be pulled further into the field.