PHYS 1444 – Section 003
Lecture #15-16

Monday October 23, 25, 2011

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Chapter 27

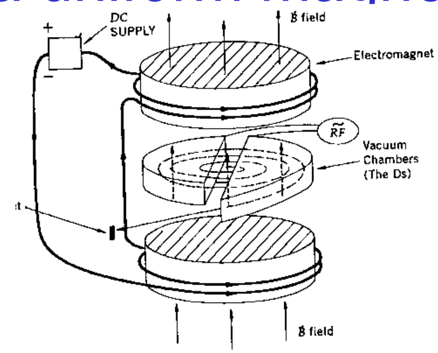
- Magnetic Dipole
- Hall Effect

Chapter 28

- Sources of Magnetic Field
- Magnetic Field Due to Straight Wire
- Forces Between Two Parallel Wires
- Ampere's Law
- Solenoid and Toroidal Magnetic Field

Cyclotron Frequency

- The time required for a particle of charge q moving w/ constant speed v to make one circular revolution in a uniform magnetic field, $\vec{B} \perp \vec{v}$, is



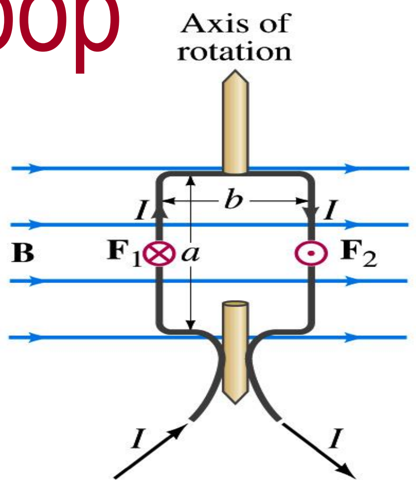
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}$$

- Since T is the period of rotation, the frequency of the rotation is

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

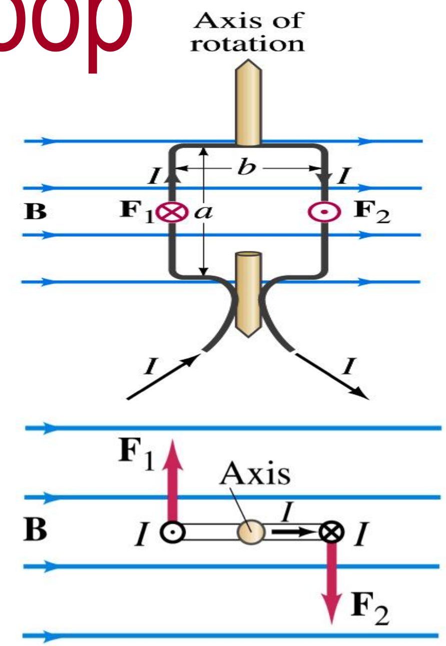
- This is the cyclotron frequency, the frequency of a particle with charge q in a cyclotron accelerator
  - While r depends on v, the frequency is independent of v and r.

Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
  - It will rotate! Why?
  - The magnetic field exerts a force on both vertical sections of wire.
  - Where is this principle used?
    - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exert a net torque in the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
  - It will not rotate further unless the direction of the current changes

Torque on a Current Loop



- So what would be the magnitude of this torque?
  - What is the magnitude of the force on the section of the wire with length a?
    - $F_a = IaB$
    - The moment arm of the coil is $b/2$
  - So the total torque is the sum of the torques by each of the forces

$$\tau = IaB\frac{b}{2} + IaB\frac{b}{2} = IabB = IAB$$

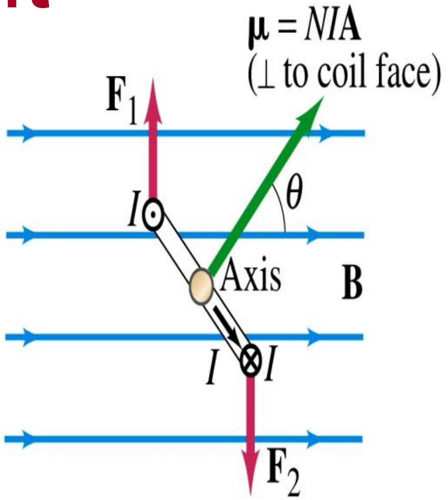
    - Where $A=ab$ is the area of the coil
  - What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

  - If the coil makes an angle θ w/ the field $\tau = NIAB\sin\theta$

Magnetic Dipole Moment

- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity N$I\mathcal{A}$ is called the **magnetic dipole moment of the coil**
  - It is a vector $\vec{\mu} = NI\vec{A}$
    - Its direction is the same as that of the area vector A and is perpendicular to the plane of the coil consistent with the right-hand rule
      - Your thumb points to the direction of the magnetic moment when your finger cups around the loop in the direction of the wire
  - Using the definition of magnetic moment, the torque can be written in vector form $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$



Magnetic Dipole Potential Energy

- Where else did you see the same form of torque?
  - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
  - The potential energy of the electric dipole is
  - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
  - The work done by the torque is
  - $U = \int \tau d\theta = \int NIAB \sin\theta d\theta = -\mu B \cos\theta + C$
  - If we chose U=0 at $\theta=\pi/2$, then C=0
  - Thus the potential energy is $U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$
    - Very similar to the electric dipole

Example 27 – 8

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius $0.529\text{x}10^{-10}$m.

What provides the centripetal force? Coulomb force

So we can obtain the speed of the electron from

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r}$$

Solving for v

$$v = \sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \sqrt{\frac{\left(8.99\times10^9\, N\cdot m^2/C^2\right)\cdot\left(1.6\times10^{-19}\,C\right)^2}{\left(9.1\times10^{-31}\,kg\right)\cdot\left(0.529\times10^{-10}\,m\right)}} = 2.19\times10^6\, m/s$$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current

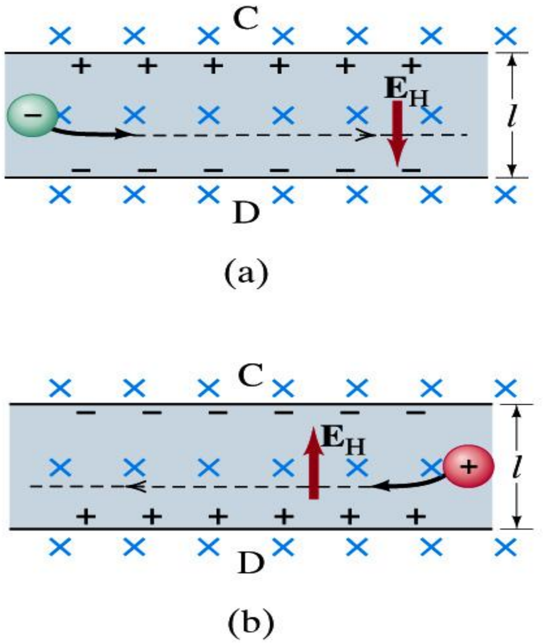
$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

Since the area of the orbit is $A=\pi r^2$, we obtain the hydrogen magnetic moment

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2} = \frac{er}{2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 mr}} = \frac{e^2}{4}\sqrt{\frac{r}{\pi\varepsilon_0 m}}$$

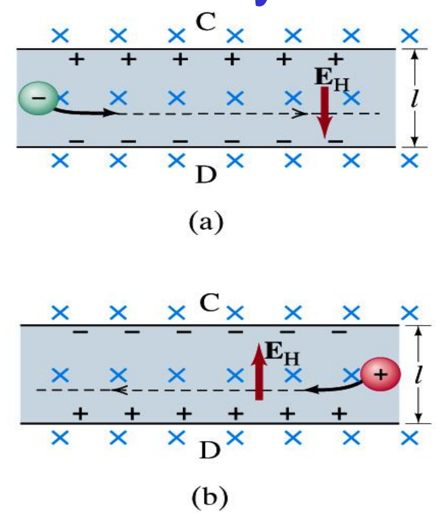
The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
  - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
    - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
  - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
  - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force
- This is called the **Hall Effect**
  - The potential difference produced is called
    - The Hall emf
  - The electric field due to the separation of charge is called the Hall field, $\mathbf{E}_H$, and it points in the direction opposite to the magnetic force



The Hall Effect

- In equilibrium, the force due to Hall field is balanced by the magnetic force ev_dB, so we obtain
- $eE_H = ev_d B$ and $E_H = v_d B$
- The Hall emf is then $\varepsilon_H = E_H l = v_d B l$
  - Where l is the width of the conductor



- What do we use the Hall effect for?
  - The current of a negative charge moving to right is equivalent to a positive charge moving to the left
  - The Hall effect can distinguish between these since the direction of the Hall field or direction of the Hall emf is opposite
  - Since the magnitude of the Hall emf is proportional to the magnetic field strength ➔ can measure the B-field strength
    - Hall probe

Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
  – How do we determine magnetic field strengths in certain situations?
  – How do two wires with electric current interact?
  – What is the general approach to finding the connection between current and magnetic field?

Magnetic Field due to a Straight Wire

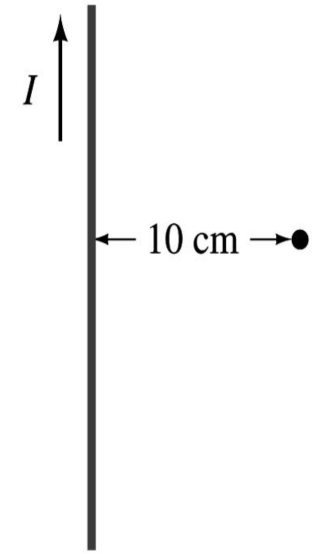
- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
  - What do you imagine the strength of the field is as a function of the distance from the wire?
    - It must be weaker as the distance increases
  - How about as a function of current?
    - Directly proportional to the current
  - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
    - This is valid as long as r << the length of the wire
  - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$B = \frac{\mu_0 I}{2\pi r}$$

  - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$

Example 28 – 1

Calculation of B near wire. A vertical electric wire in the wall of a building carries a DC current of 25A upward. What is the magnetic field at a point 10 cm to the north of this wire?



Using the formula for the magnetic field near a straight wire

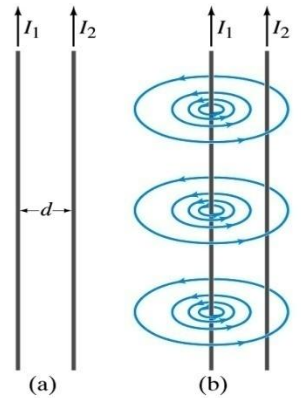
$$B = \frac{\mu_0 I}{2\pi r}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7}\ T \cdot m/A \ \cdot\ 25A}{2\pi \ \cdot\ 0.1\ m} = 5.0 \times 10^{-5}\, T$$

Force Between Two Parallel Wires

- We have learned that a wire carrying current produces a magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
  - They will exert force on each other. Repel or attract?
  - Depends on the direction of the currents

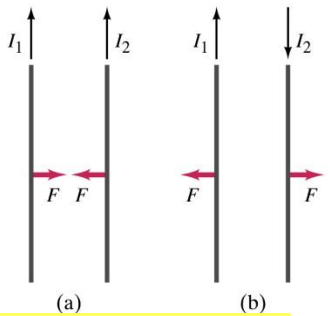


- This was first pointed out by Ampére.
- Let's consider two long parallel conductors separated by a distance d, carrying currents I_1 and I_2.
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force Between Two Parallel Wires

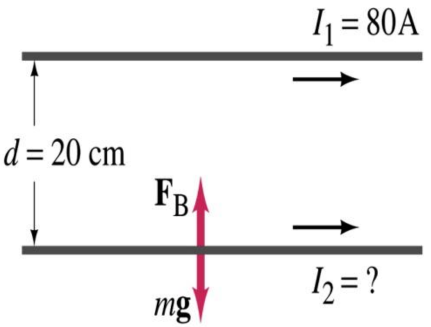
- The force F due to a magnetic field B_1 on a wire of length l, carrying a current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
  – So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi}\frac{I_1}{d}$
  – This force is only due to the magnetic field generated by the wire carrying the current I_1
    - There is a force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in opposite direction
- So the force per unit length is $$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$
- How about the direction of the force?



If the currents are in the same direction, the force is attractive. If opposite, repulsive.

Example 28 – 2

Suspending a wire with current. A horizontal wire carries a current I_1=80A DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? Downward

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{} \frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi \ 9.8\, m/s^2 \cdot 0.12\times 10^{-3}\, kg \cdot 0.20m}{4\pi \times 10^{-7}\, T\cdot m/A \cdot 80A} = 15A$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

- With this definition, the force between two wires each carrying 1A of current and separated by 1m is

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7}\ T \cdot m/A}{2\pi}\frac{1A \cdot 1A}{1m} = 2 \times 10^{-7}\ N/m$$

  - So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly 2×10^{-7} N/m.
- A Coulomb is then defined as exactly 1C=1A-s
- We do it this way since current is measured more accurately and controlled more easily than charge.

Ampére's Law

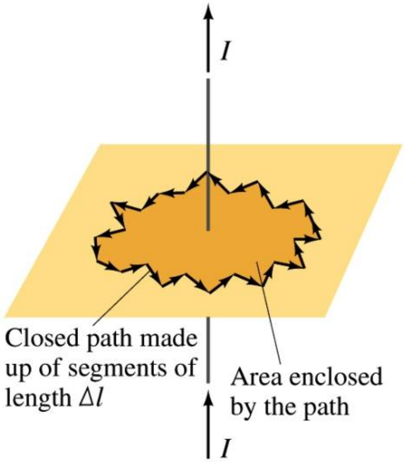
- What is the relationship between magnetic field strength and the current?

$$B = \frac{\mu_0 I}{2\pi r}$$

  – Does this work in all cases?
    - Nope!
    - OK, then when?
    - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shape of the wire?
  – French scientist André Ampére proposed such a generalized relationship

Ampére's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
  - Let's split this path into small segments each of length $\Delta\ell$.
  - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path
  - $\sum B_{\|}\Delta l = \mu_0 I_{encl}$
  - In the limit $\Delta\ell \rightarrow 0$, this relation becomes
  - $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

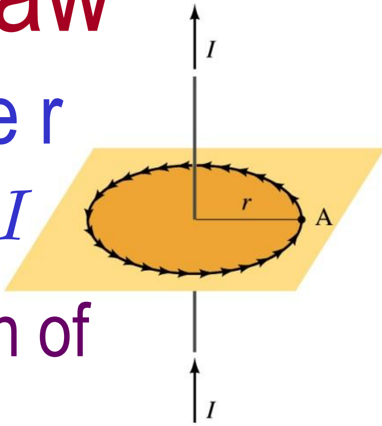


Ampére's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampére's Law



- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
  - We can apply Ampere's law to a circular path of radius r.

$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

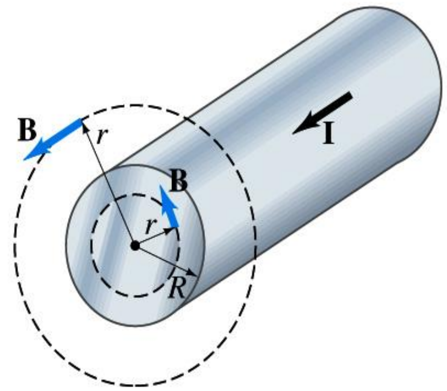
Solving for $\mathcal{B}$

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi} \frac{I}{r}$$

  - We just verified that Ampere's law works in a simple case
  - Experiments have verified that it works for other cases too
  - The importance is that it provides means to relate magnetic field to current

Example 28 – 4

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor (r>R) and (b) points inside the conductor (r<R). Assume that r, the radial distance from the axis, is much less than the length of the wire. (c) If R=2.0mm and I=60A, what is B at r=1.0mm, r=2.0mm and r=3.0mm?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangent to circles around the wire, let's choose a circular path of closed-path integral outside the wire (r>R). What is I_{encl}? $I_{encl} = I$

So using Ampere's law

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

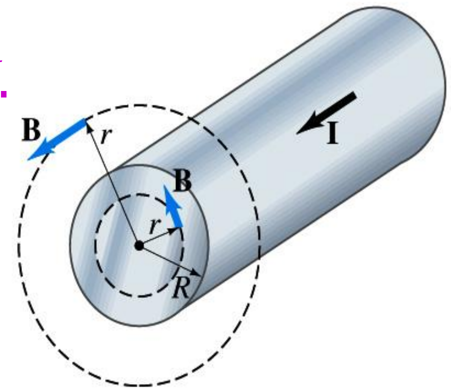
$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Example 28 – 4

For r<R, the current inside the closed path is less than I.

How much is it?

$$I_{encl} = I\frac{\pi r^2}{\pi R^2} = I\left(\frac{r}{R}\right)^2$$



So using Ampere's law

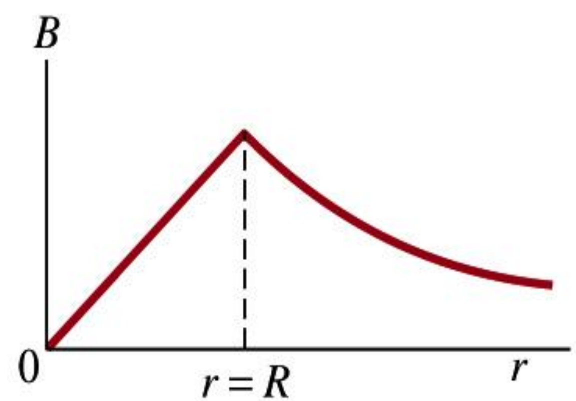
$$\mu_0 I\left(\frac{r}{R}\right)^2 = \oint \vec{B}\cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}\left(\frac{r}{R}\right)^2 = \frac{\mu_0}{2\pi}\frac{Ir}{R^2}$$

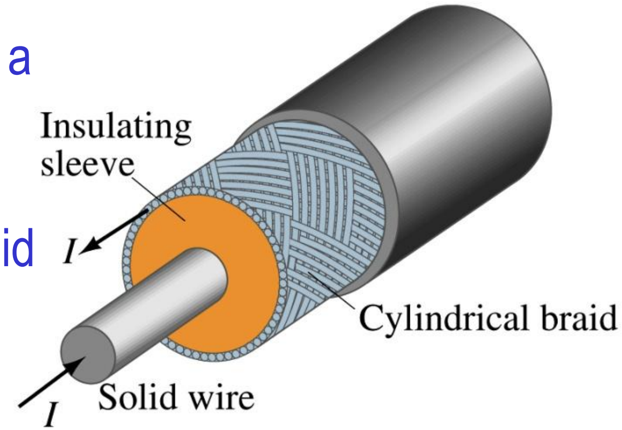
What does this mean?

The field is 0 at r=0 and increases linearly as a function of the distance from the center of the wire up to r=R then decreases as 1/r beyond the radius of the conductor.



Example 28 – 5

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

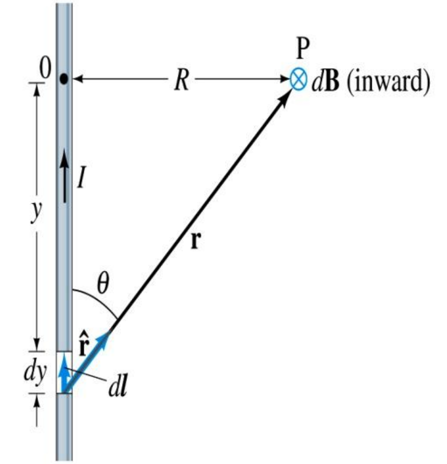
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0.$

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable is self-shielding. The outer conductor also shields against external electric fields, which could cause noise.

Example 28 – 9

B due to current I in a straight wire. For the field near a long straight wire carrying a current I, show that the Biot-Savarat law gives the same result as the simple long straight wire, $B=\mu_0 I/2\pi R$.



What is the direction of the field **B** at point P? Going into the page.

All dB at point P has the same direction based on right-hand rule.

The magnitude of B using Biot-Savart law is

$$B=\oint dB=\frac{\mu_0 I}{4\pi}\int_{-\infty}^{+\infty}\frac{\left|d\vec{l}\times\hat{r}\right|}{r^2}=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}$$

Where $dy=dl$ and $r^2=R^2+y^2$ and since $y=-R\cot\theta$ we obtain

$$dy=+R\csc^2\theta d\theta=\frac{Rd\theta}{\sin^2\theta}=\frac{Rd\theta}{\left(R/r\right)^2}=\frac{r^2 d\theta}{R}$$

Integral becomes

$$B=\frac{\mu_0 I}{4\pi}\int_{y=-\infty}^{+\infty}\frac{dy\sin\theta}{r^2}=\frac{\mu_0 I}{4\pi}\frac{1}{R}\int_{\theta=0}^{\pi}\sin\theta d\theta=-\frac{\mu_0 I}{4\pi}\frac{1}{R}\cos\theta\Big|_0^{\pi}=\frac{\mu_0 I}{2\pi}\frac{1}{R}$$

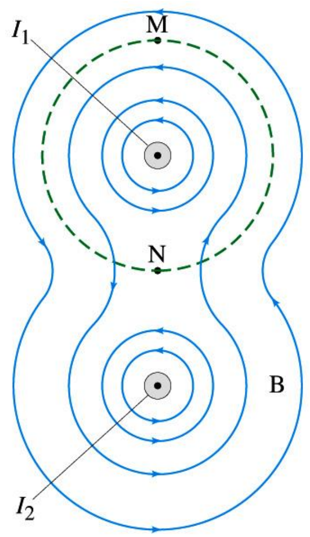
The same as the simple, long straight wire!! It works!!

Ampére's Law

- Since Ampere's law is valid in general, B in Ampere's law is not necessarily just due to the current I_{encl}.

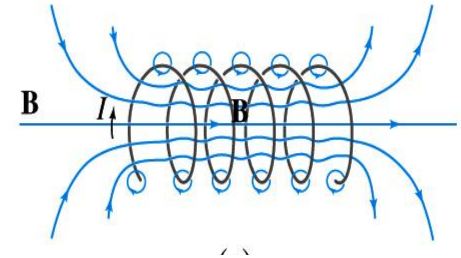
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

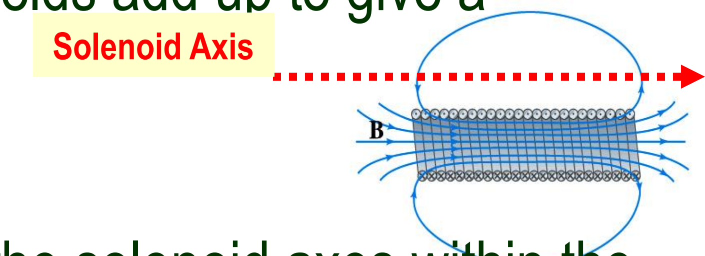
- B is the field at each point in space along the chosen path due to all sources
  - Including the current I enclosed by the path but also due to any other sources
  - How do you obtain B in the figure at any point?
    - Vector sum of the field by the two currents
  - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
  - While B for each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Solenoid and Its Magnetic Field

- What is a solenoid?
  - A long coil of wire consisting of many loops
  - If the space between loops is wide
    - The field near the wires is nearly circular
    - Between any two wires, the fields due to each loop cancel
    - Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform

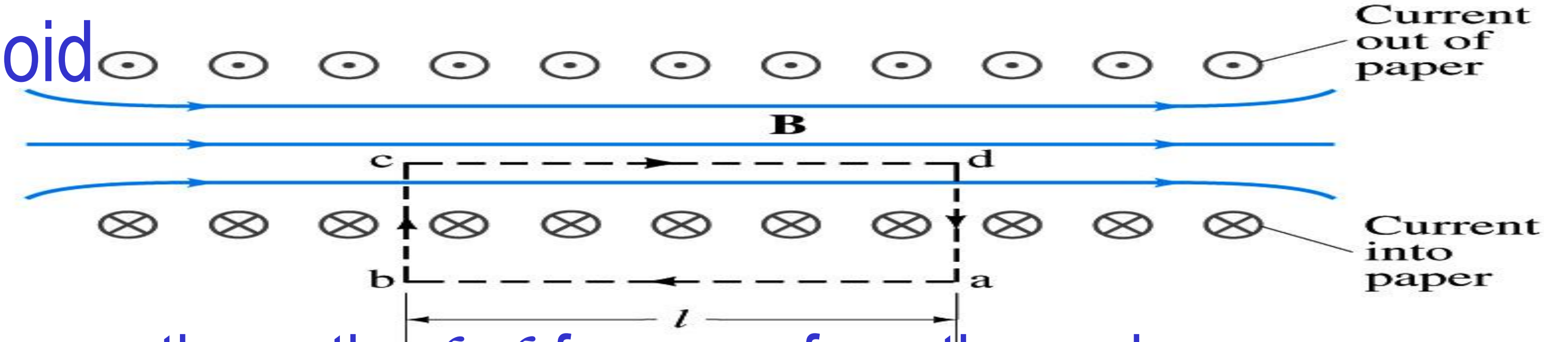




- –For long, densely packed loops
  - •The field is nearly uniform and parallel to the solenoid axes within the entire cross section
  - •The field outside the solenoid is very small compared to the field inside, except at the ends

Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid

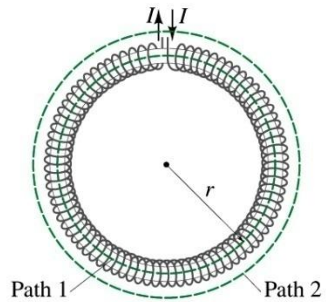


- Let's choose the path *abcd*, far away from the ends
  - We can consider four segments of the loop for integral
  - $$\oint \vec{B}\cdot d\vec{l} = \int_a^b \vec{B}\cdot d\vec{l} + \int_b^c \vec{B}\cdot d\vec{l} + \int_c^d \vec{B}\cdot d\vec{l} \quad \int_d^a \vec{B}\cdot d\vec{l}$$
  - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
  - Now the field B is perpendicular to the bc and da segments. So these integrals become 0, also.

Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
  - Where $\mathcal{N}$ is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 NI$
- If we let $n=\mathcal{N}/\ell$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $$B = \mu_0 nI$$
  - B depends on the number of loops per unit length, n, and the current I
    - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 8



(a)

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, (which is like a solenoid bent into the shape of a circle).

(a) How do you think the magnetic field lines inside the toroid look?


(b)

Since it is a bent solenoid, it should be a circle concentric with the toroid.

If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B \; 2\pi r = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

$$B = \frac{\mu_0 NI}{2\pi r}$$

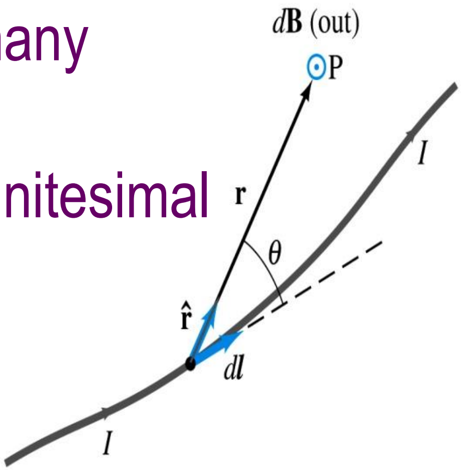
So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin ($B = \mu_0 n I$).

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.

Biot-Savart Law

- Ampere's law is useful in determining magnetic field utilizing symmetry
- But sometimes it is useful to have another method to determine the B field such as using infinitesimal current segments
  - Jean Baptiste Biot and Feilx Savart developed a law that a current I flowing in any path can be considered as many infinitesimal current elements
  - The infinitesimal magnetic field d**B** caused by the infinitesimal length $d\ell$ that carries current I is
  - $$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

Biot-Savart Law



- **r** is the displacement vector from the element $d\ell$ to the point P
- Biot-Savart law is the magnetic equivalent to Coulomb's law

The B field in the Biot-Savart law is only that due to the current