PHYS 1444 – Section 003
Lecture #21

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Chapter 30

- **Inductance**

E Field due to Magnetic Flux Change

- When electric current flows through a wire, there is an electric field in the wire that moves electrons
- We saw, however, that changing magnetic flux induces a current in the wire. What does this mean?
  - There must be an electric field induced by the changing magnetic flux.
- In other words, a changing magnetic flux produces an electric field
- This results apply not just to wires but to any conductor or any region in space

Generalized Form of Faraday's Law

- What is the relation between electric field and the potential difference $V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$
- The induced emf in a circuit is equal to the work done per unit charge by the induced electric field
- $\varepsilon = \oint \vec{E} \cdot d\vec{l}$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- The integral is taken around a path enclosing the area through which the magnetic flux Φ_B is changing.

Self Inductance

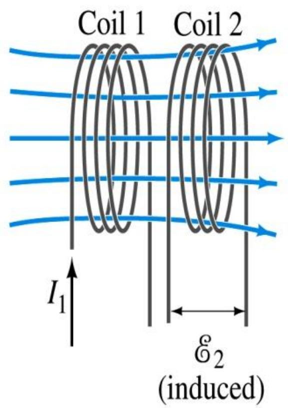
- Since the magnetic flux Φ_B passing through an N turn coil is proportional to current I in the coil, $N\Phi_B = LI$
- We define self-inductance, $\mathcal{L}$: $L = \frac{N\Phi_B}{I}$ **Self Inductance**

- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

Inductance

- Changing the magnetic flux through a circuit induces an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance



- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil 2 proportional to?
  - Rate of change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil 2 created by coil1 and N_2 is the number of closely packed loops in coil 2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21}I_1$
- The proportionality constant for this is called the Mutual Inductance and defined by $M_{21} = N_2\Phi_{21}/I_1$
- The emf induced in coil2 due to the changing current in coil1 is

$$\varepsilon_2 = -N_2\frac{d\Phi_{21}}{dt} = -\frac{d\ \ N_2\Phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$$

Mutual Inductance

- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present
  - The further apart the two coils are the less flux passes through coil 2, so M_{21} will be less.
  - Typically the mutual inductance is determined experimentally
- Just as a changing current in coil 1 will induce an emf in coil 2, a changing current in coil 2 will induce an emf in coil 1

$$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$

–We can put $M=M_{12}=M_{21}$ and obtain

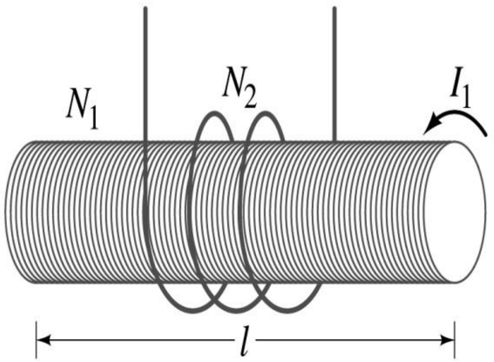
$$\varepsilon_1 = -M\frac{dI_2}{dt} \text{ and } \varepsilon_2 = -M\frac{dI_1}{dt}$$

–SI unit for mutual inductance is henry (H)

$$1H = 1V \cdot s/A = 1\Omega \cdot s$$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length l and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assume all the flux from coil1 (the solenoid) passes through coil2, and calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $$B = \frac{\mu_0 N_1 I_1}{l}$$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils 2. Thus the flux through coil2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1} \frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impede the increased current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current

Inductor

- An electrical circuit always contains some inductance but it is often negligible
  - If a circuit contains a coil of many turns, it could have a large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resistors are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a “non-inductive winding”
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Energy Stored in a Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate: $$P = I\varepsilon = IL\frac{dI}{dt}$$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LIdI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LIdI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in a Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $$U = \frac{1}{2} L I^2$$

    Energy Stored in a magnetic field inside an inductor

  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V $U = \frac{1}{2} C V^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without fringe effects
    $$L = \mu_0 N^2 A / l$$
  - The magnetic field in a solenoid is $B = \mu_0 NI / l$
  - Thus the energy stored in an inductor is
    $$U = \frac{1}{2} L I^2 = \frac{1}{2} \frac{\mu_0 N^2 A}{l} \left(\frac{Bl}{\mu_0 N} \right)^2 = \frac{1}{2} \frac{B^2}{\mu_0} Al$$
  - Thus the energy density is
    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2} \frac{B^2}{\mu_0}$$
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ

What is this?

Volume V

$$U = \frac{1}{2} \frac{B^2}{\mu_0} Al$$ E

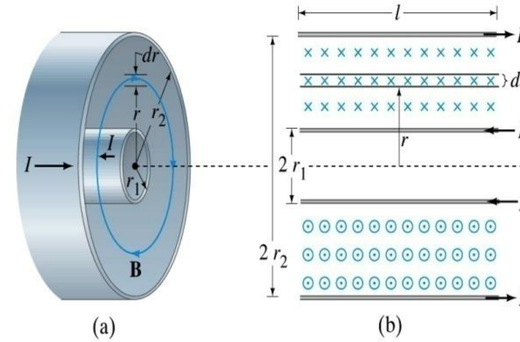
$$u = \frac{1}{2} \frac{B^2}{\mu_0}$$ E density

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi} \ln \frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi} \ln \frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest. B is highest close to $r=r_1$, near the surface of the inner conductor.