PHYS 1444 – Section 04
Lecture #22

Tuesday November 27, 2012

*Dr. **Andrew Brandt***

- AC Circuits

Last HW on ch 29-30 TBA tomorrow due Monday Dec. 3 @ 11 pm

Exam: Thursday Dec. 6, 2012 11 - 1:30 p.m.

AC Circuits – the preamble

- Do you remember how the rms and peak values for current and voltage are related?

$$V_{rms} = \frac{V_0}{\sqrt{2}} \qquad I_{rms} = \frac{I_0}{\sqrt{2}}$$

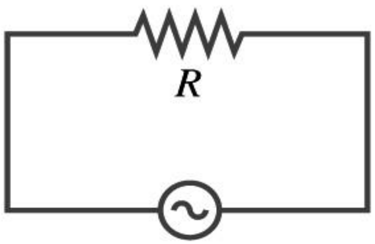
- The symbol for an AC power source is

- We assume that the voltage gives rise to current

$$I = I_0 \sin 2\pi ft = I_0 \sin \varpi t$$

where $\varpi = 2\pi f$

AC Circuit w/ Resistance only



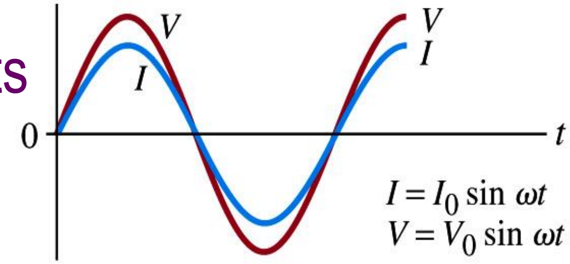
- What do you think will happen when an ac source is connected to a resistor?
- From Kirchhoff's loop rule, we obtain

$$V - IR = 0$$

- Thus

$$V = I_0 R \sin \varpi t = V_0 \sin \varpi t$$

  - where $V_0 = I_0 R$
- What does this mean?
  - Current is 0 when voltage is 0 and current is at its peak when voltage is at its peak.
  - Current and voltage are "in phase"

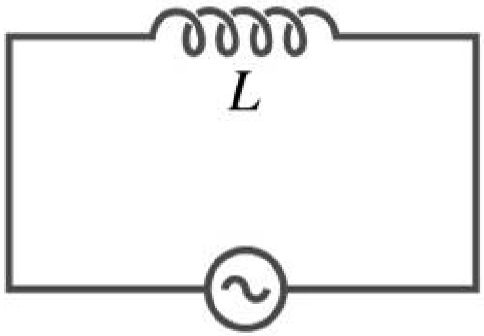


- Energy is lost via the transformation into heat at an average rate $\overline{P} = \overline{I}\,\overline{V} = I_{rms}^2 R = V_{rms}^2 / R$

AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

$$V - L\frac{dI}{dt} = 0$$



- Thus

$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  - Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$

$$V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$$

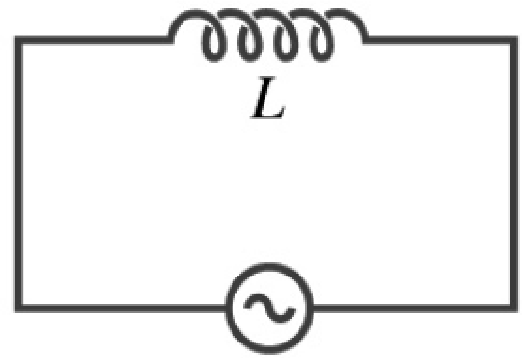
  - where $V_0 = \varpi L I_0$

- What does this mean?
  - Current and voltage are "out of phase by π/2 or 90°".
  - In other words the current reaches its peak ¼ cycle after the voltage
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the magnetic field then released back to the source

AC Circuit w/ Inductance only

- From Kirchhoff's loop rule, we obtain

$$V - L\frac{dI}{dt} = 0$$

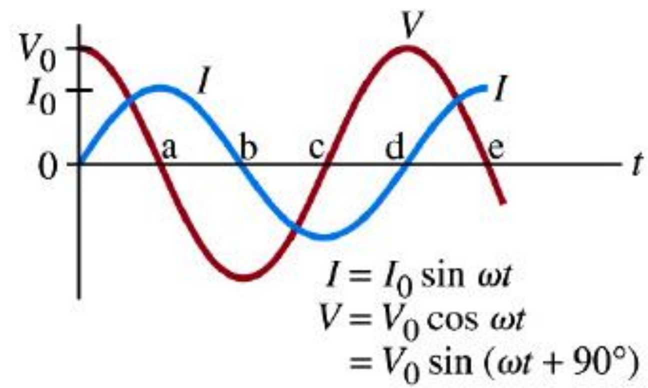


- Thus

$$V = L\frac{dI}{dt} = L\frac{d\left(I_0 \sin \varpi t\right)}{dt} = \varpi L I_0 \cos \varpi t$$

  – Using the identity $\cos\theta = \sin\left(\theta + 90^\circ\right)$

- $$V = \varpi L I_0 \sin\left(\varpi t + 90^\circ\right) = V_0 \sin\left(\varpi t + 90^\circ\right)$$
  – where $V_0 = \varpi L I_0$
  – Current and voltage are "out of phase by π/2 or 90º".



– For an inductor we can write $V_0 = I_0 X_L$

- Where X_L is the <u>inductive reactance</u> of the inductor $X_L = \varpi L$

- The relationship $V_0 = I_0 X_L$ is not generally valid since V_0 and I_0 do not occur at the same time, but $V_{rms} = I_{rms} X_L$ is valid

AC Circuit w/ Inductance only

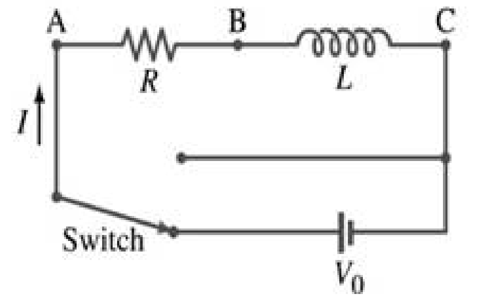
- How are the resistor and inductor different in terms of energy?
  - Inductor — Stores the energy temporarily in the magnetic field and then releases it back to the emf source
  - Resistor — Does not store energy but transforms it to thermal energy, losing it to the environment
- How are they the same?
  - They both impede the flow of charge
  - For a resistance R, the peak voltage and current are related by $V_0 = I_0 R$
  - Similarly, for an inductor we can write $V_0 = I_0 X_L$, where $X_L = \varpi L$

  and X_L is the inductive reactance of the inductor

  BUT since the voltage and current are out of phase this equation is not valid at any particular time. However $V_{rms} = I_{rms} X_L$ is valid!

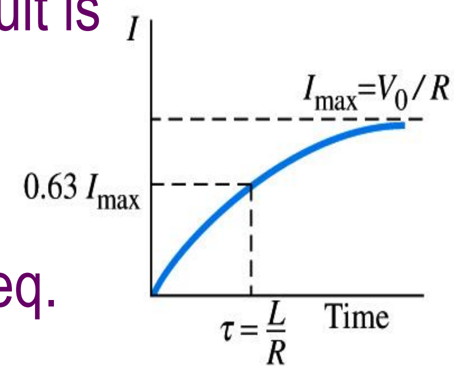
  Note the unit of X_L is Ω, while the unit of L is Henry (what is a Henry?)

LR Circuits



- This can be shown w/ Kirchhoff rule loop rules
  - The emfs in the circuit are the battery voltage V_0 and the emf $\varepsilon = -\mathcal{L}(dI/dt)$ (opposes the ΔI after the bat. switched on)
  - The sum of the potential changes through the circuit is

$$V_0 + \varepsilon - IR = \quad V_0 - L\,dI/dt - IR = 0$$



  - Where I is the current at any instance
  - By rearranging the terms, we obtain a differential eq.
  - $L\,dI/dt + IR = V_0$

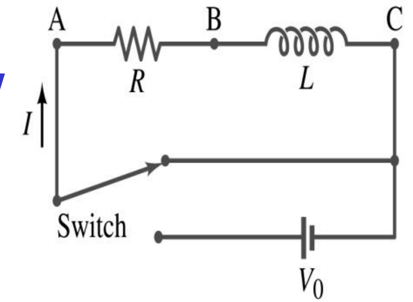
$$\int_{I=0}^{I} \frac{dI}{V_0 - IR} = \int_{t=0}^{t} \frac{dt}{L}$$

  - We can integrate just as in RC circuit
  - So the solution is $-\frac{1}{R}\ln\left(\frac{V_0 - IR}{V_0}\right) = \frac{t}{L}$

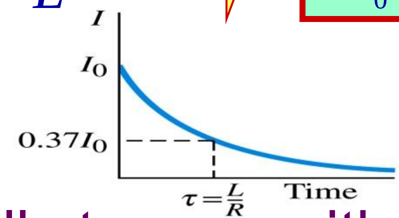
$$I = V_0\left(1 - e^{-t/\tau}\right)/R = I_{max}\left(1 - e^{-t/\tau}\right)$$

  - Where τ=L/R
    - This is the time constant τ of the LR circuit and is the time required for the current I to reach 0.63 of the maximum

Discharge of LR Circuits



- If the switch is flipped away from the battery
  - The differential equation becomes
  - $L\,dI/dt + IR = 0$
  - So the integration is $\int_{I=0}^{I} \frac{dI}{IR} = \int_{t=0}^{t} \frac{dt}{L}$ $\Rightarrow$ $\ln\frac{I}{I_0} = -\frac{R}{L}t$
  - Which results in the solution
  - $I = I_0 e^{-\frac{R}{L}t} = I_0 e^{-t/\tau}$



  - The current decays exponentially to zero with the time constant τ=L/R
  - So there always is a reaction time when a system with a coil, such as an electromagnet, is turned on or off.
  - The current in LR circuit behaves in a similar manner as for the RC circuit, except that in steady state RC current is 0, and the time constant is inversely proportional to R in LR circuit unlike the RC circuit

Example 31 – 1

Reactance of a coil. A coil has a resistance R=1.00Ω and an inductance of 0.300H. Determine the current in the coil if (a) 120 V dc is applied to it; (b) 120 V ac (rms) at 60.0Hz is applied.

Is there a reactance for dc? Nope. Why not? Since ω=0, $X_L = \varpi L = 0$

So for dc power, the current is from Kirchhoff's rule $V - IR = 0$

$$I_0 = \frac{V_0}{R} = \frac{120V}{1.00\Omega} = 120A$$

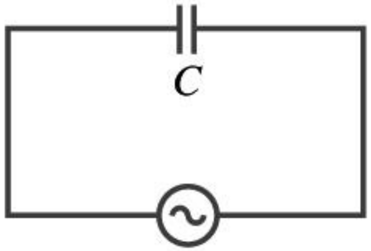
For an ac power with f=60Hz, the reactance is

$$X_L = \varpi L = 2\pi fL = 2\pi \cdot 60.0s^{-1} \cdot 0.300H = 113\Omega$$

Since the resistance can be ignored compared to the reactance, the rms current is $I_{rms} \approx \frac{V_{rms}}{X_L} = \frac{120V}{113\Omega} = 1.06A$

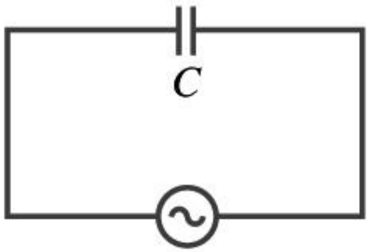
AC Circuit w/ Capacitance only

- What happens when a capacitor is connected to a dc power source?
  - The capacitor quickly charges up.
  - There is no steady current flow in the circuit
    - Since a capacitor prevents the flow of a dc current



- What do you think will happen if it is connected to an ac power source?
  - The current flows continuously. Why?
  - When the ac power turns on, charge begins to flow one direction, charging up the plates
  - When the direction of the power reverses, the charge flows in the opposite direction

AC Circuit w/ Capacitance only



- From Kirchhoff's loop rule, we obtain

$$V = \frac{Q}{C}$$

- Current at any instant is $$I = \frac{dQ}{dt} = I_0 \sin \varpi t$$

- Thus, the charge Q on the plate at any instance is

$$Q = \int_{Q=0}^{Q} dQ = \int_{t=0}^{t} I_0 \sin \varpi t dt = -\frac{I_0}{\varpi} \cos \varpi t$$

- The voltage across the capacitor is

$$V = \frac{Q}{C} = -I_0 \frac{1}{\varpi C} \cos \varpi t$$

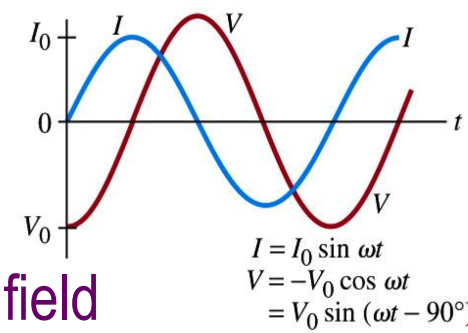
  - Using the identity $\cos\theta = -\sin(\theta - 90^\circ)$

$$V = I_0 \frac{1}{\varpi C} \sin(\varpi t - 90^\circ) = V_0 \sin(\varpi t - 90^\circ)$$

  - Where
  - $$V_0 = \frac{I_0}{\varpi C}$$

AC Circuit w/ Capacitance only

- So the voltage is $V = V_0 \sin\left(\varpi t - 90^\circ\right)$
- What does this mean?
  - Current and voltage are "out of phase by π/2 or 90°" but in this case, the voltage reaches its peak ¼ cycle after the current
- What happens to the energy?
  - No energy is dissipated
  - The average power is 0 at all times
  - The energy is stored temporarily in the electric field
  - Then released back to the source

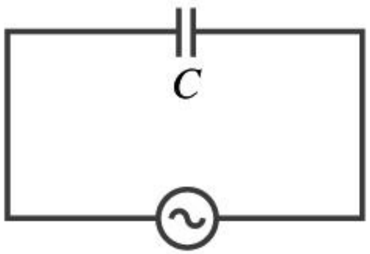


- Relationship between the peak voltage and the peak current in the capacitor can be written as $V_0 = I_0 X_C$
  - Where the capacitance reactance X_C is defined as $X_C = \frac{1}{\varpi C}$

    Infinite when ω=0.
  - Again, this relationship is only valid for rms quantities $V_{rms} = I_{rms} X_C$

Example 31 – 2

Capacitor reactance. What are the peak and rms current in the circuit in the figure if C=1.0μF and V_{rms}=120V? Calculate for *f*=60Hz.



The peak voltage is $V_0 = \sqrt{2}V_{rms} = 120V \cdot \sqrt{2} = 170V$

The capacitance reactance is

$$X_C = \frac{1}{\varpi C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \cdot 60s^{-1} \cdot 1.0 \times 10^{-6} F} = 2.7k\Omega$$

Thus the peak current is

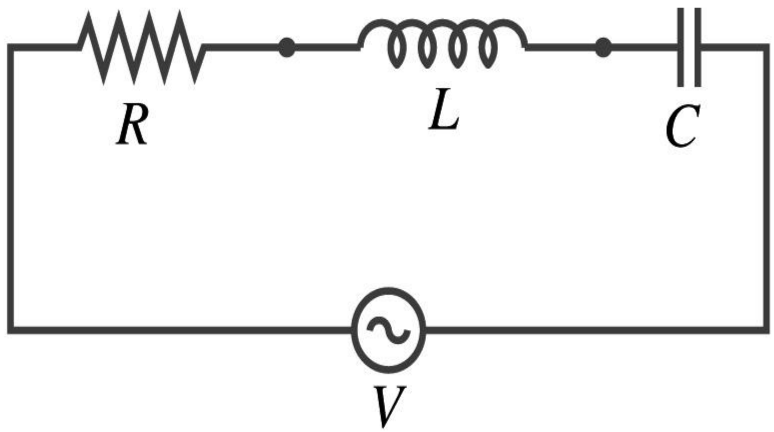
$$I_0 = \frac{V_0}{X_C} = \frac{170V}{2.7k\Omega} = 63mA$$

The rms current is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{120V}{2.7k\Omega} = 44mA$$

AC Circuit w/ LRC

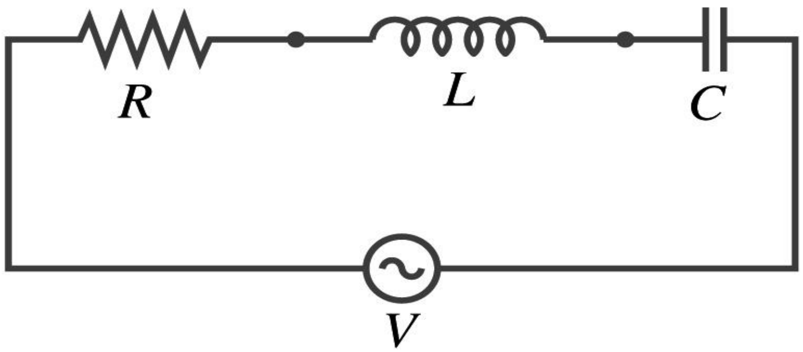
- The voltage across each element is
  - V_R is in phase with the current
  - V_L leads the current by 90°
  - V_C lags the current by 90°



- From Kirchhoff's loop rule
- $V=V_R+V_L+V_C$
  - However since they do not reach the peak voltage at the same time, the peak voltage of the source V_0 will not equal $V_{R0}+V_{L0}+V_{C0}$
  - The rms voltage also will not be the simple sum of the three

AC Circuit w/ LRC

- The current at any instant is the same at all point in the circuit
  - The currents in each elements are in phase
  - Why?
    - Since the elements are in series
  - How about the voltage?
    - They are not in phase.



- The current at any given time is

$$I = I_0 \sin \varpi t$$

- The analysis of LRC circuit can be done using the "phasor" diagram (but not by us!)