PHYS 1444 – Section 03
Lecture #23

Thursday November 29, 2012

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- Maxwells Equations

Last HW Dec. 3
@11pm Ch 29,30

Maxwell's Equations

- The development of EM theory by Oersted, Ampere and others was not done in terms of EM fields
  - The idea of fields was introduced by Faraday
- Scottish physicist James C. Maxwell unified all the phenomena of electricity and magnetism in one theory with only four equations (Maxwell's Equations) using the concept of fields
  - This theory provided the prediction of EM waves
  - As important as Newton's law since it provides dynamics of electromagnetism
  - This theory is also in agreement with Einstein's special relativity
- The biggest achievement of 19th century electromagnetic theory is the prediction and experimental verification that the electromagnetic waves can travel through empty space
  - This accomplishment
    - Opened a new world of communication
    - Yielded the prediction that the light is an EM wave
- Since all of Electromagnetism is contained in the four Maxwell's equations, this is considered as one of the greatest achievements of the human intellect

Modifying Ampere's Law

- A magnetic field is produced by an electric current

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$I =$

- This equation represents the general form of Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Extra term from Maxwell

  - This means that a magnetic field can be caused not only by an ordinary electric current but also by a changing electric flux

Displacement Current

- Maxwell interpreted the second term in the generalized Ampere's law equivalent of an electric current
  - He called this term the displacement current, I_D
  - While the other term is called as the conduction current, I
- Ampere's law then can be written as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + I_D \right)$$

  - Where

$$I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$$

  - While it is in effect equivalent to an electric current, a flow of electric charge, this actually does not have anything to do with the flow itself

Gauss' Law for Magnetism

- If there is symmetry between electricity and magnetism, there must be an equivalent law in magnetism as Gauss' Law in electricity
- The magnetic flux through a closed surface which completely encloses a volume is

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- What was Gauss' law in the electric case?
  - The electric flux through a closed surface is equal to the total net charge Q enclosed by the surface divided by ε_0.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

- Similarly, we can write Gauss' law for magnetism as

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

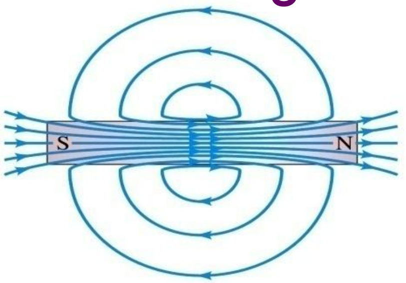
- Why is result of the integral zero?
  - There are no isolated magnetic poles, the magnetic equivalent of single electric charges

Gauss' Law for Magnetism

- What does Gauss' law in magnetism mean physically?

$$\oint \vec{B} \cdot d\vec{A} = 0$$

  - There are as many magnetic flux lines that enter the enclosed volume as leave it
  - If magnetic monopoles do not exist, there is no starting or stopping point of the flux lines
    - Electricity has sources and sinks
  - Magnetic field lines must be continuous
  - Even for bar magnets, the field lines exist both insides and outside of the magnet



Maxwell's Equations

- In the absence of dielectric or magnetic materials, the four equations developed by Maxwell are:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

Gauss' Law for electricity

A generalized form of Coulomb's law relating electric field to its sources, the electric charge

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Gauss' Law for magnetism

A magnetic equivalent of Coulomb's law, relating magnetic field to its sources. This says there are no magnetic monopoles.

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Faraday's Law

An electric field is produced by a changing magnetic field

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampére's Law

A magnetic field is produced by an electric current or by a changing electric field

Example 32 – 1

Charging capacitor. A 30-pF air-gap capacitor has circular plates of area A=100cm^2. It is charged by a 70-V battery through a 2.0-Ω resistor. At the instant the battery is connected, the electric field between the plates is changing most rapidly. At this instance, calculate (a) the current into the plates, and (b) the rate of change of electric field between the plates. (c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

Since this is an RC circuit, the charge on the plates is: $Q = CV_0 \; 1 - e^{-t/RC}$

For the initial current (t=0), we differentiate the charge with respect to time.

$$I_0 = \left.\frac{dQ}{dt}\right|_{t=0} = \left.\frac{CV_0}{RC} e^{-t/RC}\right|_{t=0} = \frac{V_0}{R} = \frac{70V}{2.0\Omega \cdot} = 35A$$

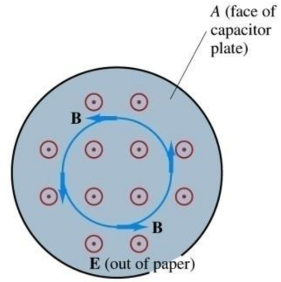
The electric field is $E = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q/A}{\varepsilon_0}$

Change of the electric field is $\dfrac{dE}{dt} = \dfrac{dQ/dt}{A\varepsilon_0} = \dfrac{35A}{8.85\times10^{-12}\, C^2/N\cdot m^2 \;\cdot\; 1.0\times10^{-2}\, m^2} = 4.0\times10^{14}\, V/m\cdot s$

Example 32 – 1

(c) Determine the magnetic field induced between the plates. Assume **E** is uniform between the plates at any instant and is zero at all points beyond the edges of the plates.

The magnetic field lines generated by changing electric field is perpendicular to E and is circular due to symmetry



Whose law can we use to determine B?

Extended Ampere's Law w/ $I_{encl}=0$!

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

We choose a circular path of radius r, centered at the center of the plane, following the B.

For $r<r_{plate}$, the electric flux is $\Phi_E = EA = E\pi r^2$ since E is uniform throughout the plate

So from Ampere's law, we obtain $B\cdot\left(2\pi r\right) = \mu_0\varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \mu_0\varepsilon_0\pi r^2 \frac{dE}{dt}$

Solving for B $\rightarrow$ $$B = \mu_0\varepsilon_0 \frac{r}{2}\frac{dE}{dt}$$ For $r<r_{plate}$

Since we assume E=0 for $r>r_{plate}$, the electric flux beyond the plate is fully contained inside the surface. $\Phi_E = EA = E\pi r_{plate}^2$

So from Ampere's law, we obtain $B\cdot\left(2\pi r\right) = \mu_0\varepsilon_0 \frac{d\left(E\pi r_{plate}^2\right)}{dt} = \mu_0\varepsilon_0\pi r_{plate}^2 \frac{dE}{dt}$

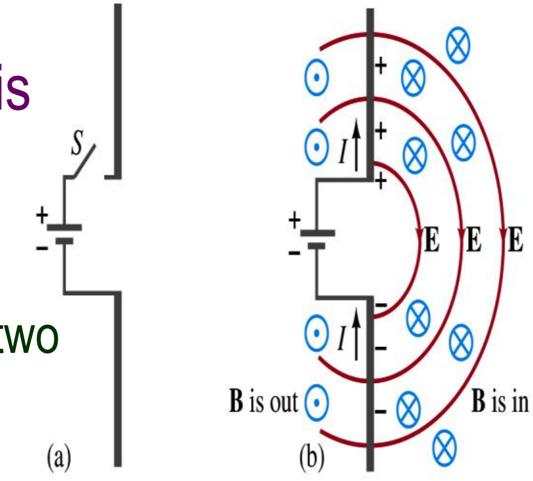
Solving for B $\rightarrow$ $$B = \frac{\mu_0\varepsilon_0 r_{plate}^2}{2r}\frac{dE}{dt}$$ For $r>r_{plate}$

Maxwell's Amazing Leap of Faith

- According to Maxwell, a magnetic field will be produced even in empty space if there is a changing electric field
  - He then took this concept one step further and concluded that
    - If a changing magnetic field produces an electric field, the electric field is also changing in time.
    - This changing electric field in turn produces a magnetic field that also changes
    - This changing magnetic field then in turn produces the electric field that changes
    - This process continues
  - With the manipulation of the equations, Maxwell found that the net result of this interacting changing fields is a wave of electric and magnetic fields that can actually propagate (travel) through space

Production of EM Waves

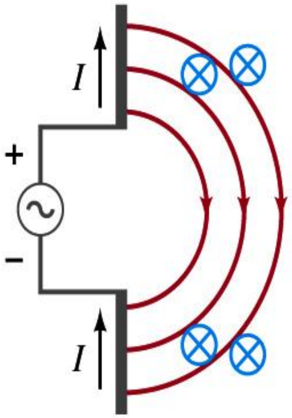
- Consider two conducting rods connected to a DC power source
  - What do you think will happen when the switch is closed?
    - The rod connected to the positive terminal acquires a positive charge and the other a negative one
    - Then an electric field will be generated between the two rods
    - Since there is current that flows through the rods, a magnetic field around them will be generated

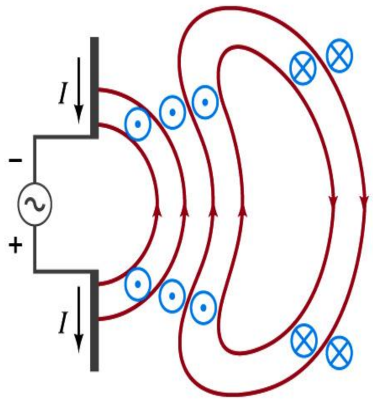


- How far would the electric and magnetic fields extend?
  - In the static case, the field extends indefinitely
  - When the switch is closed, the fields are formed near the rods quickly but the stored energy in the fields won't propagate w/ infinite speed

Production of EM Waves

- What happens if the antenna is connected to an ac power source?
  - When the connection is initially made, the rods are charging up quickly w/ the current flowing in one direction as shown in the figure
    - The field lines form as in the dc case
    - The field lines propagate away from the antenna
  - Then the direction of the voltage reverses
    - New field lines in the opposite direction forms
    - While the original field lines still propagate farther away from the rod
      - Since the original field propagates through empty space, the field lines must form a closed loop (no charge exist)
    - Since changing electric and magnetic fields produce changing magnetic and electric fields, the fields moving outward are self-supporting and do not need antenna with flowing charge
  - The field far from the antenna is called the **radiation field**
  - Both electric and magnetic fields form closed loops perpendicular to each other





Properties of Radiation Fields

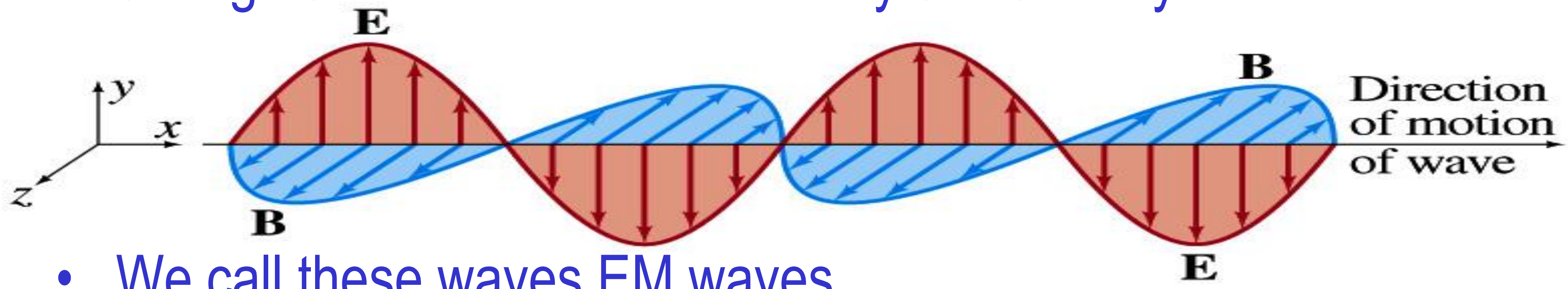
- The fields are propogated throughout all space on both sides of the antenna
- The field strengths are greatest in the direction perpendicular to the oscillating charge while along the parallel direction the fields are zero
- The magnitudes of **E** and **B** in the radiation field decrease with distance ~1/r
- The energy carried by the EM wave is proportional to the square of the amplitude, E^2 or B^2
  - So the intensity of wave decreases as $1/r^2$

Properties of Radiation Fields

- The electric and magnetic fields at any point are perpendicular to each other and to the direction of motion
- The fields alternate in direction
  - The field strengths vary from maximum in one direction, to 0 and to maximum in the opposite direction
- The electric and magnetic fields are in phase
- Very far from the antenna, the field lines are pretty flat over a reasonably large area
  - Called plane waves

EM Waves

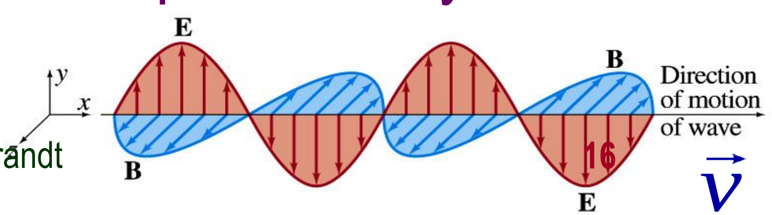
- If the voltage of the source varies sinusoidally, the field strengths of the radiation field vary sinusoidally



- We call these waves EM waves
- They are transverse waves
- EM waves are always waves of fields
  - Since these are fields, they can propagate through empty space
- In general accelerating electric charges give rise to electromagnetic waves
- This prediction from Maxwell's equations was experimentally proven (posthumously) by Heinrich Hertz through the discovery of radio waves

EM Waves and Their Speeds

- Let's consider a region of free space. What's a free space?
  - An area of space where there are no charges or conduction currents
  - In other words, far from emf sources so that the wave fronts are essentially flat or not distorted over a reasonable area
  - What are these flat waves called?
    - Plane waves
    - At any instance **E** and **B** are uniform over a large plane perpendicular to the direction of propagation
  - So we can also assume that the wave is traveling in the x-direction w/ velocity, **v**=v**i**, and that **E** is parallel to y axis and **B** is parallel to z axis `



Maxwell's Equations in free space

- In a region of free space with Q=0 and I=0, Maxwell's four equations become simpler

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} \quad \xrightarrow{Q_{encl}=0} \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\oint \vec{B}\cdot d\vec{A} = 0 \quad \xrightarrow{\text{No Change}} \quad \oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \quad \xrightarrow{\text{No Change}} \quad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \xrightarrow{I_{encl}=0} \quad \oint \vec{B}\cdot d\vec{l} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt}$$

One can observe the symmetry between electricity and magnetism.

The last equation is the most important one for EM waves and their propagation!!

EM Waves from Maxwell's Equations

- If the wave is sinusoidal w/ wavelength λ and frequency *f*, this traveling wave can be written as

$$E = E_y = E_0 \sin \;\; kx - \omega t$$

$$B = B_z = B_0 \sin \;\; kx - \omega t$$

  – Where

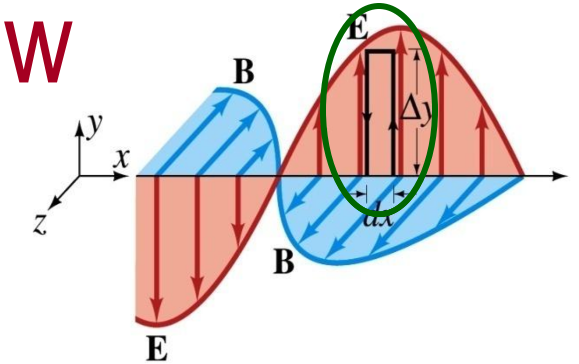
$$k = \frac{2\pi}{\lambda} \qquad \varpi = 2\pi f$$

Thus

$$f\lambda = \frac{\omega}{k} = v$$

  – What is v?
    - It is the speed of the traveling wave
  – What are E_0 and B_0?
    - The amplitudes of the EM wave. Maximum values of **E** and **B** field strengths.

From Faraday's Law



- Let's apply Faraday's law

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

  – to the rectangular loop of height Δy and width dx

- $\vec{E}\cdot d\vec{l}$ along the top and bottom of the loop is 0. Why?
  – Since **E** is perpendicular to d*l*.
  – So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{E}\cdot d\vec{l} = \vec{E}\cdot d\vec{x} + \; \vec{E} + d\vec{E} \; \cdot \Delta\vec{y} + \vec{E}\cdot d\vec{x}' + \; \vec{E}\cdot\Delta\vec{y}' =$$

$$= 0 \; + \; E + dE \;\; \Delta y \; - 0 \; - E\Delta y = dE\Delta y$$

  – For the right-hand side of Faraday's law, the magnetic flux through the loop changes as

$$-\frac{d\Phi_B}{dt} = \frac{dB}{dt}dx\Delta y$$

Thus

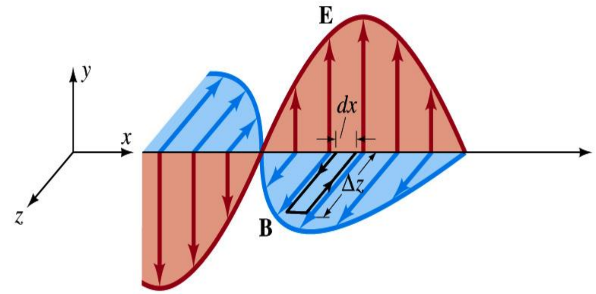
$$dE\Delta y = -\frac{dB}{dt}dx\Delta y$$

$$\frac{dE}{dx} = -\frac{dB}{dt}$$

Since E and B depend on x and t

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

From the Modified Ampére's Law



- Let's apply Maxwell's 4th equation

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

  – to the rectangular loop of length Δz and width dx

- $\vec{B}\cdot d\vec{l}$ along the x-axis of the loop is 0
  – Since **B** is perpendicular to dℓ
  – So the result of the integral through the loop counterclockwise becomes

$$\oint \vec{B}\cdot d\vec{l} = B\Delta Z - \ B + dB \ \ \Delta Z = -dB\Delta Z$$

  – For the right-hand side of the equation is

$$\mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 \varepsilon_0 \frac{dE}{dt} dx\Delta z$$

Thus

$$-dB\Delta z = \mu_0 \varepsilon_0 \frac{dE}{dt} dx\Delta z$$

  – $$\frac{dB}{dx} = -\mu_0 \varepsilon_0 \frac{dE}{dt}$$

Since E and B depend on x and t

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Relationship between **E**, **B** and **v**

- Let's now use the relationship from Faraday's law $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$
- Taking the derivatives of E and B as given by their traveling wave form, we obtain

$$\frac{\partial E}{\partial x}=\frac{\partial}{\partial x}\left(E_0\sin\left(kx-\omega t\right)\right)=kE_0\cos\left(kx-\omega t\right)$$

$$\frac{\partial B}{\partial t}=\frac{\partial}{\partial t}\left(B_0\sin\left(kx-\omega t\right)\right)=-\omega B_0\cos\left(kx-\omega t\right)$$

Since $\frac{\partial E}{\partial x}=-\frac{\partial B}{\partial t}$ **We obtain** $kE_0\cos\left(kx-\omega t\right)=\omega B_0\cos\left(kx-\omega t\right)$

Thus $\frac{E_0}{B_0}=\frac{\omega}{k}=v$

– Since E and B are in phase, we can write $E/B=v$
  - This is valid at any point and time in space. What is v?
    – The velocity of the wave

Speed of EM Waves

- Let's now use the relationship from Apmere's law $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$
- Taking the derivatives of E and B as given their traveling wave form, we obtain

$$\frac{\partial B}{\partial x} = \frac{\partial}{\partial x}\left(B_0 \sin(kx - \omega t)\right) = kB_0 \cos(kx - \omega t)$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}\left(E_0 \sin(kx - \omega t)\right) = -\omega E_0 \cos(kx - \omega t)$$

Since $\frac{\partial B}{\partial x} = -\varepsilon_0 \mu_0 \frac{\partial E}{\partial t}$ **We obtain** $kB_0 \cos(kx - \omega t) = \varepsilon_0 \mu_0 \omega E_0 \cos(kx - \omega t)$

Thus $\frac{B_0}{E_0} = \frac{\varepsilon_0 \mu_0 \omega}{k} = \varepsilon_0 \mu_0 v$

– However, from the previous page we obtain $E_0/B_0 = v = \frac{1}{\varepsilon_0 \mu_0 v}$

– Thus $v^2 = \frac{1}{\varepsilon_0 \mu_0}$

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \frac{1}{\sqrt{\left(8.85 \times 10^{-12}\, C^2/N \cdot m^2\right) \cdot \left(4\pi \times 10^{-7}\, T \cdot m/A\right)}} = 3.00 \times 10^8\, m/s$$

The speed of EM waves is the same as the speed of light! EM waves behaves like light!

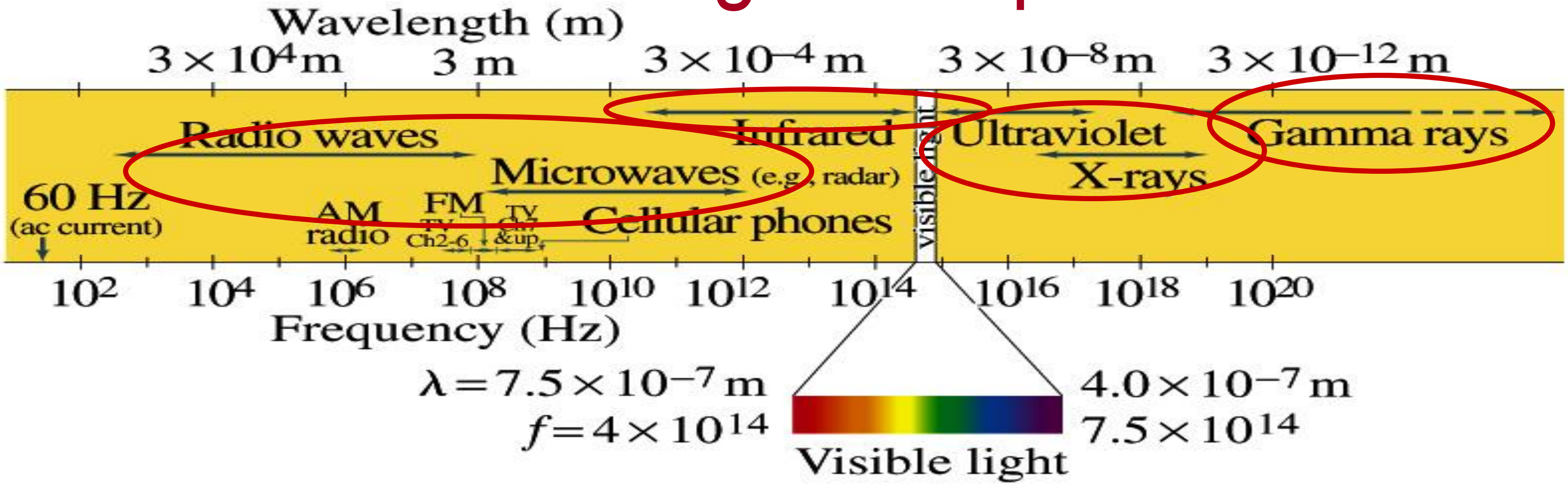
Light as EM Wave

- People knew some 60 years before Maxwell that light behaves like a wave, but …
  - They did not know what kind of waves they are.
    - Most importantly what is it that oscillates in light?
- Heinrich Hertz first generated and detected EM waves experimentally in 1887 using a spark gap apparatus
  - Charge was rushed back and forth in a short period of time, generating waves with frequency about 10^9Hz (these are called radio waves)
  - He detected using a loop of wire in which an emf was produced when a changing magnetic field passed through
  - These waves were later shown to travel at the speed of light

Light as EM Wave

- The wavelengths of visible light were measured in the first decade of the 19th century
  - The visible light wave length were found to be between 4.0×10^{-7}m (400nm) and 7.5×10^{-7}m (750nm)
  - The frequency of visible light is $f\lambda = c$
    - Where f and λ are the frequency and the wavelength of the wave
      - What is the range of visible light frequency?
      - 4.0×10^{14}Hz to 7.5×10^{14}Hz
    - c is 3×10^{8}m/s, the speed of light
- EM Waves, or EM radiation, are categorized using EM spectrum

Electromagnetic Spectrum



- Low frequency waves, such as radio waves or microwaves can be easily produced using electronic devices
- Higher frequency waves are produced in natural processes, such as emission from atoms, molecules or nuclei
- Or they can be produced from acceleration of charged particles
- Infrared radiation (IR) is mainly responsible for the heating effect of the Sun
  - The Sun emits visible lights, IR and UV
    - The molecules of our skin resonate at infrared frequencies so IR is preferentially absorbed and thus creates warmth

Example 32 – 2

Wavelength of EM waves. Calculate the wavelength (a) of a 60-Hz EM wave, (b) of a 93.3-MHz FM radio wave and (c) of a beam of visible red light from a laser at frequency 4.74×10^{14}Hz.

What is the relationship between speed of light, frequency and the wavelength? $c = f\lambda$

Thus, we obtain $\lambda = \frac{c}{f}$

For f=60Hz $\lambda = \frac{3 \times 10^8 \; m/s}{60 s^{-1}} = 5 \times 10^6 \, m$

For f=93.3MHz $\lambda = \frac{3 \times 10^8 \; m/s}{93.3 \times 10^6 \, s^{-1}} = 3.22 m$

For f=4.74×10^{14}Hz $\lambda = \frac{3 \times 10^8 \; m/s}{4.74 \times 10^{14} \, s^{-1}} = 6.33 \times 10^{-7} \, m$