PHYS 1444 – Section 003
Lecture #3

Tuesday August 30, 2012

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Chapter 21

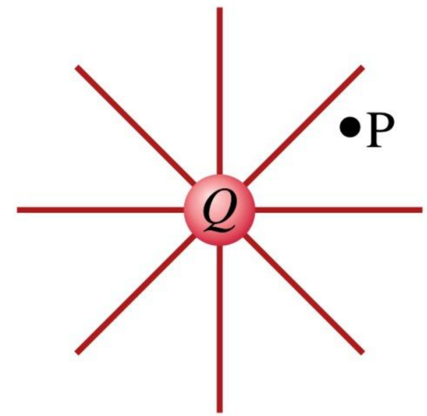
--Coulomb's Law

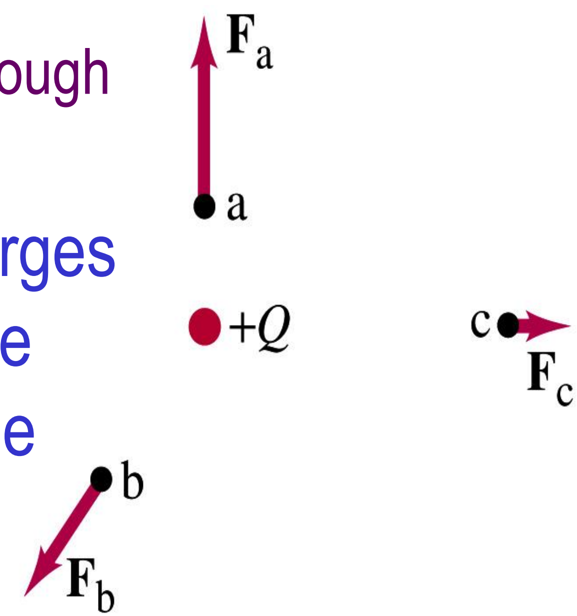
--Electric Field

--Electric Dipole

The Electric Field

- Both gravitational and electric forces act over a distance without touching objects ➔ What kind of forces are these?
  - Field forces
- Michael Faraday developed the idea of a field.
  - Faraday argued that the electric field extends outward from every charge and permeates through all space.
- The field due to a charge or a group of charges can be inspected by placing a small positive test charge in the vicinity and measuring the force on it.





The Electric Field

- The electric field at any point in space is defined as the force exerted on a tiny positive test charge divided by magnitude of the test charge
  - Electric force per unit charge

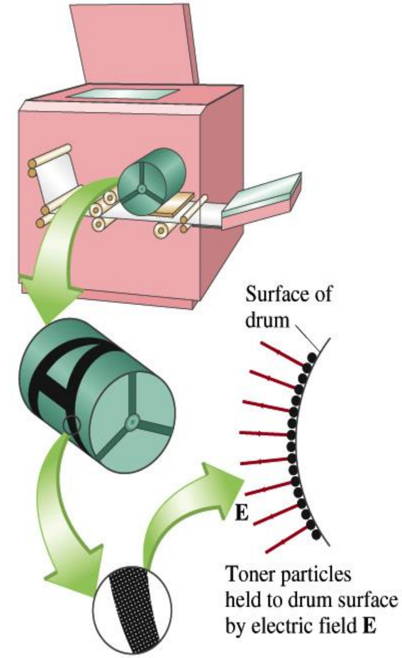
$$\vec{E} = \frac{\vec{F}}{q}$$

- What kind of quantity is the electric field?
  - Vector quantity. Why?
- What is the unit of the electric field?
  - N/C
- What is the magnitude of the electric field at a distance r from a single point charge Q?

$$E = \frac{F}{q} = \frac{kQq/r^2}{q} = \frac{kQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

Example 21 – 5

- **Electrostatic copier**. An electrostatic copier works by selectively arranging positive charges (in a pattern to be copied) on the surface of a nonconducting drum, then gently sprinkling negatively charged dry toner (ink) onto the drum. The toner particles temporarily stick to the pattern on the drum and are later transferred to paper and "melted" to produce the copy. Suppose each toner particle has a mass of 9.0×10^{-16}kg and carries an average of 20 extra electrons to provide an electric charge. Assuming that the electric force on a toner particle must exceed twice its weight in order to ensure sufficient attraction, compute the required electric field strength near the surface of the drum.



The electric force must be the same as twice the gravitational force on the toner particle.

So we can write $F_e = qE = 2F_g = 2mg$

Thus, the magnitude of the electric field is

$$E = \frac{2mg}{q} = \frac{2\cdot\left(9.0\times10^{-16}\,kg\right)\cdot\left(9.8\,m/s^2\right)}{20\left(1.6\times10^{-19}\,C\right)} = 5.5\times10^{3}\,N/C.$$

Direction of the Electric Field

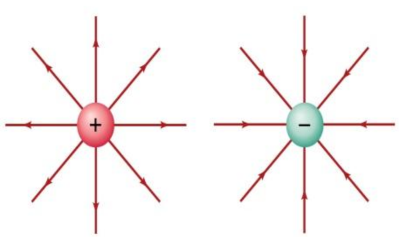
- If there are several charges, the individual fields due to each charge are added vectorially to obtain the total field at any point.

$$\vec{E}_{Tot} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 + \ldots$$

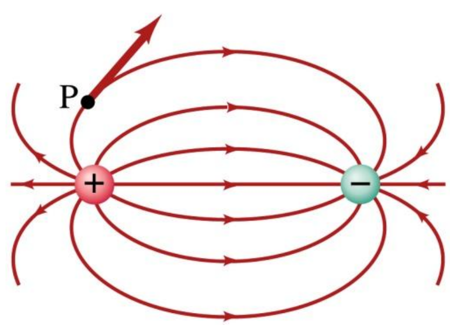
- This superposition principle of electric field has been verified experimentally
- For a given electric field **E** at a given point in space, we can calculate the force **F** on any charge q, **F**=q**E**.
  - How does the direction of the force and the field depend on the sign of the charge q?
  - The two are in the same directions if $q>0$
  - The two are in opposite directions if $q<0$

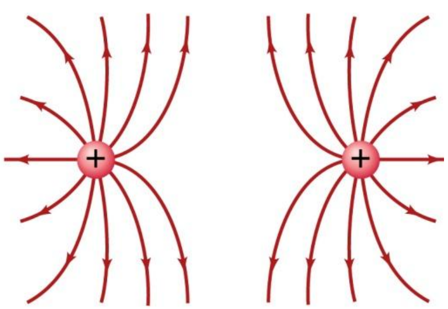
Field Lines

- The electric field is a vector quantity. Thus, its magnitude can be represented by the length of the vector, with the arrowhead indicating the direction.
- Electric field lines are drawn to indicate the direction of the force due to the given field on a positive test charge.
  - Number of lines crossing a unit area perpendicular to E is proportional to the magnitude of the electric field.
  - The closer the lines are together, the stronger the electric field in that region.
  - Start on positive charges and end on negative charges.

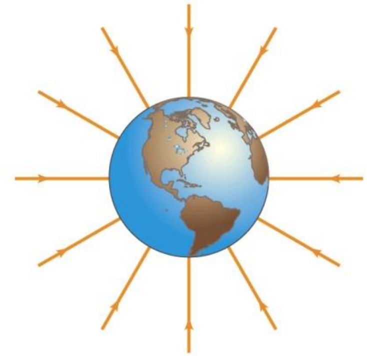


(a) (b)





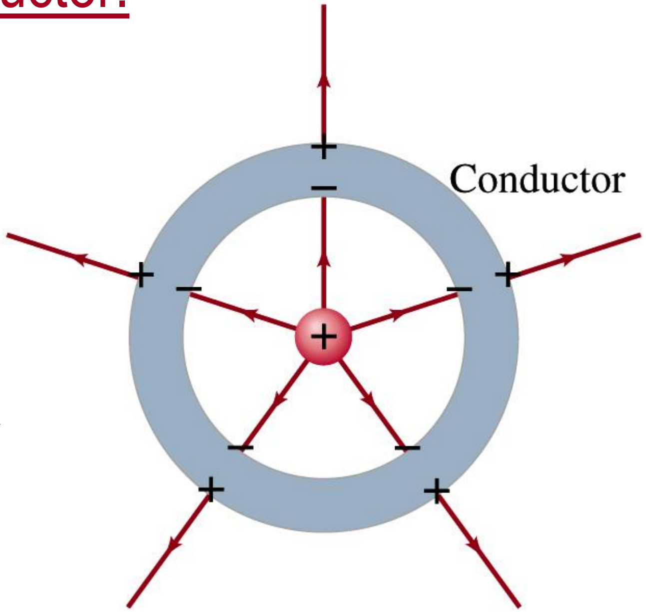
Earth's G-field lines



Electric Fields and Conductors

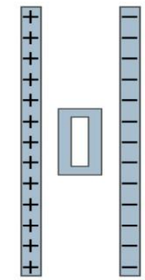
- The electric field inside a conductor is ZERO in a static situation (charge is at rest) Why?
  - If there were an electric field within a conductor, there would be a force on its free electrons.
  - The electrons would move until they reach positions where the electric field become zero.
  - Electric field can exist inside a non-conductor.

- Consequences of the above
  - Any net charge on a conductor distributes itself on the surface.
  - Although no field exists inside (the material of) a conductor, fields can exist outside the conductor due to induced charges on either surface
  - The electric field is always perpendicular to the outside surface of a conductor.

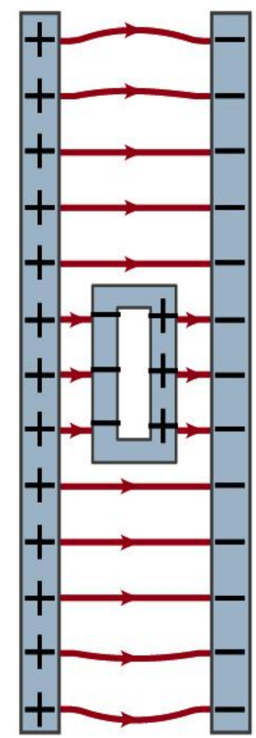


Example 21-13

- **Shielding, and safety in a storm.** A hollow metal box is placed between two parallel charged plates. What is the field in the box?



- If the metal box were solid
  - The free electrons in the box would redistribute themselves along the surface (the field lines would not penetrate into the metal).
- The free electrons do the same in hollow metal boxes just as well as for solid metal boxes.
- Thus a conducting box is an effective device for shielding. ➔ Faraday cage
- So what do you think will happen if you were inside a car when the car was struck by lightning?



Example 21 – 14

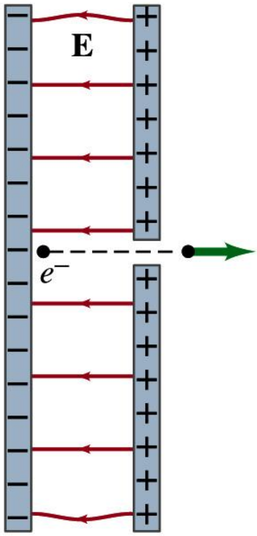
- **Electron accelerated by electric field**. An electron (mass m = 9.1×10^{-31} kg) is accelerated in a uniform field E (E = 2.0×10^{4} N/C) between two parallel charged plates.

  The separation of the plates is 1.5 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate.

  (a) With what speed does it leave the hole?

  (b) Can the gravitational force can be ignored?

  Assume the hole is so small that it does not affect the uniform field between the plates.



The magnitude of the force on the electron is F=qE and is directed to the right. The equation to solve this problem is

$$F = qE = ma$$

The magnitude of the electron's acceleration is $a = \frac{F}{m} = \frac{qE}{m}$

Between the plates the field **E** is uniform, thus the electron undergoes a uniform acceleration

$$a = \frac{eE}{m_e} = \frac{1.6\times10^{-19}C \quad 2.0\times10^{4}N/C}{9.1\times10^{-31}kg} = 3.5\times10^{15}\,m/s^2$$

Example 21 – 14

Since the travel distance is 1.5×10^{-2}m, using one of the kinetic eq. of motions,

$$v^2 = v_0^2 + 2ax \quad \therefore v = \sqrt{2ax} = \sqrt{2\cdot 3.5\times10^{15}\cdot 1.5\times10^{-2}} = 1.0\times10^7\ m/s$$

Since there is no electric field outside the conductor, the electron continues moving with this speed after passing through the hole.

(b) Can the gravitational force can be ignored? Assume the hole is so small that it does not affect the uniform field between the plates.

The magnitude of the electric force on the electron is

$$F_e = qE = eE = \left(1.6\times10^{-19}C\right)\left(2.0\times10^4 N/C\right) = 3.2\times10^{-15} N$$

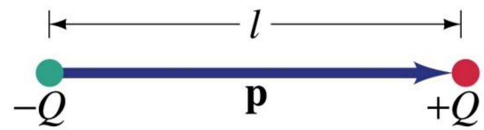
The magnitude of the gravitational force on the electron is

$$F_G = mg = 9.8\,m/s^2 \times \left(9.1\times10^{-31}kg\right) = 8.9\times10^{-30} N$$

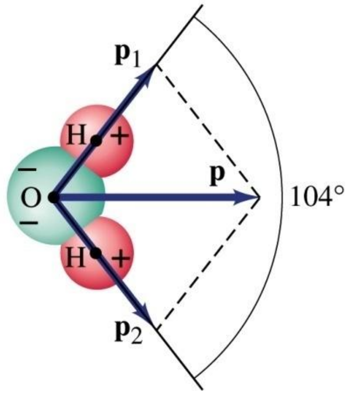
Thus the gravitational force on the electron is negligible compared to the electromagnetic force.

Electric Dipoles

- An electric dipole is the combination of two equal charges of opposite sign, +Q and –Q, separated by a distance ℓ, which behaves as one entity.
- The quantity $Q\ell$ is called the electric dipole moment and is represented by the symbol p.

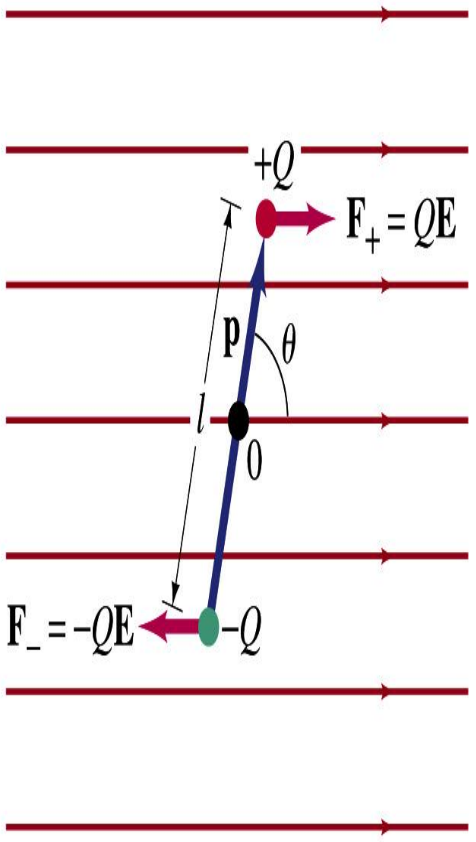


  - The dipole moment is a vector quantity
  - The magnitude of the dipole moment is $Q\ell$. Unit? **C-m**
  - Its direction is from the negative to the positive charge.
  - Many of diatomic molecules like CO have a dipole moment. ➔ These are referred as polar molecules.
    - Symmetric diatomic molecules, such as O_2, do not have dipole moment.
  - The water molecule also has a dipole moment which is the vector sum of two dipole moments between Oxygen and each of Hydrogen atoms.

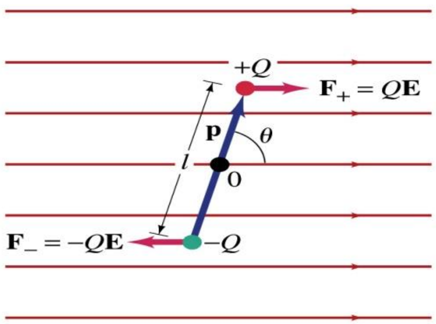


Dipoles in an External Field

- Let's consider a dipole placed in a uniform electric field **E**.
  - What do you think will happen to the dipole in the figure?
    - Forces will be exerted on the charges.
      - The positive charge will get pushed toward right while the negative charge will get pulled toward left.
    - What is the net force acting on the dipole?
      - Zero
    - So will the dipole move?
      - Yes, it will.
    - Why?
      - There is torque applied on the dipole.



Dipoles in an External Field, cnt'd



- How much is the torque on the dipole?
  - Do you remember the formula for torque?
    - $\vec{\tau} = \vec{r} \times \vec{F}$ I thought you could!
  - The magnitude of the torque exerting on each of the charges with respect to the rotational axis at the center is
    - $\tau_{+Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(\frac{l}{2}\right) QE \sin\theta\right| = \frac{l}{2}QE\sin\theta$
    - $\tau_{-Q} = \left|\vec{r} \times \vec{F}\right| = rF\sin\theta = \left|\left(-\frac{l}{2}\right) -QE \sin\theta\right| = \frac{l}{2}QE\sin\theta$
  - Thus, the total torque is
    - $\tau_{Total} = \tau_{+Q} + \tau_{-Q} = \frac{l}{2}QE\sin\theta + \frac{l}{2}QE\sin\theta = lQE\sin\theta = pE\sin\theta$
  - So the torque on a dipole in vector notation is $\vec{\tau} = \vec{p} \times \vec{E}$
- The effect of the torque is to try to turn the dipole so that the dipole moment is parallel to **E**.

Potential Energy of a Dipole in an External Field

- What is the work done on the dipole by the electric field to change the angle from θ_1 to θ_2?

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_1}^{\theta_2} -\tau \, d\theta$$

Why negative?

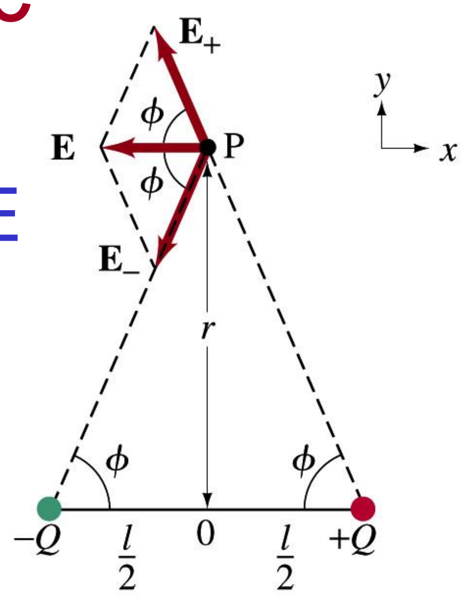
Because τ and θ are opposite directions to each other.

- The torque is $\tau = pE\sin\theta$.
- Thus the work done on the dipole by the field is

$$W = \int_{\theta_1}^{\theta_2} -pE\sin\theta \, d\theta = pE \left[\cos\theta\right]_{\theta_1}^{\theta_2} = pE\left(\cos\theta_2 - \cos\theta_1\right)$$

- What happens to the dipole's potential energy, U, when positive work is done on it by the field?
  - It decreases.
- If we choose U=0 when θ_1=90 degrees, then the potential energy at θ_2=θ becomes $U = -W = -pE\cos\theta = -\vec{p} \cdot \vec{E}$

Electric Field from a Dipole



- Let's consider the case in the picture.
- There are fields due to both charges. The total E field from the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{r^2 + \left(l/2\right)^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2 + \left(l/2\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2 + l^2/4}$$

- Now we must work out the x and y components of the total field.
  - Sum of the two y components is
    - Zero since they are the same but in opposite direction
  - So the magnitude of the total field is the same as the sum of the two x-components:

$$E = 2E_{+}\cos\phi = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r^2 + l^2/4}\frac{l}{2\sqrt{r^2 + l^2/4}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2 + l^2/4\right)^{3/2}}$$

Example 21 – 16

- **Dipole in a field**. The dipole moment of a water molecule is 6.1×10^{-30}C-m. A water molecule is placed in a uniform electric field with magnitude 2.0×10^{5}N/C.

  (a) What is the magnitude of the maximum torque that the field can exert on the molecule?

  (b) What is the potential energy when the torque is at its maximum?

  (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when θ=90 degrees. Thus the magnitude of the maximum torque is

$$\tau = pE\sin\theta = pE =$$

$$= 6.1\times10^{-30}\,C\cdot m \quad 2.5\times10^{5}\,N/C \;= 1.2\times10^{-24}\,N\cdot m$$

Example 21 – 16

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

And τ is at its maximum at θ=90 degrees, the potential energy, U, is

$$U = -pE\cos\theta = -pE\cos\ 90^\circ\ = 0$$

Is the potential energy at its minimum at θ=90 degrees? No

Why not? Because U will become negative as θ increases.

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when cosθ= -1, θ=180 degrees.

Why is this different than the position where the torque is maximized?

The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field, to reach the equilibrium position at θ=0.

Torque is maximized when the field is perpendicular to the dipole, θ=90.

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $F = ma$	Torque $\tau = I\alpha$
Work	Work $W = Fd\cos\theta$	Work $W = \tau\theta$
Power	$P = \vec{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$