PHYS 1444 – Section 003
Lecture #4

Tuesday Sep. 4, 2012

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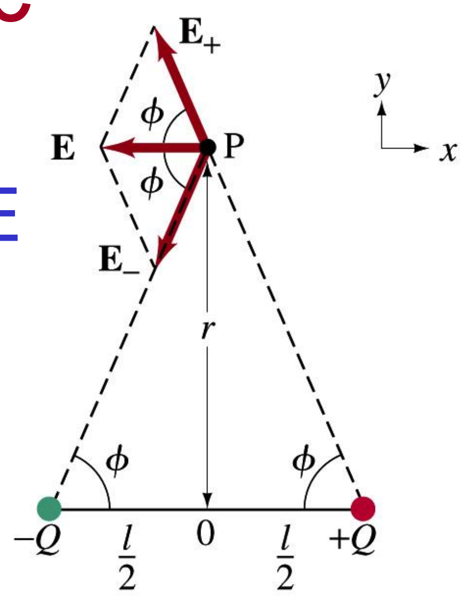
- Chapter 21: Dipoles
- Chapter 22:
  - Electric Flux
  - Gauss' Law

Electric Field from a Dipole



- Let's consider the case in the picture.
- There are fields due to both charges. The total E field from the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

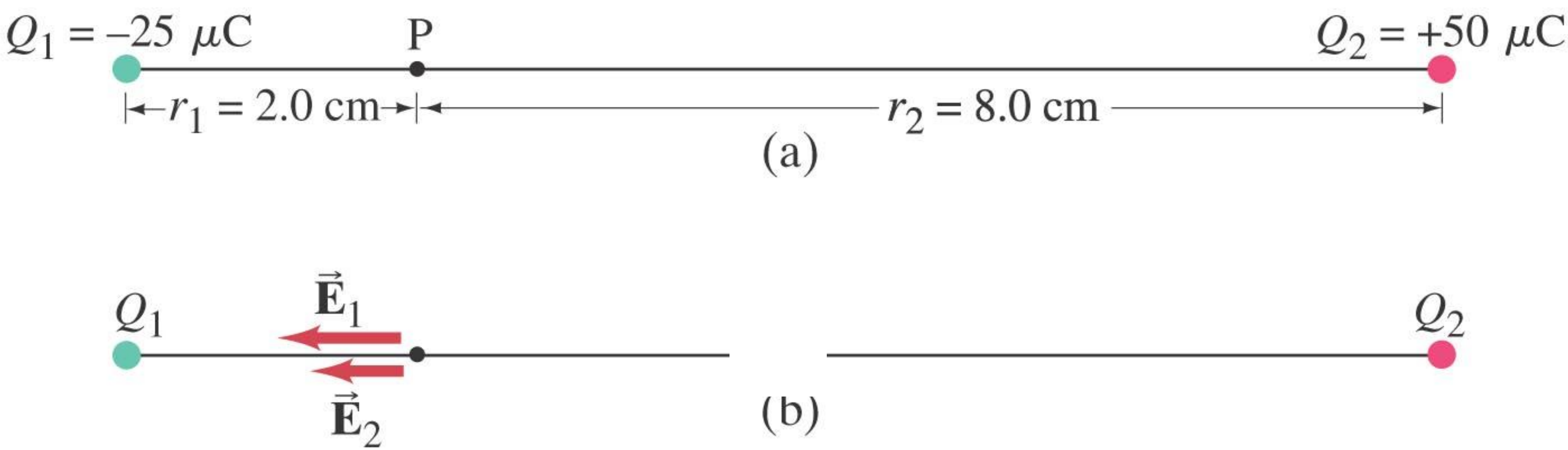
$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{r^2 + (l/2)^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2 + (l/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2 + l^2/4}$$

- Now we must work out the x and y components of the total field.
  - Sum of the two y components is
    - Zero since they are the same but in opposite direction
  - So the magnitude of the total field is the same as the sum of the two x-components:

$$E = 2E_+\cos\phi = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r^2 + l^2/4}\frac{l}{2\sqrt{r^2 + l^2/4}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left(r^2 + l^2/4\right)^{3/2}}$$

Example 21-7

Two point charges are separated by a distance of 10.0 cm. One has a charge of -25 μC and the other +50 μC. (a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge. (b) If an electron (mass = 9.11 x 10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?



Solution: a. The electric fields add in magnitude, as both are directed towards the negative charge. E = 6.3 x 10^8 N/C. b. We don't know the relative lengths of E_1 and E_2 until we do the calculation. The acceleration is the force (charge times field) divided by the mass, and will be opposite to the direction of the field (due to the negative charge of the electron). Substitution gives a = 1.1 x 10^{20} m/s^2.

21-7 Electric Field Calculations for Continuous Charge Distributions

A continuous distribution of charge may be treated as a succession of infinitesimal (point) charges. The total field is then the integral of the infinitesimal fields due to each bit of charge:

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{r^2}.$$

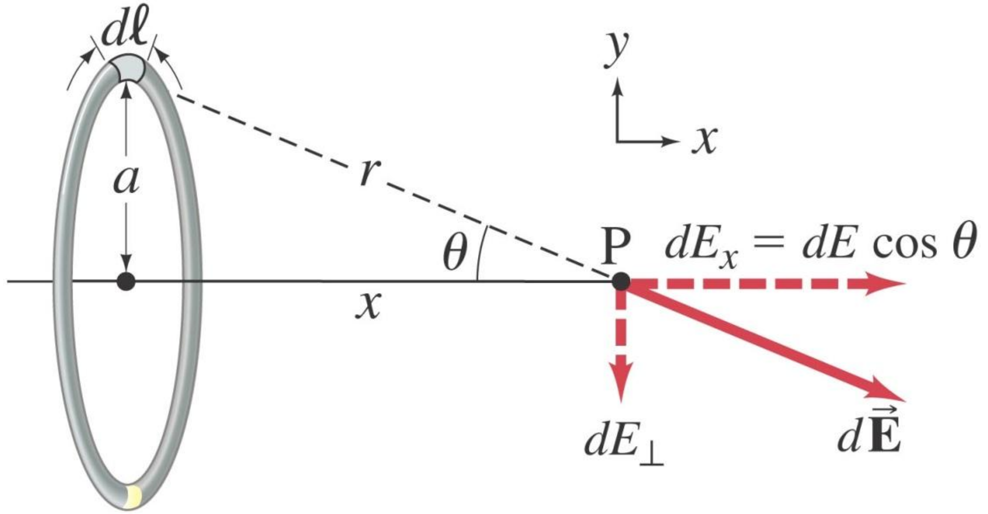
$$\vec{\mathbf{E}} = \int d\vec{\mathbf{E}}.$$

Remember that the electric field is a vector; you will need a separate integral for each component.

21-7 Electric Field for Continuous Charge Distributions

Example 21-9: A ring of charge.

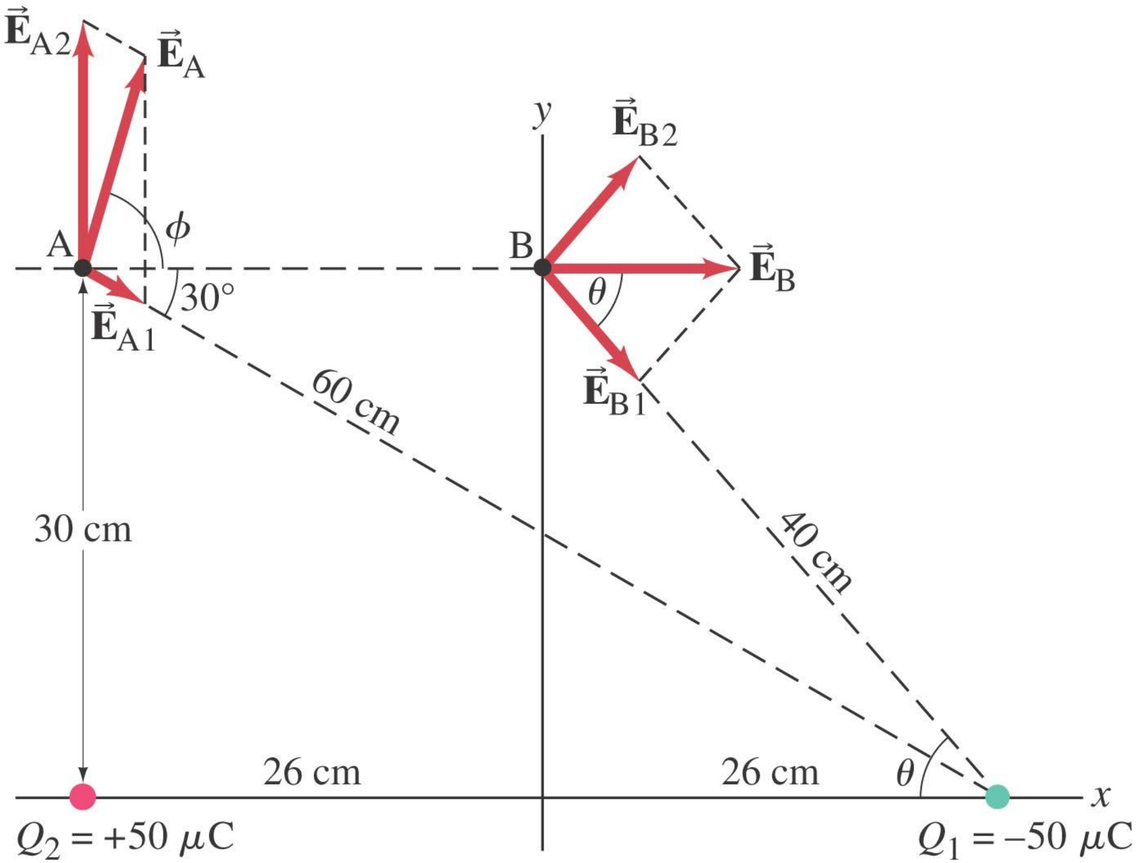
A thin, ring-shaped object of radius *a* holds a total charge +*Q* distributed uniformly around it. Determine the electric field at a point P on its axis, a distance *x* from the center. Let λ be the charge per unit length (C/m).



Solution: Because P is on the axis, the transverse components of E must add to zero, by symmetry. The longitudinal component of dE is dE cos θ, where $\cos\theta = x/(x^2 + a^2)^{1/2}$. Write $dQ = \lambda dl$, and integrate dl from 0 to $2\pi a$. Answer: $E = (1/4\pi\varepsilon_0)(Qx/[x^2 + a^2]^{3/2})$

Example 21-8

Calculate the total electric field (a) at point A and (b) at point B in the figure due to both charges, Q_1 and Q_2.



Solution: The geometry is shown in the figure. For each point, the process is: calculate the magnitude of the electric field due to each charge; calculate the x and y components of each field; add the components; recombine to give the total field.

a. E = 4.5 x 10^6 N/C, 76 above the x axis.

b. E = 3.6 x 10^6 N/C, along the x axis.

Example 21 – 16

- **Dipole in a field**. The dipole moment of a water molecule is 6.1×10^{-30} C-m. A water molecule is placed in a uniform electric field with magnitude 2.0×10^{5} N/C.
  (a) What is the magnitude of the maximum torque that the field can exert on the molecule?
  (b) What is the potential energy when the torque is at its maximum?
  (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when θ=90 degrees. Thus the magnitude of the maximum torque is

$$\tau = pE\sin\theta = pE =$$

$$= \left(6.1\times10^{-30}\,C\cdot m\right)\left(2.5\times10^{5}\,N/C\right) = 1.2\times10^{-24}\,N\cdot m$$

Example 21 – 16

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p}\cdot\vec{E} = -pE\cos\theta$

And τ is at its maximum at θ=90 degrees, the potential energy, U, is

$$U = -pE\cos\theta = -pE\cos\ 90^\circ\ = 0$$

Is the potential energy at its minimum at θ=90 degrees? No

Why not? Because U will become negative as θ increases.

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when cosθ= -1, θ=180 degrees.

Why is this different than the position where the torque is maximized?

The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field, to reach the equilibrium position at θ=0.

Torque is maximized when the field is perpendicular to the dipole, θ=90.

Similarity Between Linear and Rotational Motions

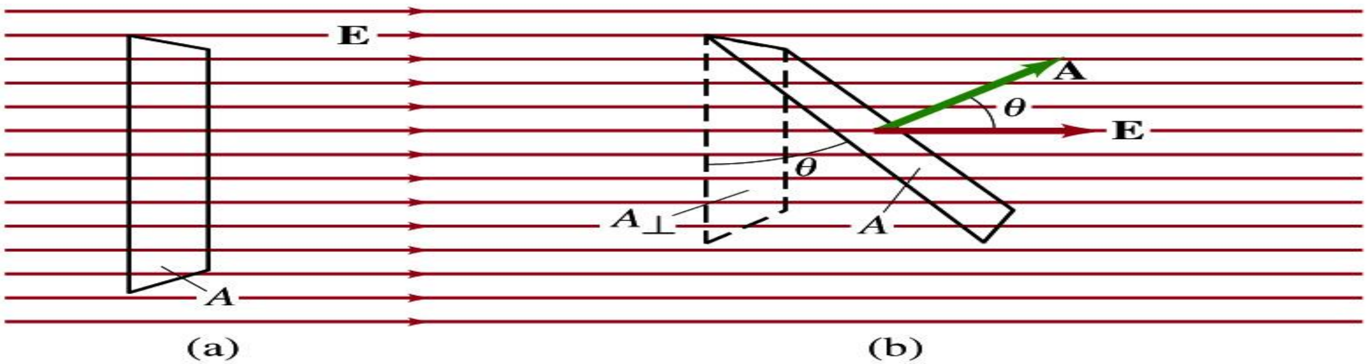
All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $F = ma$	Torque $\tau = I\alpha$
Work	Work $W = Fd\cos\theta$	Work $W = \tau\theta$
Power	$P = \vec{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$

Gauss' Law

- Gauss' law states the relationship between electric charge and electric field.
  - More general and elegant form of Coulomb's law.
- The electric field from a distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field

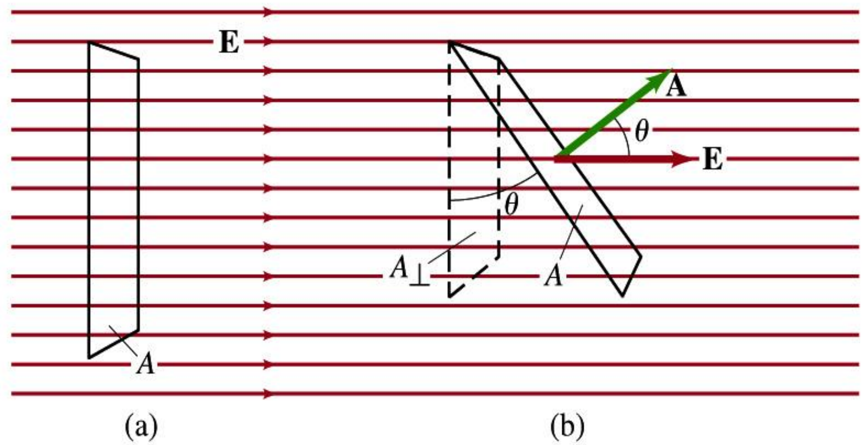
Electric Flux



- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux is defined as
  - Φ_E=EA, if the field is perpendicular to the surface
  - Φ_E=EAcosθ, if the field makes an angle θ with the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
  - Total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_\perp = \Phi_E$

Example 22 – 1

- **Electric flux**. (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux if the angle is 30 degrees?



The electric flux is

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$$

So when (a) θ=0, we obtain

$$\Phi_E = EA\cos\theta = EA = 200N/C \cdot 0.1 \times 0.2\mathrm{m}^2 = 4.0\,\mathrm{N}\cdot\mathrm{m}^2/C$$

And when (b) θ=30 degrees (1,2,√3) we obtain

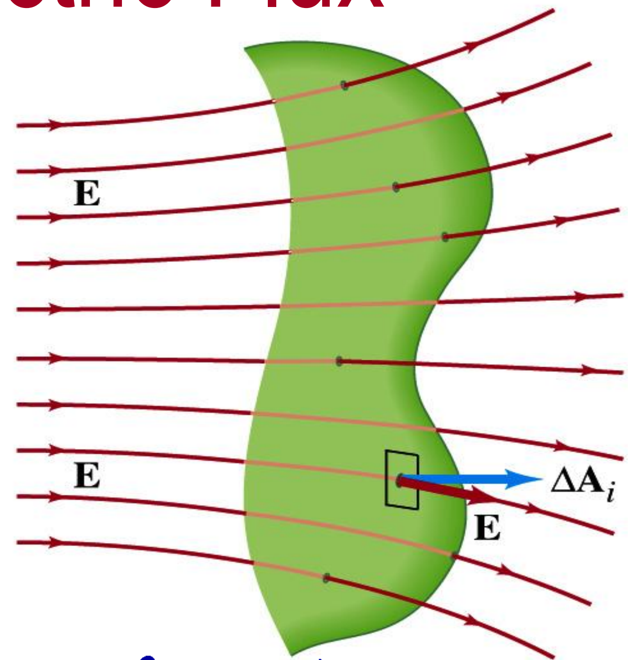
$$\Phi_E = EA\cos 30^\circ = 200N/C \cdot 0.1 \times 0.2\mathrm{m}^2 \cos 30^\circ = 3.5\,\mathrm{N}\cdot\mathrm{m}^2/C$$

Generalization of the Electric Flux

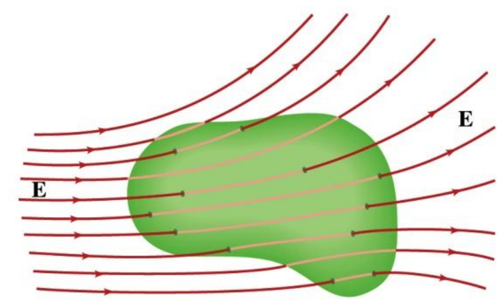
- Let's consider a surface of area A that has an irregular shape, and furthermore, that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta \mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.
- Then the electric flux through the entire surface is

$$\Phi_E \approx \sum_{i=1}^{n} \vec{E}_i \cdot \Delta \vec{A}_i$$

- In the limit where $\Delta \mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.



$$\Phi_E = \int \vec{E}_i \cdot d\vec{A}$$ open surface



$$\Phi_E = \oint E_i \cdot dA$$ closed surface