PHYS 1444 – Section 003
Lecture #7

Tuesday Sept. 18, 2012
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- Chapter 23:
  - Electric Potential due to Point Charges
  - Shape of the Electric Potential
  - V due to Charge Distributions
  - Equi-potential Lines and Surfaces
  - Electric Potential Due to Electric Dipole
  - E determined from V

The Plan

- HW on Ch 23 and 24 due Tues 9/25 at 9pm, available this afternoon
- I will lecture again on Thursday
- Finish Ch 23 today, Ch 24 on Thursday

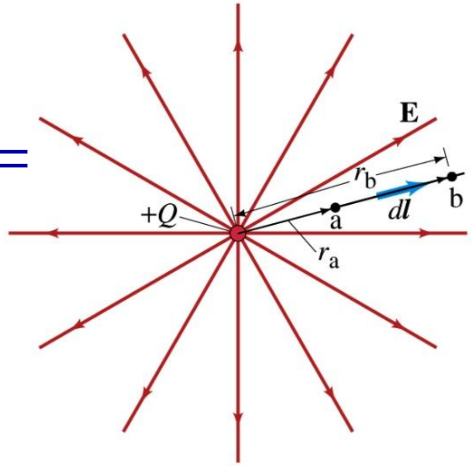
Electric Potential due to Point Charges

- What is the electric field due to a point charge Q at a distance r?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = k\frac{Q}{r^2}$$

- Electric potential due to the field E for moving from point r_a to r_b away from the charge Q is

$$V_b - V_a = -\int_{r_a}^{r_b} \vec{E}\cdot d\vec{l} = -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{\hat{r}}{r^2}\cdot\hat{r}dr =$$



$$= -\frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{1}{r^2}dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

Notice how the integral is carried out in the radial direction.

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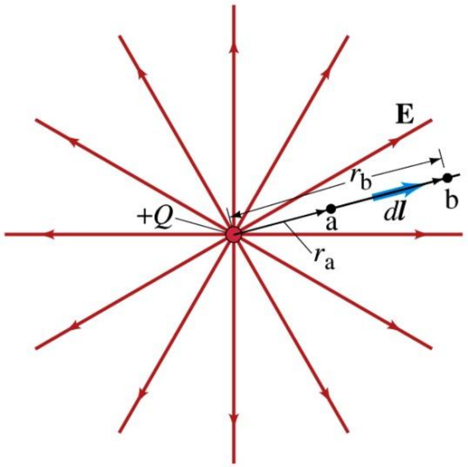
Electric Potential due to Point Charges

- Since only the differences in potential have physical meaning, we can choose $V_b = 0$ at $r_b = \infty$.
- The electrical potential V at a distance r from a single point charge is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



- So the absolute potential from a single point charge depends only on the magnitude of the point charge and the distance from it

Properties of the Electric Potential

- What are the differences between the electric potential and the electric field?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

  - Electric potential
    - Electric potential energy per unit charge
    - Inversely proportional to the distance
    - Simply add the potential from each of the charges to obtain the total potential from multiple charges, since potential is a scalar quantity
  - Electric field

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

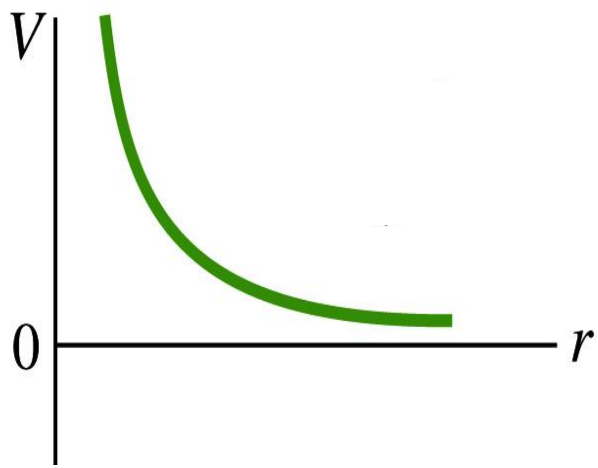
    - Electric force per unit charge
    - Inversely proportional to the square of the distance
    - Need vector sums to obtain the total field from multiple charges
- Potential for a positive charge is large near the charge and decreases to 0 at large distances.
- Potential for the negative charge is small (large magnitude but negative) near the charge and increases with distance to 0

Shape of the Electric Potential

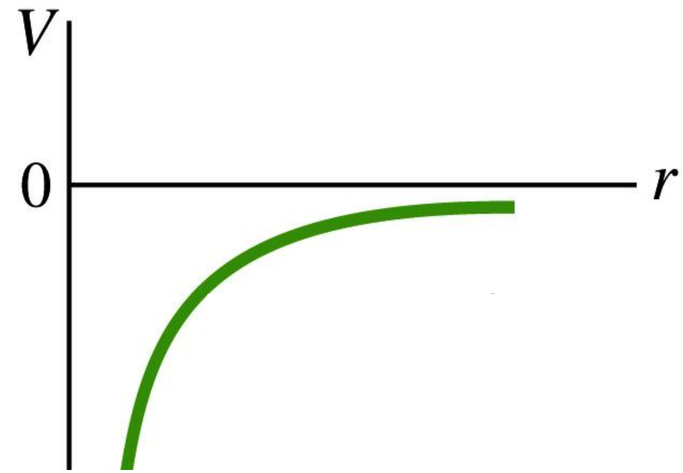
- So, how does the electric potential look as a function of distance?
  - What is the formula for the potential by a single charge?

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Positive Charge



Negative Charge



A uniformly charged sphere would have the same potential as a single point charge.

What does this mean? **Uniformly charged sphere behaves like all the charge is on the single point in the center.**

Example 23 – 6

Work to bring two positive charges close together: What minimum work is required by an external force to bring a charge q=3.00 μC from a great distance away ($\mathrm{r} = \infty$) to a point 0.500 m from a charge Q=20.0 μC?

What is the work done by the electric field in terms of potential energy and potential?

$$W = -qV_{ba} = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - \frac{Q}{r_a}\right)$$

Since $r_b = 0.500m, r_a = \infty$ we obtain

$$W = -\frac{q}{4\pi\varepsilon_0}\left(\frac{Q}{r_b} - 0\right) = -\frac{q}{4\pi\varepsilon_0}\frac{Q}{r_b} = -\frac{8.99\times10^9 N\cdot m^2/C^2 \cdot 3.00\times10^{-6}C \quad 20.00\times10^{-6}C}{0.500m} = -1.08J$$

In other words, the external force must input work of +1.08J to bring the charge 3.00μC from infinity to 0.500m from the 20.0μC charge.

Electrostatic Potential Energy: Two charges

- What is the electrostatic potential energy of a configuration of charges? (Choose V=0 at r=∞)
  - If there are no other charges around, a single point charge Q_1 in isolation has no potential energy and feels no electric force
- If a second point charge Q_2 is to a distance r_{12} from Q_1 ,the potential at the position of Q_2 is $V = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_{12}}$
- The potential energy of the two charges relative to V=0 at r= ∞ is

$$U = Q_2 V = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r_{12}}$$

  - This is the work that needs to be done by an external force to bring Q_2 from infinity to a distance r_{12} from Q_1.
  - It is also a negative of the work needed to separate them to infinity.

Electrostatic Potential Energy: Three Charges

- So what do we do for three charges?
- Work is needed to bring all three charges together
  - There is no work needed to bring Q_1 to a certain place without the presence of any other charge
  - The work needed to bring Q_2 to a distance to Q_1 is
  - The work need to bring Q_3 to a distance to Q_1 and Q_2 is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r_{12}}$$

$$U_3 = U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r_{23}}$$

- So the total electrostatic potential of the three charge system is

$$U = U_{12} + U_{13} + U_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1Q_2}{r_{12}} + \frac{Q_1Q_3}{r_{13}} + \frac{Q_2Q_3}{r_{23}}\right) \quad V = 0 \text{ at } r = \infty$$

Electric Potential from Charge Distributions

- Let's consider that there are n individual point charges in a given space and V=0 at $r = \infty$
- Then the potential due to the charge Q_i at a point a, distance r_{ia} from Q_i is

$$V_{ia} = \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- Thus the total potential V_a by all n point charges is

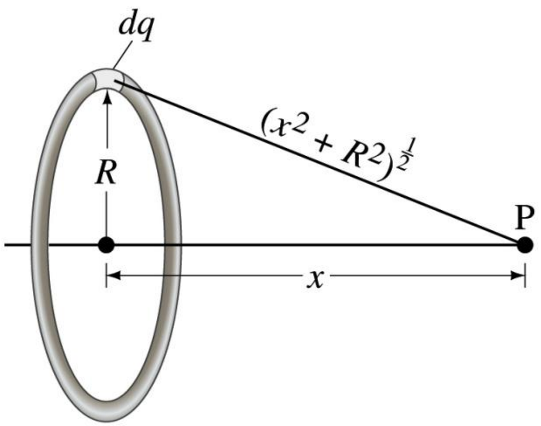
$$V_a = \sum_{i=1}^{n} V_{ia} = \sum_{i=1}^{n} \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}}$$

- For a continuous charge distribution, we obtain

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$

Example 23 – 8

- **Potential due to a ring of charge**: A thin circular ring of radius R carries a uniformly distributed charge Q. Determine the electric potential at a point P on the axis of the ring a distance x from its center.



- Each point on the ring is at the same distance from the point P. What is the distance?

$$r = \sqrt{R^2 + x^2}$$

- So the potential at P is

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0 r}\int dq =$$

$$\frac{1}{4\pi\varepsilon_0\sqrt{x^2+R^2}}\int dq = \frac{Q}{4\pi\varepsilon_0\sqrt{x^2+R^2}}$$

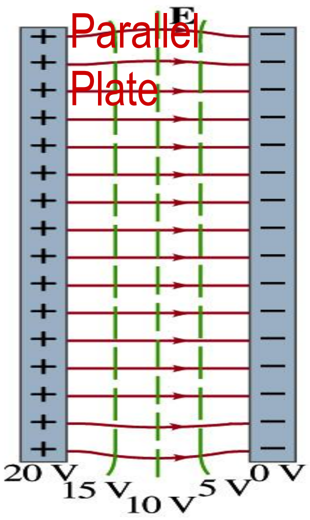
What's this?

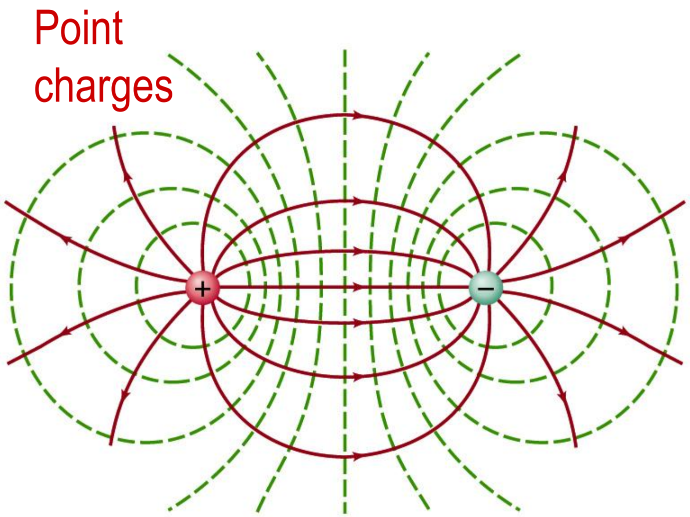
Equi-potential Surfaces

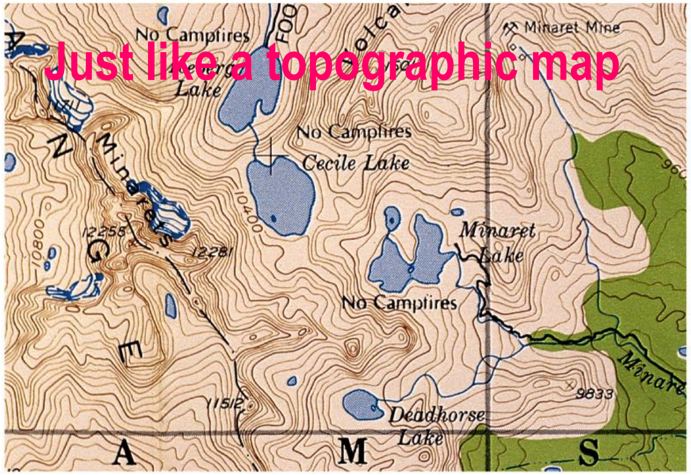
- Electric potential can be visualized using equipotential lines in 2-D or equipotential surfaces in 3-D
- Any two points on equipotential surfaces (lines) have the same potential
- What does this mean in terms of the potential difference?
  - The potential difference between the two points on an equipotential surface is 0.
- How about the potential energy difference?
  - Also 0.
- What does this mean in terms of the work to move a charge along the surface between these two points?
  - No work is necessary to move a charge between these two points.

Equi-potential Surfaces

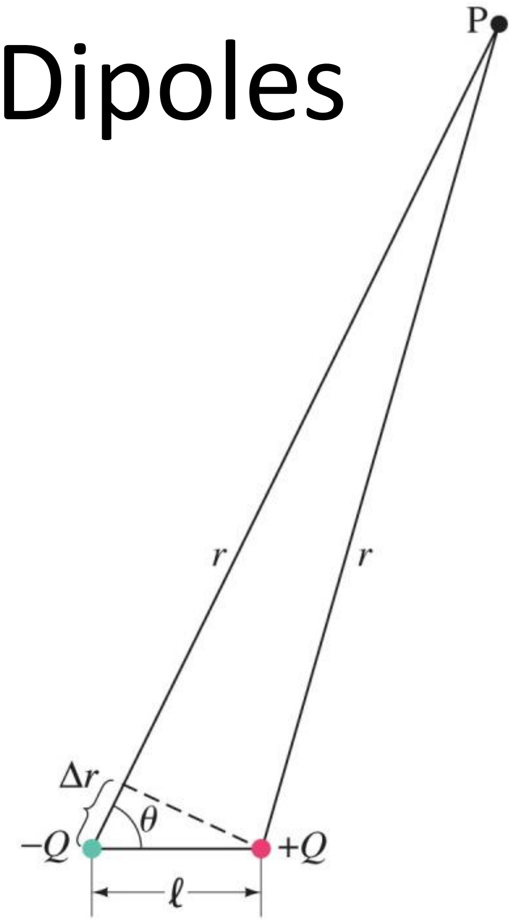
- An equipotential surface (line) must be perpendicular to the electric field. Why?
  - If there are any parallel components to the electric field, it would require work to move a charge along the surface.
- Since the equipotential surface (line) is perpendicular to the electric field, we can draw these surfaces or lines easily.
- There can be no electric field inside a conductor in static case, thus the entire volume of a conductor must be at the same potential.
- So the electric field must be perpendicular to the conductor surface.







Potential due to Electric Dipoles



$\Delta r = \ell \cos\theta$

$$V = \sum \frac{Q_i}{4\pi\varepsilon_0}\frac{1}{r_{ia}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} + \frac{-Q}{r+\Delta r}\right) =$$

$$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r+\Delta r}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{\Delta r}{r(r+\Delta r)}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{r^2} =$$

V due to dipole a distance r from the dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

E Determined from V

- Potential difference between two points is

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$$

- So we can write

$$E_l = -\frac{dV}{dl}$$

  – What are dV and E_ℓ?
    - dV is the infinitesimal potential difference between two points separated by the distance $d\ell$
    - E_ℓ is the field component along the direction of $d\ell$.

$$\vec{E} = -\vec{\nabla} V = -\left(\vec{i}\,\frac{\partial}{\partial x} + \vec{j}\,\frac{\partial}{\partial y} + \vec{k}\,\frac{\partial}{\partial z} \right) V$$

Electrostatic Potential Energy: electron Volt

- What is the unit of electrostatic potential energy?
  - Joules
- Joules is a very large unit in dealing with electrons, atoms or molecules
- For convenience a new unit, electron volt (eV), is defined
  - 1 eV is defined as the energy acquired by a particle carrying the charge equal to that of an electron (q=e) when it moves across a potential difference of 1V.
  - How many Joules is 1 eV then? $1eV = 1.6\times10^{-19}C\cdot 1V = 1.6\times10^{-19}J$
- eV however is **<u>not a standard SI unit</u>**. You must convert the energy to Joules for computations.