PHYS 1444 – Section 003
Lecture #8

Thursday Sep. 20, 2012
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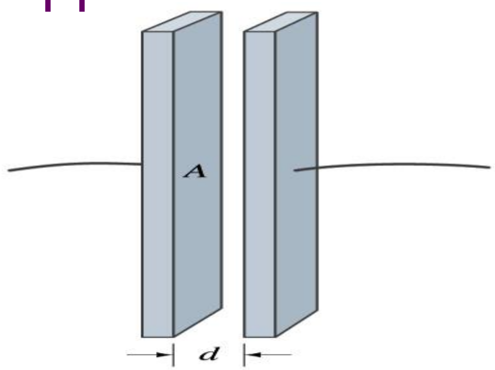
- Chapter 24
  - Capacitors and Capacitance
  - Capacitors in Series and Parallel
  - Energy Stored in Capacitors
  - Dielectrics

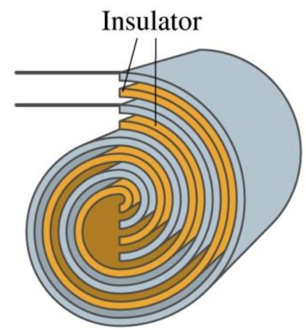
Capacitors (or Condensers)

- What is a capacitor?
  - A device that can store electric charge without letting the charge flow
- What does it consist of?
  - Usually consists of two oppositely charged conducting objects (plates or sheets) placed near each other without touching
  - Why can't they touch each other?
    - The charges will neutralize each other
- Can you give some examples?
  - Camera flash, surge protectors, computer keyboard, binary circuits…
- How is a capacitor different than a battery?
  - Battery provides potential difference by storing energy (usually chemical energy) while the capacitor stores charge but very little energy.

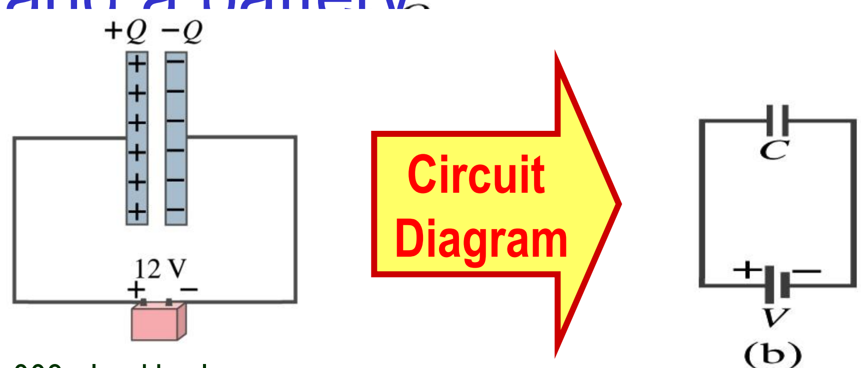
Capacitors

- A simple capacitor consists of a pair of parallel plates of area $\mathcal{A}$ separated by a distance d.
  - A cylindrical capacitors are essentially parallel plates wrapped around as a cylinder.



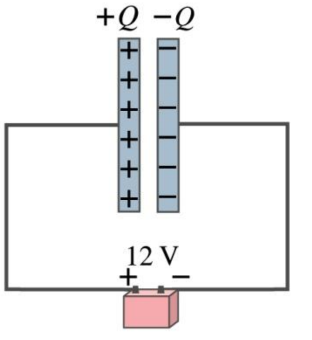


- Symbols for a capacitor and a battery:
  - Capacitor -||-
  - Battery (+) -|l- (-)



Capacitors

- What do you think will happen if a battery is connected (voltage is applied) to a capacitor?
  - The capacitor gets charged quickly, one plate positive and the other negative with an equal amount. of charge



- Each battery terminal, the wires and the plates are conductors. What does this mean?
  - All conductors are at the same potential.
  - the full battery voltage is applied across the capacitor plates.
- So for a given capacitor, the amount of charge stored in the capacitor is proportional to the potential difference V_{ba} between the plates. How would you write this formula?

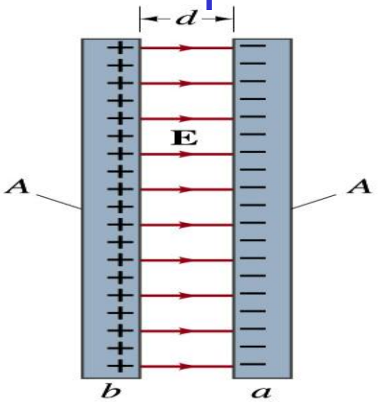
$$Q = CV_{ba}$$

C is a property of a capacitor so does not depend on Q or V.

  - C is a proportionality constant, called capacitance of the device.
  - What is the unit? C/V or Farad (F) Normally use μF or pF.

Determination of Capacitance

- C can be determined analytically for capacitors w/ simple geometry and air in between.
- Let's consider a parallel plate capacitor.
  - Plates have area A each and separated by d.
    - d is smaller than the length, so E is uniform.
  - For parallel plates $E=\sigma/\varepsilon_0$, where σ is the surface charge density.



- E and V are related $V_{ba} = -\int_a^b \vec{E} \cdot d\vec{l}$
- Since we take the integral from the lower potential point *a* to the higher potential point *b* along the field line, we obtain
- $V_{ba} = V_b - V_a = -\int_a^b Edl\cos 180^\circ = +\int_a^b Edl = \int_a^b \frac{\sigma}{\varepsilon_0} dl = \int_a^b \frac{Q}{\varepsilon_0 A} dl = \frac{Q}{\varepsilon_0 A}\int_a^b dl = \frac{Q}{\varepsilon_0 A}\left(b-a\right) = \frac{Qd}{\varepsilon_0 A}$
- So from the formula:
  - What do you notice?

$$C = \frac{Q}{V_{ba}} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

C only depends on the area (A) and the separation (d) of the plates and the permittivity of the medium between them.

Example 24 – 1

Capacitor calculations: (a) Calculate the capacitance of a capacitor whose plates are 20 cm x 3.0 cm and are separated by a 1.0 mm air gap. (b) What is the charge on each plate if the capacitor is connected to a 12 V battery? (c) What is the electric field between the plates? (d) Estimate the area of the plates needed to achieve a capacitance of 1F, given the same air gap.

(a) Using the formula for a parallel plate capacitor, we obtain

$$C = \frac{\varepsilon_0 A}{d} =$$

$$= \left(8.85\times10^{-12}\, C^2/N\cdot m^2\right)\frac{0.2\times0.03m^2}{1\times10^{-3}m} = 53\times10^{-12}\, C^2/N\cdot m = 53pF$$

(b) From Q=CV, the charge on each plate is

$$Q = CV = \left(53\times10^{-12}\, C^2/N\cdot m\right)\left(12V\right) = 6.4\times10^{-10}C = 640pC$$

Example 24 – 1

(c) Using the formula for the electric field in two parallel plates

$$E=\frac{\sigma}{\varepsilon_0}=\frac{Q}{A\varepsilon_0}=\frac{6.4\times10^{-10}\,C}{6.0\times10^{-3}\,m^2\times8.85\times10^{-12}\,C^2/N\cdot m^2}=1.2\times10^4\ N/C=1.2\times10^4\,V/m$$

Or, since $V=Ed$ we can obtain $E=\frac{V}{d}=\frac{12V}{1.0\times10^{-3}\,m}=1.2\times10^4\,V/m$

(d) Solving the capacitance formula for A, we obtain

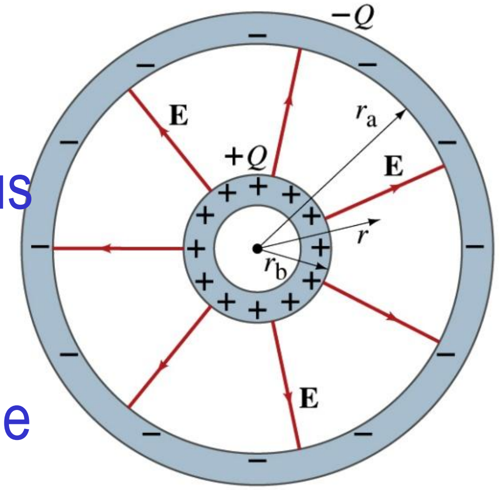
$$C=\frac{\varepsilon_0 A}{d}$$

Solve for A

$$A=\frac{Cd}{\varepsilon_0}=\frac{1F\cdot1\times10^{-3}\,m}{9\times10^{-12}\,C^2/N\cdot m^2}\approx10^8\,m^2\approx100km^2$$

About 40% the area of Arlington ($256km^2$).

Example 24 – 3



Spherical capacitor: A spherical capacitor consists of two thin concentric spherical conducting shells, of radius r_a and r_b, as in the figure. The inner shell carries a uniformly distributed charge Q on its surface and the outer shell an equal but opposite charge –Q. Determine the capacitance of this configuration.

Using Gauss' law, the electric field outside a uniformly charged conducting sphere is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

So the potential difference between a and b is

$$V_{ba} = -\int_a^b \vec{E}\cdot d\vec{l} =$$

$$= -\int_a^b E\cdot dr = -\int_a^b \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r}\right)_{r_a}^{r_b} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b}-\frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a-r_b}{r_b r_a}\right)$$

Thus capacitance is

$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left(\frac{r_a-r_b}{r_b r_a}\right)} = \frac{4\pi\varepsilon_0 r_b r_a}{r_a-r_b}$$

Capacitor Cont'd

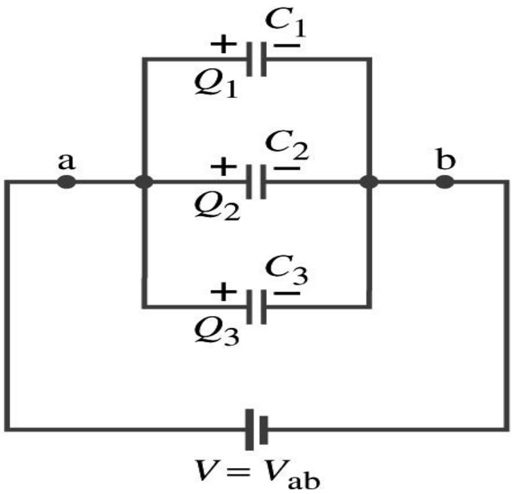
- A single isolated conductor can be said to have a capacitance, C.
- C can still be defined as the ratio of the charge to absolute potential V on the conductor.
  - So Q=CV.
- The potential of a single conducting sphere of radius r_b can be obtained as

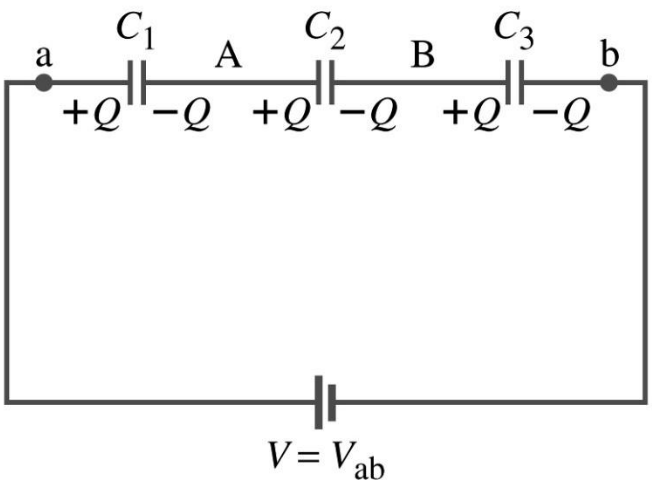
$$V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = \frac{Q}{4\pi\varepsilon_0 r_b} \quad \text{where} \quad r_a \to \infty$$

- So its capacitance is $C = Q/V = 4\pi\varepsilon_0 r_b$
- Although it has capacitance, a single conductor is not considered to be a capacitor, as a second nearby charge is required to store charge

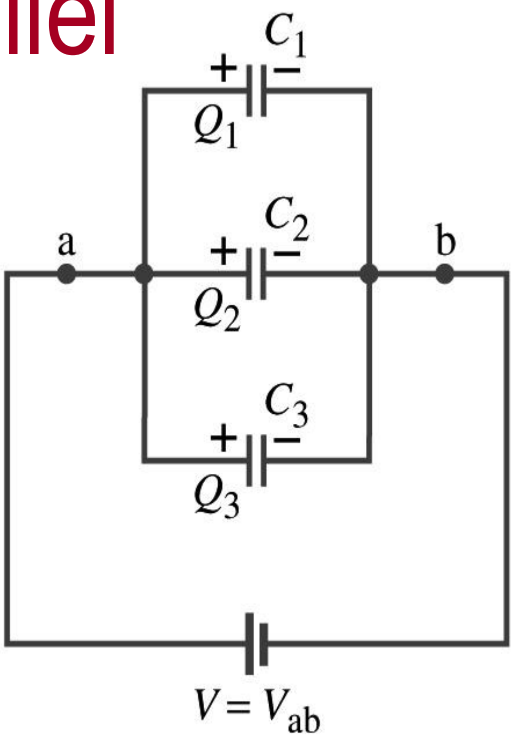
Capacitors in Series or Parallel

- Capacitors are used in many electric circuits
- What is an electric circuit?
  - A closed path of conductors, usually wires connecting capacitors and other electrical devices, in which
    - charges can flow
    - there is a voltage source such as a battery
- Capacitors can be connected in various ways.
  - In parallel and in Series or in combination





Capacitors in Parallel



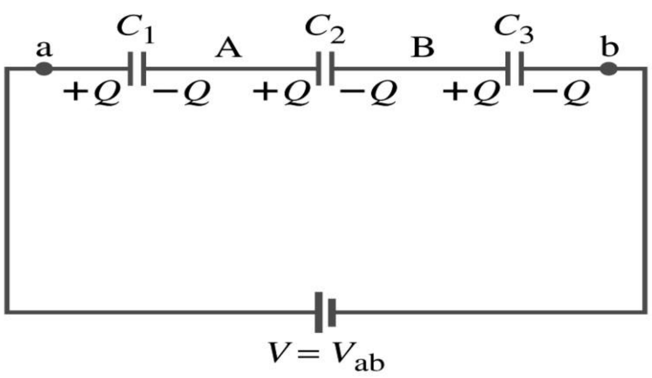
- Parallel arrangement provides the **<u>same voltage</u>** across all the capacitors.
  - Left hand plates are at V_a and right hand plates are at V_b
  - So each capacitor plate acquires charges given by the formula
    - $Q_1=C_1V$, $Q_2=C_2V$, and $Q_3=C_3V$
- The total charge Q that must leave battery is then
  - $Q=Q_1+Q_2+Q_3=V(C_1+C_2+C_3)$
- Consider that the three capacitors behave like a single "equivalent" one
  - $Q=C_{eq}V= V(C_1+C_2+C_3)$

For capacitors in parallel the capacitance is the sum of the individual capacitors

- Thus the equivalent capacitance in parallel is $C_{eq} = C_1 + C_2 + C_3$

Capacitors in Series

- Series arrangement is more "interesting"
  - When battery is connected, +Q flows to the left plate of C_1 and –Q flows to the right plate of C_3
  - This induces opposite sign charges on the other plates.
  - Since the capacitor in the middle is originally neutral, charges get induced to neutralize the induced charges
  - So the charge on each capacitor is the same value, Q. (**Same charge**)



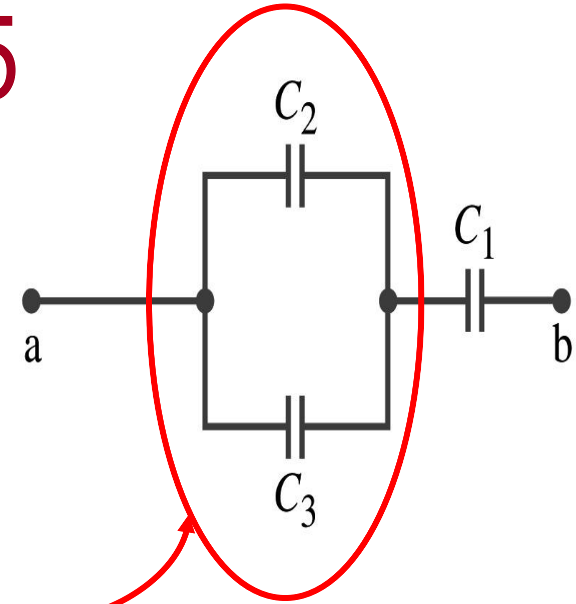
- Consider that the three capacitors behave like an equivalent one
  - $Q=C_{eq}V \rightarrow V=Q/C_{eq}$
- The total voltage V across the three capacitors in series must be equal to the sum of the voltages across each capacitor.
  - $V=V_1+V_2+V_3=(Q/C_1+Q/C_2+Q/C_3)$
- Putting all these together, we obtain:
- $V=Q/C_{eq}=Q(1/C_1+1/C_2+1/C_3)$
- Thus the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

The total capacitance is smaller than the smallest C!!!

Example 24 – 5

Equivalent Capacitor: Determine the capacitance of a single capacitor that will have the same effect as the combination shown in the figure. Take $C_1=C_2=C_3=C$.



We should do these first!!

How? These are in parallel so the equivalent capacitance is:

$$C_{eq} = C_3 + C_2 = 2C$$

Now the equivalent capacitor is in series with C1.

$$\frac{1}{C_{eq}} = \frac{1}{C_{eq1}} + \frac{1}{C_2} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C}$$

Solve for C_{eq}

$$C_{eq} = \frac{2C}{3}$$

Electric Energy Storage

- A charged capacitor stores energy.
  - The stored energy is the work done to charge it.
- The net effect of charging a capacitor is removing one type of charge (+ or -) from a plate and moving it to the other plate.
  - Battery does this when it is connected to a capacitor.
- Capacitors do not charge immediately.
  - Initially when the capacitor is uncharged, no work is necessary to move the first bit of charge. Why?
    - Since there is no charge, there is no field that the external work needs to overcome.
  - When some charge is on each plate, it requires work to add more charge due to electric repulsion.

Electric Energy Storage

- What work is needed to add a small amount of charge (dq) when the potential difference across the plates is V? $dW = Vdq$
- Since V=q/C, the work needed to store total charge Q is

$$W = \int_0^Q V dq = \frac{1}{C}\int_0^Q q dq = \frac{Q^2}{2C}$$

- Thus, the energy stored in a capacitor when the capacitor carries charges +Q and –Q is

$$U = \frac{Q^2}{2C}$$

- Since Q=CV, we can rewrite

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Example 24 – 7

Energy store in a capacitor: A camera flash unit stores energy in a 150μF capacitor at 200V. How much electric energy can be stored?

Use the formula for stored energy. Umm.. Which one?

What do we know from the problem? C and V

So we use the one with C and V: $U = \frac{1}{2}CV^2$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(150\times10^{-6}F\right)\left(200V\right)^2 = 3.0J$$

How do we get J from FV²? $FV^2 = \left(\frac{C}{V}\right)V^2 = CV = C\left(\frac{J}{C}\right) = J$

Electric Energy Density

- The energy stored in a capacitor can be considered as being stored in the electric field between the two plates
- For a uniform field E between two plates, $V=Ed$ and $C=\varepsilon_0 A/d$
- Thus the stored energy is

$$U=\frac{1}{2}CV^2=\frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2=\frac{1}{2}\varepsilon_0 E^2 Ad$$

- Since Ad is the gap volume, we can obtain the energy density, stored energy per unit volume, as

$$u=\frac{1}{2}\varepsilon_0 E^2$$

Valid for plates with a vacuum gap

Electric energy stored per unit volume in any region of space is proportional to the square of the electric field in that region.

Dielectrics

- Capacitors generally have an insulating sheet of material, called a dielectric, between the plates to
  - Increase the breakdown voltage above that in air
  - Allows the plates get closer together without touching
    - Increases capacitance (recall $C=\varepsilon_0 A/d$)
  - Also increases the capacitance by the dielectric constant

$$C = KC_0$$

    - Where C_0 is the intrinsic capacitance when the gap is vacuum, and K or κ is the dielectric constant

Dielectrics

- The value of dielectric constant varies depending on material (Table 24 – 1)
  - K for vacuum is 1.0000
  - K for air is 1.0006 (this is why permittivity of air and vacuum are used interchangeably.)
- The maximum electric field before breakdown occurs is the dielectric strength. What is its unit?
  - V/m
- The capacitance of a parallel plate capacitor with a dielectric (K) filling the gap is

$$C = KC_0 = K\varepsilon_0 \frac{A}{d}$$

Dielectrics

- A new quantity, the permittivity of dielectric, is defined as **$\varepsilon = K\varepsilon_0$**
- The capacitance of a parallel plate with a dielectric medium filling the gap is

$$C = \varepsilon \frac{A}{d}$$

- The energy density stored in an electric field E in a dielectric is

$$u = \frac{1}{2} K \varepsilon_0 E^2 = \frac{1}{2} \varepsilon E^2$$

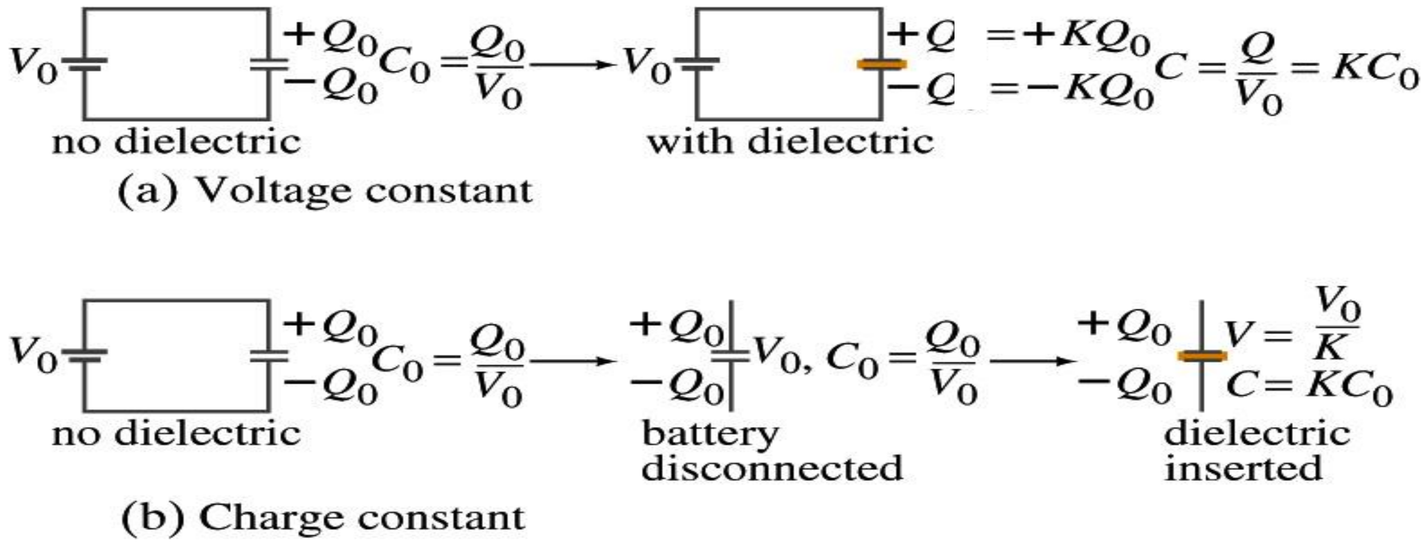
Valid for any space with dielectric of permittivity ε.

Effect of a Dielectric Material

- Let's consider the two cases below:

Case #1 :
constant V

Case #2 :
constant Q



- Constant voltage: Experimentally observed that the total charge on each plate of the capacitor increases by K as the dielectric material is inserted between the gap ➔ $Q=KQ_0$
  – The capacitance increased to $C=Q/V_0=KQ_0/V_0=KC_0$
- Constant charge: Voltage found to drop by a factor K ➔ $V=V_0/K$
  – The capacitance increased to $C=Q_0/V=KQ_0/V_0=KC_0$