CHAPTER 12

The Atomic Nucleus

- 12.6 Radioactive Decay
- 12.7 Alpha+ Beta Decay
12.6: Radioactive Decay

Marie Curie and her husband Pierre discovered polonium and radium in 1898.
- The simplest decay form is that of a gamma ray, which represents the nucleus changing from an excited state to lower energy state.
- Other modes of decay include emission of $\alpha$ particles, $\beta$ particles, protons, neutrons, and fission.

The disintegrations or decays per unit time (activity).

$$
\text{Activity} = - \frac{dN}{dt} = R
$$

where $dN / dt$ is negative because total number $N$ decreases with time.
Radioactive Decay

- SI unit of activity is the becquerel: 1 Bq = 1 decay / s.
- Recent use is the Curie (Ci) $3.7 \times 10^{10}$ decays / s.

If $N(t)$ is the number of radioactive nuclei in a sample at time $t$, and $\lambda$ (decay constant) is the probability per unit time that any given nucleus will decay:

$$R = \lambda N(t)$$
$$dN(t) = -R \, dt = -\lambda N(t) \, dt$$

$$\int \frac{dN}{N} = -\int \lambda \, dt$$

$$\ln N = -\lambda t + \text{constant}$$

$$N(t) = e^{\lambda t + \text{constant}}$$

If we let $N(t = 0) \equiv N_0$

$$N(t) = N_0 e^{-\lambda t} \quad \text{----- radioactive decay law}$$
Radioactive Decay

- The activity $R$ is
  \[ R = \lambda N(t) = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \]
  where $R_0$ is the initial activity at $t = 0$.

- It is common to refer to the half-life $t_{1/2}$ or the mean lifetime $\tau$ rather than its decay constant.

\[
N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}}
\]

\[
\ln\left(\frac{1}{2}\right) = \ln(e^{-\lambda t_{1/2}}) = -\lambda t_{1/2}
\]

- The half-life is
  \[
t_{1/2} = \frac{-\ln(1/2)}{\lambda} = \frac{\ln(2)}{\lambda} = 0.693
  \]

- The mean lifetime is
  \[
  \tau = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln(2)}
  \]
Radioactive Decay

- The number of radioactive nuclei as a function of time
When a nucleus decays, all the conservation laws must be observed:

- Mass-energy
- Linear momentum
- Angular momentum
- Electric charge
- **Conservation of nucleons**
  - The total number of nucleons (A, the mass number) must be conserved in a low-energy nuclear reaction or decay.
Alpha, Beta, and Gamma Decay

- Let the radioactive nucleus $^{A}_{Z}X$ be called the parent and have a mass $M\left(^{A}_{Z}X\right)$.

- Two or more products can be produced in the decay.
- Let the lighter one be $M_y$ and the mass of the heavier one (daughter) be $M_D$.
- The conservation of energy equation is $M\left(^{A}_{Z}X\right) = M_D + M_y + Q/c^2$

where $Q$ is the energy released (disintegration energy) and equal to the total kinetic energy of the reaction products; Note $Q=-B$

- If $B > 0$, a nuclide is bound and stable; $Q = \left[ M\left(^{A}_{Z}X\right) - M_D - M_y \right] c^2$
- If $Q > 0$, a nuclide is unbound, unstable, and may decay.
- If $Q < 0$, decay emitting nucleons do not occur.
Alpha Decay

- The nucleus $^4\text{He}$ has a binding energy of 28.3 MeV.
- If the last two protons and two neutrons in a nucleus are bound by less than 28.3 MeV, then the emission of an alpha particle (alpha decay) is possible.

$$^{A\ Z}X \rightarrow ^{A-4\ Z-2}D + \alpha$$

$$Q = \left[ M\left(^{A\ Z}X\right) - M\left(^{A-4\ Z-2}D\right) - M\left(^4\text{He}\right) \right] c^2$$

- If $Q > 0$, alpha decay is possible.

EX. $^{230}_{92}\text{U} \rightarrow \alpha + ^{226}_{90}\text{Th}$

The appropriate masses are

$M\left(^{230}_{92}\text{U}\right) = 230.033927 \text{u}; M\left(^4\text{He}\right) = 4.002603 \text{u}; M\left(^{226}_{90}\text{Th}\right) = 226.024891 \text{u}$
Alpha Decay

- Insert into Eq. (12.31)

\[ Q = \left[ M(\text{^{230}U}) - M(\text{^{226}Th}) - M(\text{^{4}He}) \right] c^2 \]

\[ = [230.033927 \text{ u } - 226.024891 \text{ u } - 4.002603 \text{ u}] c^2 \left( \frac{931.5 \text{ MeV}}{c^2 \cdot \text{u}} \right) = 6.0 \text{ MeV} \]

- In order for alpha decay to occur, two neutrons and two protons group together within the nucleus prior to decay and the alpha particle has difficulty in overcoming the nuclear attraction from the remaining nucleons to escape.

The potential energy diagram of an alpha particle
Alpha Decay

- The barrier height $V_B$ is greater than 20 MeV.
- The kinetic energies of alpha particles emitted from nuclei range from 4-10 MeV.

It is impossible classically for the alpha particle to escape the nucleus, but the alpha particles are able to tunnel through the barrier.

A higher energy $E_2$ has much higher probability of escape than does a lower energy $E_1$.

There is a correlation between lower energies and greater difficulty of escaping (longer lifetimes).
Alpha Decay

- Assume the parent nucleus is initially at rest so that the total momentum is zero.
- The final momenta of the daughter $p_D$ and alpha particle $p_\alpha$ have the same magnitude and opposite directions.
Beta Decay

- Unstable nuclei may move closer to the line of stability by undergoing beta decay.

- The decay of a free neutron is

\[ n \rightarrow p + \beta^- \]

- The beta decay of $^{14}$C (unstable) to form $^{14}$N, a stable nucleus, can be written as

\[ ^{14}_6\text{C} \rightarrow ^{14}_7\text{N} + \beta^- \]
Beta Decay

- There was a problem in neutron decay, the spin $\frac{1}{2}$ neutron cannot decay to two spin $\frac{1}{2}$ particles, a proton and an electron. $^{14}$C has spin 0, $^{14}$N has spin 1, and the electron has spin $\frac{1}{2}$. We cannot combine spin $\frac{1}{2}$ & 1 to obtain a spin 0.

- Wolfgang Pauli suggested a neutrino $\nu$ that must be produced in beta decay. It has spin quantum number $\frac{1}{2}$, charge 0, and carries away the additional energy missing in Fig. (12.14).
Beta Decay

- An occasional electron is detected with the kinetic energy $K_{\text{max}}$ required to conserve energy, but in most cases the electron’s kinetic energy is less than $K_{\text{max}}$.

  the neutrino has little or no mass, and its energy may be all kinetic.

- Neutrinos have no charge and do not interact electromagnetically.
- They are not affected by the strong force of the nucleus.
- They are the weak interaction.
- The electromagnetic and weak forces are the electroweak force.
\section*{$\beta^-$ Decay}

- There are antineutrinos $\bar{\nu}$.
- The beta decay of a free neutron of $^{14}\text{C}$ is written as
  \[ n \rightarrow p + \beta^- + \bar{\nu} \quad \beta^- \text{ decay} \]
  \[ ^{14}\text{C} \rightarrow ^{14}\text{N} + \beta^- + \bar{\nu} \quad \beta^- \text{ decay} \]
- In the general beta decay of the parent nuclide $^{A\,Z}\,X$ to the daughter $^{A\,Z+1}\,D$, the reaction is $^{A\,Z}\,X$
  \[ ^{A\,Z}\,X \rightarrow ^{A\,Z+1}\,D + \beta^- + \bar{\nu} \quad \beta^- \text{ decay} \]
- The disintegration energy $Q$ is
  \[ Q = \left[ M\left(^{A\,Z}\,X\right) - M\left(^{A\,Z+1}\,D\right) \right] c^2 \quad \beta^- \text{ decay} \]
- In order for $\beta^-$ to occur, we must have $Q > 0$.
- The nucleus $A$ is constant, but $Z$ charges to $Z + 1$. 
**β⁺ Decay**

- What happens for unstable nuclides with too many protons?
- Positive electron (positron) is produced.
- Positron is the antiparticle of the electron.
- A free proton does not decay when $t_{1/2} > 10^{32}$ y.
- The nucleus $^{14}$O is unstable and decays by emitting a positron to become stable $^{14}$N. $^{14}$O $\rightarrow$ $^{14}$N $+$ $\beta^+$ $+$ $\nu$ $\quad \beta^+$ decay

- The general $\beta^+$ decay is

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}D + \beta^+ + \nu \quad \beta^+$ decay

- The disintegration energy $Q$ is

$$Q = \left[ M\left(^{A}_{Z}X\right) - M\left(^{A}_{Z-1}D\right) - 2m_e \right]c^2 \quad \beta^+$ decay
Electron Capture

- Classically, inner K-shell and L-shell electrons are tightly bound and their orbits are highly elliptical, these electrons spend a time passing through the nucleus, thereby the possibility of atomic electron capture.
- The reaction for a proton is $p + e^- \rightarrow n + \nu$
- The general reaction is
  $$\frac{A}{Z} X + e^- \rightarrow \frac{A}{Z-1} D + \nu$$  \hspace{1cm} \text{Electron capture}
- The disintegration energy $Q$ is
  $$Q = \left[ M\left(\frac{A}{Z} X\right) - M\left(\frac{A}{Z-1} D\right) \right] c^2$$  \hspace{1cm} \text{Electron capture}
Radioactive Carbon Dating

- Radioactive $^{14}\text{C}$ is produced in our atmosphere by the bombardment of $^{14}\text{N}$ by neutrons produced by cosmic rays.

\[ n + ^{14}\text{N} \rightarrow ^{14}\text{C} + p \]

- When living organisms die, their intake of $^{14}\text{C}$ ceases, and the ratio of $^{14}\text{C} / ^{12}\text{C} (= R)$ decreases as $^{14}\text{C}$ decays. The period just before 9000 years ago had a higher $^{14}\text{C} / ^{12}\text{C}$ ratio by factor of about 1.5 than it does today.

- Because the half-life of $^{14}\text{C}$ is 5730 years, it is convenient to use the $^{14}\text{C} / ^{12}\text{C}$ ratio to determine the age of objects over a range up to 45,000 years ago.
CHAPTER 13

Nuclear Interactions and Applications

- 13.1 Nuclear Reactions
- 13.4 Fission
- 13.5 Fission Reactors
- 13.6 Fusion
- 13.7 Special Applications

Ernest Lawrence, upon hearing the first self-sustaining chain reaction would be developed at the University of Chicago in 1942 rather than at his University of California, Berkeley lab said, “You’ll never get the chain reaction going here. The whole tempo of the University of Chicago is too slow.”

- Quoted by Arthur Compton in Atomic Quest
13.1: Nuclear Reactions

- First nuclear reaction was a nitrogen target bombarded with alpha particles, which emitted protons. The reaction is written as:

\[ \alpha + ^{14}_7 \text{N} \rightarrow p + ^{17}_8 \text{O} \]

- The first particle is the projectile and the second is the (nitrogen) target. These two nuclei react to form proton projectiles and the residual oxygen target.
13.4: Fission

- In fission a nucleus separates into two fission fragments; one fragment is typically somewhat larger than the other.
- Fission occurs for heavy nuclei because of the increased Coulomb forces between the protons.
- For a spherical nucleus of with mass number $A \sim 240$, the attractive short-range nuclear forces offsets the Coulomb repulsive term. As a nucleus becomes nonspherical, the surface energy is increased, and the effect of the short-range nuclear interactions is reduced.
- Nucleons on the surface are not surrounded by other nucleons, and the unsaturated nuclear force reduces the overall nuclear attraction. For a certain deformation, a critical energy is reached, and the fission barrier is overcome.

- Spontaneous fission can occur for nuclei with $Z^2 / A \geq 49$ ($Z \approx 115$, $A \approx 270$)
Induced Fission

- Fission may also be *induced* by a nuclear reaction. A neutron absorbed by a heavy nucleus forms a highly excited compound nucleus that may quickly fission.

- An induced fission example is

  \[ n + \frac{235}{92}U \rightarrow \frac{236}{92}U^* \rightarrow \frac{99}{40}Zr + \frac{134}{52}Te + 3n \]

- The fission products have a ratio of \( N/Z \) much too high to be stable for their \( A \) value.

- There are many possibilities for the \( Z \) and \( A \) of the fission products.

- Symmetric fission (products with equal \( Z \)) is possible, but the most probable fission is asymmetric (one mass larger than the other).
Thermal Neutron Fission

- Fission fragments are highly unstable because they are so neutron rich.
- Prompt neutrons are emitted simultaneously with the fissioning process. Even after prompt neutrons are released, the fission fragments undergo beta decay, releasing more energy.
- Most of the ~200 MeV released in fission goes to the kinetic energy of the fission products, but the neutrons, beta particles, neutrinos, and gamma rays typically carry away 30–40 MeV of the kinetic energy.
Chain Reactions

- Because several neutrons are produced in fission, these neutrons may subsequently produce other fissions. This is the basis of the **self-sustaining chain reaction**.
- If slightly more than one neutron, on the average, results in another fission, the chain reaction becomes **critical**.
- A sufficient amount of mass is required for a neutron to be absorbed, called the **critical mass**.
- If less than one neutron, on the average, produces another fission, the reaction is **subcritical**.
- If more than one neutron, on the average, produces another fission, the reaction is **supercritical**.
  - An atomic bomb is an extreme example of a supercritical fission chain reaction.
Chain Reactions

- A critical-mass fission reaction can be controlled by absorbing neutrons. A self-sustaining controlled fission process requires that not all the neutrons are *prompt*. Some of the neutrons are *delayed* by several seconds and are emitted by daughter nuclides. These delayed neutrons allow the control of the nuclear reactor.

- *Control rods* regulate the absorption of neutrons to sustain a controlled reaction.
Several components are important for a controlled nuclear reactor:

1) Fissionable fuel
2) Moderator to slow down neutrons
3) Control rods for safety and to control criticality of reactor
4) Reflector to surround moderator and fuel in order to contain neutrons and thereby improve efficiency
5) Reactor vessel and radiation shield
6) Energy transfer systems if commercial power is desired

Two main effects can “poison” reactors: (1) neutrons may be absorbed without producing fission [for example, by neutron radiative capture], and (2) neutrons may escape from the fuel zone.
Core Components

- Fission neutrons typically have 1–2 MeV of kinetic energy, and because the fission cross section increases as $1/v$ at low energies, slowing down the neutrons helps to increase the chance of producing another fission. A moderator is used to elastically scatter the high-energy neutrons and thus reduce their energies. A neutron loses the most energy in a single collision with a light stationary particle. Hydrogen (in water), carbon (graphite), and beryllium are all good moderators.

- The simplest method to reduce the loss of neutrons escaping from the fissionable fuel is to make the fuel zone larger. The fuel elements are normally placed in regular arrays within the moderator.
Core Components

- The delayed neutrons produced in fission allow the mechanical movement of the rods to control the fission reaction. A “fail-safe” system automatically drops the control rods into the reactor in an emergency shutdown.

- If the fuel and moderator are surrounded by a material with a very low neutron capture cross section, there is a reasonable chance that after one or even many scatterings, the neutron will be backscattered or “reflected” back into the fuel area. Water is often used both as moderator and reflector.
The most common method is to pass hot water heated by the reactor through some form of heat exchanger.

In *boiling water reactors* (BWRs) the moderating water turns into steam, which drives a turbine producing electricity.

In *pressurized water reactors* (PWRs) the moderating water is under high pressure and circulates from the reactor to an external heat exchanger where it produces steam, which drives a turbine.

Boiling water reactors are inherently simpler than pressurized water reactors. However, the possibility that the steam driving the turbine may become radioactive is greater with the BWR. The two-step process of the PWR helps to isolate the power generation system from possible radioactive contamination.
Types of Reactors

- *Power* reactors produce commercial electricity.
- *Research* reactors are operated to produce high neutron fluxes for neutron-scattering experiments.
- *Heat production* reactors supply heat in some cold countries.
- Some reactors are designed to produce *radioisotopes*.
- Several *training* reactors are located on college campuses.
Nuclear Reactor Problems

- The danger of a serious accident in which radioactive elements are released into the atmosphere or groundwater is of great concern to the general public.
- Thermal pollution both in the atmosphere and in lakes and rivers used for cooling may be a significant ecological problem.
- A more serious problem is the safe disposal of the radioactive wastes produced in the fissioning process, because some fission fragments have a half-life of thousands of years.
- Two widely publicized accidents at nuclear reactor facilities—one at Three Mile Island in Pennsylvania in 1979, the other at Chernobyl in Ukraine in 1986—have significantly dampened the general public’s support for nuclear reactors.
- Large expansion of nuclear power can succeed only if four critical problems are overcome: lower costs, improved safety, better nuclear waste management, and lower proliferation risk.
Breeder Reactors

- A more advanced kind of reactor is the *breeder* reactor, which produces more fissionable fuel than it consumes.

- The chain reaction is:

\[
n + ^{238}_{92}U \rightarrow ^{239}_{92}U^* \rightarrow \gamma + ^{239}_{92}U \\
\quad \quad \quad \quad \quad \quad \beta^- + ^{239}_{93}Np + \bar{\nu} \rightarrow \beta^- + ^{239}_{94}Pu + \bar{\nu}
\]

- The plutonium is easily separated from uranium by chemical means.

- *Fast breeder reactors* have been built that convert $^{238}U$ to $^{239}Pu$. The reactors are designed to use fast neutrons.

- Breeder reactors hold the promise of providing an almost unlimited supply of fissionable material.

- One of the downsides of such reactors is that plutonium is highly toxic, and there is concern about its use in unauthorized weapons production.
If two light nuclei fuse together, they also form a nucleus with a larger binding energy per nucleon and energy is released. This reaction is called **nuclear fusion**.

The most energy is released if two isotopes of hydrogen fuse together in the reaction.

\[
^2\text{H} + ^3\text{H} \rightarrow n + ^4\text{He} \quad Q = 17.6 \text{ MeV}
\]
Formation of Elements

- The **proton-proton chain** includes a series of reactions that eventually converts four protons into an alpha particle.

- As stars form due to gravitational attraction of interstellar matter, the heat produced by the attraction is enough to cause protons to overcome their Coulomb repulsion and fuse by the following reaction:

\[
^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + \beta^+ + \nu \quad Q = 0.42 \text{ MeV}
\]

- The deuterons are then able to combine with \(^1\text{H}\) to produce \(^3\text{He}\):

\[
^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma \quad Q = 5.49 \text{ MeV}
\]

- The \(^3\text{He}\) atoms can then combine to produce \(^4\text{He}\):

\[
^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H} \quad Q = 12.86 \text{ MeV}
\]
Formation of Elements

- As the reaction proceeds, however, the temperature increases, and eventually $^{12}$C nuclei are formed by a process that converts three $^4$He into $^{12}$C.
- Another cycle due to carbon is also able to produce $^4$He. The series of reactions responsible for the carbon or CNO cycle are

\[
\begin{align*}
^1\text{H} + ^{12}\text{C} & \rightarrow ^{13}\text{N} + \gamma \\
^1\text{H} + ^{13}\text{C} & \rightarrow ^{14}\text{N} + \gamma \\
^1\text{H} + ^{14}\text{N} & \rightarrow ^{15}\text{O} + \gamma \\
^1\text{H} + ^{15}\text{N} & \rightarrow ^{12}\text{C} + ^4\text{He}
\end{align*}
\]

- Proton-proton and CNO cycles are the only nuclear reactions that can supply the energy in stars.
Nuclear Fusion on Earth

- Among the several possible fusion reactions, three of the simplest involve the three isotopes of hydrogen.

\[
\begin{align*}
{^2}\text{H} + {^2}\text{H} & \rightarrow n + {^3}\text{He} \quad Q = 3.3 \text{ MeV} \\
{^2}\text{H} + {^2}\text{H} & \rightarrow p + {^3}\text{H} \quad Q = 4.0 \text{ MeV} \\
{^2}\text{H} + {^3}\text{H} & \rightarrow n + {^4}\text{He} \quad Q = 17.6 \text{ MeV}
\end{align*}
\]

- Three main conditions are necessary for controlled nuclear fusion:
  1) The temperature must be hot enough to allow the ions, for example, deuterium and tritium, to overcome the Coulomb barrier and fuse their nuclei together. This requires a temperature of 100–200 million K.
  2) The ions have to be confined together in close proximity to allow the ions to fuse. A suitable ion density is \(2–3 \times 10^{20} \text{ ions/m}^3\).
  3) The ions must be held together in close proximity at high temperature long enough to avoid plasma cooling. A suitable time is 1–2 s.
Over 1100 radioisotopes are available for clinical use.

Radioisotopes are used in tomography, a technique for displaying images of practically any part of the body to look for abnormal physical shapes or for testing functional characteristics of organs. By using detectors (either surrounding the body or rotating around the body) together with computers, three-dimensional images of the body can be obtained.

They use single-photon emission computed tomography, positron emission tomography, and magnetic resonance imaging.
Archaeology

- Investigators can now measure a large number of trace elements in many ancient specimens and then compare the results with the concentrations of components having the same origin.

- Radioactive dating indicates that humans had a settlement near Clovis, New Mexico 12,000 years ago. Several claims have surfaced in the past few years, especially from South America, that dispute this earliest finding, but no conclusive proof has been confirmed.

- The Chauvet Cave, discovered in France in 1995, is one of the most important archaeological finds in decades. More than 300 paintings and engravings and many traces of human activity, including hearths, flintstones, and footprints, were found. These works are believed, from $^{14}$C radioactive dating, to be from the Paleolithic era, some 32,000 years ago.
Neutron activation is a nondestructive technique that is becoming more widely used to examine oil paintings. A thermal neutron beam from a nuclear reactor is spread broadly and evenly over the painting. Several elements within the painting become radioactive. X-ray films sensitive to beta emissions from the radioactive nuclei are subsequently placed next to the painting for varying lengths of time. This method is called an autoradiograph.

It was used to examine Van Dyck’s Saint Rosalie Interceding for the Plague-Stricken of Palermo, from the New York Metropolitan Museum of Art collection and revealed an over-painted self-portrait of Van Dyck himself.
Crime Detection

- The examination of gunshots by measuring trace amounts of barium and antimony from the gunpowder has proven to be 100 to 1000 times more sensitive than looking for the residue itself.

- Scientists are also able to detect toxic elements in hair by neutron activation analysis.