1. HW4 on ch 5 will be assigned Friday 2/19 and due Monday 3/1
2. HW5 on ch 6 will be assigned Friday 2/26 and due Monday 3/8
3. Monday 3/8 will be review
4. Weds 3/10 test on ch 1-6
CHAPTER 5

Wave Properties of Matter and Quantum Mechanics I

- 5.1 X-Ray Scattering
- 5.2 De Broglie Waves
- 5.3 Electron Scattering
- 5.4 Wave Motion
- 5.5 Waves or Particles?
- 5.6 Uncertainty Principle
- 5.7 Probability, Wave Functions, and the Copenhagen Interpretation
- 5.8 Particle in a Box

I thus arrived at the overall concept which guided my studies: for both matter and radiations, light in particular, it is necessary to introduce the corpuscle concept and the wave concept at the same time.

- Louis de Broglie, 1929
Bragg’s Law

- William Bragg interpreted the x-ray scattering as the reflection of the incident x-ray beam from a unique set of planes of atoms within the crystal.
- There are two conditions for constructive interference of the scattered x rays:

1) The angle of incidence must equal the angle of reflection of the outgoing wave.
2) The difference in path lengths must be an integral number of wavelengths.

- Bragg’s Law:
  \[ n\lambda = 2d \sin \theta \]
  \( (n = \text{integer}) \)
5.2 De Broglie Waves

• Particle properties of waves supported by data (Ch. 3)
• De Broglie in the 1920’s made a bold proposal without strong evidence that moving particles could have wavelike properties (Nobel Prize when verified) [Rubbia]
• Photon: \[ E = pc = hf \Rightarrow p = \frac{hf}{c} = \frac{h}{\lambda} \therefore \lambda = \frac{h}{p} \]
• De Broglie suggested that this is a general expression, so for a relativistic particle with \( p = \gamma mv \) this implies that \( \lambda = \frac{h}{\gamma mv} \). This is the general expression for the de Broglie Wavelength
• Large momentum implies short wavelength (accelerators with large momentum thus probe short wavelengths/small distances; ex. search for compositeness)
• Wave and particle aspects cannot be observed at the same time. Which properties are most evident depends on how the \( dB\lambda \) compares with particle’s dimensions, and the dimensions of what the particle interacts with
De Broglie Example

• Tiger says to his caddie, “I wonder if my golf ball has wavelike properties?
• He hits a 46 g golf ball with a velocity of 30 m/s (swoosh).
• Find the $dB\lambda$; what do you expect?
• $v \ll c \Rightarrow \gamma = 1$
  $$\lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{sec}}{0.046 \text{ kg} \cdot 30 \text{ m/sec}} = 4.8 \times 10^{-34} \text{ m}$$
• The wavelength is so small relevant to the dimensions of the golf ball that it has no wave like properties
• What about an electron with $v = 10^7 \text{ m/sec}$?
• This large velocity is still not relativistic so
  $$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.1 \times 10^{-31} \text{ kg} \times 10^7 \text{ m/s}} = 7.3 \times 10^{-11} \text{ m}$$
• Now, the radius of a hydrogen atom (proton + electron) is $5 \times 10^{-11} \text{ m}$
• Thus, the wave character of the electron is the key to understanding atomic structure and behavior
De Broglie Example

- What is the kinetic energy of a proton with a 1 fm wavelength
- Rule of thumb, need relativistic calculation unless \( pc \ll m_p c^2 = .938 GeV \)

\[
pc = \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \text{ ev} \cdot \text{s} \cdot 3 \times 10^8 \text{ m/s}}{1 \times 10^{-15} \text{ m}} = 1.24 GeV
\]

- This implies a relativistic calculation is necessary so can’t use \( KE = p^2/2m \)

\[
E = \sqrt{(mc^2)^2 + p^2 c^2} = \sqrt{(0.938)^2 + (1.24)^2} = 1.55 GeV
\]

\[
KE = E - mc^2 = 1.55 - 0.938 = 0.617 GeV = 617 MeV
\]
Diffraction: Davisson and Germer

- Davisson and Germer in U.S. and Thomson in UK confirmed de Broglie hypothesis in 1927 by demonstrating diffraction (wave phenomena) of electron beams off crystal
- Classically electrons scattered in all directions with minor dependence on intensity and energy
- Initial results for scattering off block of nickel favored classical interpretation!
- Then accident allowed air in to setup which oxidized the surface of the nickel block, so they baked (heated) it to get rid of impurities
- Suddenly results differed: distinct maxima and minima with position dependent on incident electron energy

Figure 3.8 The diffraction of the de Broglie waves by the target is responsible for the results of Davisson and Germer.
Davisson and Germer (cont.)

- How can the results be explained and why did the change occur?
- Heating causes nickel to form a single large crystal instead of many small ones
- Electron “waves” diffracted off single crystal
- Bragg equation for diffractive maxima: \( n\lambda = 2d \sin \theta \)
- \( d = 0.091 \) from x-ray diffraction measurement
- \( \theta = 65^\circ \) angle of incidence and scattering relative to Bragg planes (families of parallel planes in crystal—see Sec. 2.6) for n=1 maxima
- Obtain \( \lambda = 0.165 \text{nm} \) …and the point is?
- We can combine \( \lambda = \frac{h}{mv} \) with \( v = \sqrt{\frac{2KE}{m}} \) to obtain \( \lambda = \frac{h}{\sqrt{2mKE}} \)
- Plugging in gives \( \lambda = 0.166 \text{nm} \) ! Hello Stockholm!
- Why non-relativistic? 54 eV small compared to 0.511 MeV
Bohr Atom

- Allowed orbits are integer number of de Broglie wavelengths  \( n\lambda = 2\pi r_n \)

- Non-integer number of wavelengths is discontinuous, so not physical
Bohr Atom Derivation with DBλ

- Consider n=1, the circular orbit case: for this to be self-consistent \( n\lambda = 2\pi r_n \) implies that \( \lambda = 2\pi r_1 \)

- \( \lambda = \frac{h}{mv} \) with \( v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}} \) yields \( \lambda = \frac{h}{e} \sqrt{\frac{4\pi\varepsilon_0 r}{m}} \)

- \( \lambda = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{sec}}{1.6 \times 10^{-19} \text{ C}} \sqrt{\frac{5.3 \times 10^{-11} m}{9 \times 10^9 \frac{N\cdot m^2}{C^2} \cdot 9.1 \times 10^{-31} \text{ kg}}} \) so finally

\[ \lambda = 33 \times 10^{-11} m = 2\pi r_1 = a_0 = 5.3 \times 10^{-11} m \]
What kind of wave is it?

- Sound waves are waves of pressure, light waves are EM fields, what are dB Waves?
- Matter waves: \( \Psi(x, y, z, t) \) the wave function (sigh) is related to the likelihood of finding a particle in a place at a certain time
- Although the wave function itself has no physical significance,
- \( |\Psi|^2 \) (the square of the absolute value of the wave function represents the probability density: the probability of experimentally finding a body described by the wave function \( \Psi(x, y, z, t) \) at a given point \((x, y, z)\) at a given time \(t\) is proportional to \( |\Psi|^2 \))
- If this is non-zero, then there is a chance to find the particle in that place
- The wave function describes the particle as spread out in space, but the particle itself is either found at a certain time or place … or not
- You can’t find 20% of an electron, but you can have \( |\Psi|^2 = 0.2 \)
- Calculating this wave function is the purview of Quantum Mechanics (Ch. 6+7)
Phase Velocity

• How fast do dB waves travel?
• Might guess wave has same velocity as particle
• One speed associated with a wave is the phase velocity\n  \[ v_p = f \lambda \]
• We have an expression \[ \lambda = h / \gamma mv \], but what’s \( f \)?
• \[
E = hf = \gamma mc^2
\]
  so \[
  f = \frac{\gamma mc^2}{h}
\]
• Thus \[
  v_p = \frac{\gamma mc^2}{h} \cdot \frac{h}{\gamma mv} = \frac{c^2}{v}
\]
• Oops, with \( v < c \) this means that \( v_p > c \), so phase velocity cannot be correct speed of particle represented by de Broglie waves.
Wave Equation

• Want to develop an expression for displacement $y$ as a function of position and time
• Consider a string
• If I oscillate the string, each point on the string will move up and down with time
• So if I define the maximum displacement in $y$ as amplitude $A$ at $x=0$ at a time $t=0$, then I can describe the displacement as a function of time at $x=0$ using:

$$y = A \cos 2\pi ft$$

• At $t=0$ $x=A$, at $t=T/4$ $x=0$, at $t=T/2$ $x=-A$ etc.
• This equation can give the displacement at a given point at any time
• But I want a description at any point at any time
• Note that the wave travels a distance $x=v_p t$ in a time $t$, so the displacement in $y$ at a position $x$ at a time $t$ is the same as the displacement at $x=0$ at an earlier time $t - x/v_p$ (may help to picture a single pulse)
• so substituting $t - x/v_p$ for $t$, gives a general expression

$$y = A \cos 2\pi f\left(t - \frac{x}{v_p}\right)$$
Wave Equation

- Wave Equation: \[ y = A \cos 2\pi f (t - \frac{x}{v_p}) \]

- General Wave Equation: \[ y = A \cos(\omega t - kx) \]

- where \( \omega = 2\pi f \) is the angular frequency (a particle moving in a circle \( f \) times /second sweeps over \( 2\pi f \) radians), and

- wave number \( k = \frac{2\pi}{\lambda} \) is the number of radians in a one m long wave train, since \( 2\pi \) radians is one wavelength (or you can look at it as how many wavelengths fit in \( 2\pi \): that is, the smaller the wavelength the bigger the wave number)