Time Dilation Example

- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay \( t_0 = 2.2 \ \mu\text{sec} \)
- For a muon incident on Earth with \( v=0.998c \), an observer on Earth would see what lifetime of the muon?
- \( 2.2 \ \mu\text{sec} \)
- Moving clocks run slow so when an outside observer measures, they see a longer time than the muon itself sees. (It’s who is the observer that matters not who is actually moving)
More about Muons

• They are typically produced in atmosphere about 6 km above surface of Earth and frequently have velocities that are a substantial fraction of speed of light, \( v = 0.998 \, c \) for example

\[
vt_0 = 2.994 \times 10^8 \, \frac{m}{sec} \times 2.2 \times 10^{-6} \, sec = 0.66 \, km
\]

• How do they reach the Earth?
• Standing on the Earth, we see the muon as moving so it has a longer life (moving clocks run slow) and we see it living \( t = 35 \, \mu \text{sec} \) not 2.2 \( \mu \text{sec} \), so it can travel 16 times further than 0.66 km that it “thinks” it can travel, or about 10 km, so it can easily cover the 6 km necessary to reach the ground.
• But riding on a muon, the trip takes only 2.2 \( \mu \text{sec} \), so how do they reach the ground???
• Muon-rider sees the ground moving, so the length contracts and is only

\[
\frac{L_0}{\gamma} = 6 / 16 = 0.38 \, km
\]

so muon can go .66 km, but reaches the ground in only .38 km
Velocity Addition Example

- Lance is riding his bike at 0.8c relative to observer. He throws a ball at 0.7c in the direction of his motion. What speed does the observer see?

\[ v_x = \frac{v_x' + v}{1 + \frac{v v_x'}{c^2}} \]

\[ v_x = \frac{.7c + .8c}{1 + \frac{.7c \times .8c}{c^2}} = 0.962c \]
Relativistic Momentum Example

- A meteor with mass of 1 kg travels 0.4c
- Find its momentum, what if it were going twice as fast? compare with classical case

\[
a) \frac{v}{c} = 0.4 \quad \gamma = 1.09 \quad p = \gamma mv = 1.09 \times 1\text{kg} \times 0.4 \times 3 \times 10^8 \text{ kg m} \text{sec} = 1.31 \times 10^8 \text{ kg m sec} \\

b) \frac{v}{c} = 0.8 \quad \gamma = 1.67 \quad p = 1.67 \times 1 \times 0.8 \times 3 \times 10^8 = 4.01 \times 10^8 \text{ kg m sec} \\

c) p = mv = 1.2 \times 10^8 \text{ kg m sec} \\

d) = 2.4 \times 10^8 \text{ kg m sec}
\]
Relativistic Energy

\[ E = \gamma mc^2 = KE + mc^2 \quad KE = (\gamma - 1)mc^2 \quad E = \sqrt{(mc^2)^2 + p^2c^2} \]

- Ex. 1.6: A stationary bomb (at rest) explodes into two fragments each with 1.0 kg mass that move apart at speeds of 0.6 c relative to original bomb. Find the original mass M.

\[ E_i = Mc^2 + KE_i = Mc^2 + 0 = E_f = \gamma_1 m_1 c^2 + \gamma_1 m_2 c^2 \]

\[ Mc^2 = \frac{2m_1c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow M = 2\text{kg} / 0.8 = 2.5\text{kg} \]

- What if the bomb were not stationary?
Properties of Photoelectric Effect

- Existence of photoelectric effect was not a surprise, but the details were surprising
  1) Very little time (nanoseconds) between arrival of light pulse and emission of electron
  2) Electron energy independent of intensity of light
  3) At higher frequency $\nu$ get higher energy electrons

Minimum frequency ($\nu_0$) required for photoelectric effect depends on material:

$$\text{slope} = \frac{\Delta E}{\Delta \nu} = h$$

(Planck’s constant)
Einstein Explains P.E. Effect

- Einstein explained P.E. effect: energy of light not distributed evenly over classical wave but in discrete regions called quanta and later photons
  1) EM wave concentrated in photon so no time delay between incident photon and p.e. emission
  2) All photons of same frequency have same energy $E = h\nu$, so changing intensity changes number ($I = Nh\nu$, where $N$ is rate/area) but not energy
  3) Higher frequency gives higher energy

\[ h = 6.626 \times 10^{-34} \text{ J} \cdot \text{sec} \]

- Electrons have maximum KE when all energy of photon given to electron.
- $\phi$ is work function or minimum energy required to liberate electron from material ($\phi = h \nu_0$)

\[ KE_{\text{max}} = h\nu - \phi = h\nu - h\nu_0 \]
Example of PE for Iron

• a) Find $\phi$ given

$$\nu_0 = 1.1 \times 10^{15} \text{ Hz}$$

$$\phi = h\nu_0$$

$$\phi = 4.14 \times 10^{-15} \text{ eV} \cdot \text{sec} \times 1.1 \times 10^{15} \text{ s}^{-1} \quad \phi = 4.5 \text{ eV}$$

• b) If p.e.’s are produced by light with a wavelength of 250 nm, what is stopping potential?

$$KE_{\text{max}} = eV_0 = h\nu - \phi = \frac{1.24 \times 10^{-6} \text{ eV} \cdot m}{250 \times 10^{-9} \text{ m}} - 4.5 \text{ eV}$$

=4.96-4.5 =0.46 eV (is this your final answer?)

NO! it is $V_0 = 0.46$V (not eV)

x-ray is inverse photo-electric effect, can neglect binding energy, since x-ray is very energetic
Photon Energy Loss

$\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi)$

Compton scattering

Pair production

$\geq 2m_e c^2 = .511 MeV \times 2$

Figure 2.27 X- and gamma rays interact with matter chiefly through the photoelectric effect, Compton scattering, and pair production. Pair production requires a photon energy of at least 1.02 MeV.

Figure 2.28 The relative probabilities of the photoelectric effect, Compton scattering, and pair production as functions of energy in carbon (a light element) and lead (a heavy element).
De Broglie  

\[ E = pc = hv \implies p = \frac{hv}{c} = \frac{h}{\lambda} \implies \lambda = \frac{h}{p} \]

- Tiger example
- He hits a 46 g golf ball with a velocity of 30 m/s (swoosh).
- Find the wavelength \( \lambda \); what do you expect?
- \( v \ll c \Rightarrow \gamma = 1 \)

\[ \lambda = \frac{h}{mv} = \frac{6.6 \times 10^{-34} \text{ J} \cdot \text{sec}}{0.046 \text{ kg} \cdot 30 \text{ m/sec}} = 4.8 \times 10^{-34} \text{ m} \]

- The wavelength is so small relevant to the dimensions of the golf ball that it has no wave like properties
- What about an electron with \( v = 10^7 \text{ m/sec} \)?
- This large velocity is still not relativistic so

\[ \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{sec}}{9.1 \times 10^{-31} \text{ kg} \times 10^7 \text{ m/sec}} = 7.3 \times 10^{-11} \text{ m} \]

- Now, the radius of a hydrogen atom (proton + electron) is \( 5 \times 10^{-11} \text{ m} \)
- Thus, the wave character of the electron is the key to understanding atomic structure and behavior
De Broglie Example 3.2

• What is the kinetic energy of a proton with a 1 fm wavelength
• Rule of thumb, need relativistic calculation unless
  \( pc \ll m_p c^2 = .938 GeV \)

\[
p c = \frac{hc}{\lambda} = \frac{4.136 \times 10^{-15} \text{ ev} \cdot \text{s} \cdot 3 \times 10^8 \text{ m/s}}{1 \times 10^{-15} \text{ m}} = 1.24 \text{ GeV}
\]

• Is \( pc \) = energy? Units are right, but \( pc \neq \text{energy} \)

• Since \( pc > m_p c^2 \) need relativistic calculation

• This implies a relativistic calculation is necessary so can’t use \( KE = \frac{p^2}{2m} \)

\[
E = \sqrt{mc^2 + p^2 c^2} = \sqrt{(0.938)^2 + (1.24)^2} = 1.55 \text{ GeV}
\]

\[
KE = E - mc^2 = 1.55 - 0.938 = 0.617 \text{ GeV} = 617 \text{ MeV}
\]
Wave Velocities

- Definition of phase velocity: \( v_p = v \lambda = 2\pi v \lambda / 2\pi = \frac{\omega}{k} \)
- Definition of group velocity: \( v_g = \frac{\Delta \omega}{\Delta k} = \frac{d \omega}{dk} \)
- For light waves: \( v_g = v_p = c \)
- For de Broglie waves \( v_g = \frac{d \omega}{dk} \)
Particle in a Box Still

- General expression for non-rel Kinetic Energy: \( KE = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} \)
- With no potential energy in this model, and applying constraint on wavelength gives:
  \[ E_n = \frac{h^2}{2m(2L/n)^2} = \frac{n^2h^2}{8mL^2} \]
- Each permitted \( E \) is an energy level, and \( n \) is the quantum number
- General Conclusions:
  1) Trapped particle cannot have arbitrary energy like a free particle—only specific energies allowed depending on mass and size of box
  2) Zero energy not allowed! \( v=0 \) implies infinite wavelength, which means particle is not trapped
  3) \( h \) is very small so quantization only noticeable when \( m \) and \( L \) are also very very small
Example 3.5

- 10 g marble in 10 cm box, find energy levels

\[ E_n = \frac{n^2 \hbar^2}{8mL^2} = \frac{n^2 (6.63 \times 10^{-34} \text{J}\cdot\text{s})^2}{8 \cdot 10^{-2} \text{kg} \times (10^{-1} \text{m})^2} = 5.5 \times 10^{-64} n^2 \text{J} \]

- For \( n=1 \) \( E_{\text{min}} = 5.5 \times 10^{-64} \text{J} \) and \( v = 3.3 \times 10^{-31} \text{m/s} \)
- Looks suspiciously like a stationary marble!!
- At reasonable speeds \( n=10^{30} \)
- Quantum effects not noticeable for classical phenomena
Uncertainty Principle

a) For a narrow wave group the position is accurately measured, but wavelength and thus momentum cannot be precisely determined.

b) Conversely for extended wave group it is easy to measure wavelength, but position uncertainty is large.

- Werner Heisenberg 1927, it is impossible to know exact momentum and position of an object at the same time.

\[ \Delta x \Delta p \gg \frac{\hbar}{2} \quad \Delta E \Delta t \gg \frac{\hbar}{2} \]
Rutherford Scattering

- The actual result was very different—although most events had small angle scattering, many wide angle scatters were observed.
- “It was almost as incredible as if you fired a 15 inch shell at a piece of tissue paper and it came back at you.”
- Implied the existence of the nucleus.
- We perform similar experiments at Fermilab and CERN to look for fundamental structure.

\[ N(\theta) = \frac{K}{\sin^4 \frac{\theta}{2} K E_\alpha^2} \]
Spectral Lines

• For Hydrogen Atom (experimental observation):
  \[ \frac{1}{\lambda} = R \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \]

  where \( n_f \) and \( n_i \) are final and initial quantum states
  • \( R=\text{Rydberg Constant} \quad 1.097 \times 10^7 \text{ m}^{-1} = 0.01097 \text{ nm}^{-1} \)

  • Balmer Series \( n_f = 2 \) and \( n_i = 3, 4, 5 \) visible wavelengths in Hydrogen spectrum 656.3, 486.3, … 364.6 (limit as \( n \rightarrow \infty \))

Figure 4.11 The spectral series of hydrogen. The wavelengths in each series are related by simple formulas.
Bohr Energy Levels

\[ E_n = \frac{-e^2}{8\pi\epsilon_0 r_m} = \frac{E_1}{n^2} \]

How much energy is required to raise an electron from the ground state of a Hydrogen atom to the n=3 state?

\[ \Delta E = E_f - E_i = \frac{E_1}{n_f^2} - \frac{E_1}{n_i^2} = E_1 \left(\frac{1}{3^2} - \frac{1}{1}\right) = -13.6 \left(\frac{8}{9}\right) = 12.1eV \]

Rydberg atom has r=0.01 mm what is n? E?

\[ r_n = n^2 r_1 \quad n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{1 \times 10^{-5}}{5.3 \times 10^{-11}}} = 435 \quad E_n = \frac{E_1}{n^2} = 7.19 \times 10^{-5} eV \]

Bohr atom explains energy levels

\[ \Delta E = E_f - E_i = \frac{E_1}{n_f^2} - \frac{E_1}{n_i^2} = h\nu = hc/\lambda \quad \frac{1}{\lambda} = \frac{-E_i}{ch} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \]

Correspondence principle: For large n Quantum Mechanics → Classical Mechanics
Expectation Values

• Complicated QM way of saying average
• After solving Schrodinger Eq. for particle under particular condition $\Psi$ contains all info on a particle permitted by uncertainty principle in the form of probabilities.
• This is simply the value of $x$ weighted by its probabilities and summed over all possible values of $x$, we generally write this as

$$\langle x \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)x\Psi(x,t)dx$$
Eigenvalue Examples

• If operator is \( \hat{G} = \frac{d^2}{dx^2} \) and wave function is \( \psi = e^{2x} \) find eigenvalue

\[
\hat{G}\psi = \frac{d^2}{dx^2}(e^{2x}) = \frac{d}{dx}\left[\frac{d}{dx} e^{2x}\right] = \frac{d}{dx}(2e^{2x}) = 4e^{2x} \implies \hat{G}\psi = 4\psi
\]

• So eigenvalue is 4

• How about sin(kx)?

• eigenvalue is \(-k^2\)

• How about \(x^4\)?

• Not an eigenvector since \( \hat{G}\psi = \frac{d^2}{dx^2}(x^4) = 12x^2 \neq cx^4 \)