

HW3

1. 2. $S = (p_1 + p_2)^2$
 $S = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2$

IN LAB frame $\vec{p}_2 = 0$ $E_2 = m_2 c^2$

$$S_{LAB} = E_1^2 + 2E_1 m_2 c^2 + (m_2 c^2)^2 - p_1^2 c^2$$

$$\text{since } E_1^2 - p_1^2 c^2 = (m_1 c^2)^2$$

$$S_{LAB} = (m_1 c^2)^2 + 2E_1 m_2 c^2 + (m_2 c^2)^2$$

$$\text{For } E_1 \gg m_1 c^2 \text{ or } m_2 c^2$$

$$S_{LAB} \approx 2E_1 m_2 c^2$$

$$\boxed{\sqrt{S_{LAB}} = \sqrt{2E_1 m_2 c^2}}$$

IN CM frame $\vec{p}_1 + \vec{p}_2 = 0$

$$S_{CM} = E_1^2 + 2E_1 E_2 + E_2^2$$

$$\text{for } E_1 = E_2$$

$$S_{CM} = 4E^2$$

$$\boxed{\sqrt{S_{CM}} = 2E}$$

3. $\sqrt{S} = 14 \text{ TeV} = \sqrt{2E_1 m_2 c^2}$

$$E_1 = \frac{S}{2m_2 c^2} = \frac{14^2 \text{ TeV}^2}{2 \times 0.938 \text{ GeV}} = 104 \text{ TeV} \times \frac{\text{TeV}}{\text{GeV}} \times \frac{1}{1000}$$

$$E_1 = \boxed{1.04 \times 10^5 \text{ TeV}}$$

4. Fermi 1.7 $p_e = 0.511 \text{ MeV}/c$

$$\boxed{E} = \sqrt{p^2 c^2 + (mc^2)^2} = \sqrt{(0.511)^2 + (0.511)^2} = \sqrt{2} \cdot 0.511 \text{ MeV} = \boxed{0.723 \text{ MeV}}$$

$$E = \gamma mc^2 \Rightarrow \boxed{\gamma} = \sqrt{2} = \boxed{1.41}$$

$$\boxed{KE} = (\gamma - 1) mc^2 = \boxed{0.212 \text{ MeV}}$$

$$(E = KE + mc^2 \Rightarrow \gamma mc^2 = KE + mc^2)$$

$$KE = (\gamma - 1) mc^2$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \gamma^2 = \frac{1}{1 - \beta^2}$$

$$1 - \beta^2 = \frac{1}{\gamma^2}$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} = \frac{\gamma^2 - 1}{\gamma^2} \Rightarrow \boxed{\beta} = \sqrt{\gamma^2 - 1} / \gamma = \frac{1}{\sqrt{2}} = \boxed{0.707}$$

HW 4 Solutions

$$\begin{aligned}
 1) \quad \rho &= \frac{M}{V} = \frac{Z m_p + (A-Z) m_n}{\frac{4}{3} \pi R^3} \approx \frac{A m_p}{\frac{4}{3} \pi (1.2 \text{ fm } A^{1/3})^3} = \frac{m_p}{\frac{4}{3} \pi (1.2)^3} \\
 &= \frac{938 \text{ MeV}/c^2}{7.24 \text{ fm}^3} = 130 \frac{\text{MeV}/c^2}{\text{fm}^3} \quad (\text{independent of } A) \\
 &= 130 \frac{\text{MeV}}{c^2 \text{ fm}^3} \times \frac{10^6 \text{ eV}}{\text{MeV}} \times \frac{1.6 \times 10^{-12} \text{ erg}}{\text{eV}} \times \frac{(\text{fm})^3}{(10^{-13} \text{ cm})^3} \times \frac{1}{9 \times 10^{20} \frac{\text{cm}^2}{\text{s}^2}} \\
 &= 23 \times \frac{10^{-6}}{10^{-19}} \frac{\text{erg}}{\text{cm}^3 \text{ s}^2} = \boxed{2.3 \times 10^{14} \frac{\text{g}}{\text{cm}^3}} \\
 &\quad \frac{\text{g} \cdot \text{cm}^2}{\text{s}^2} / \text{cm}^3 \text{ s}^2 = \frac{\text{g}}{\text{cm}^3} \\
 \text{or} \quad &2.3 \times 10^{14} \frac{\text{g}}{\text{cm}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times \frac{(100 \text{ cm})^3}{\text{m}^3} \\
 &= \boxed{2.3 \times 10^{17} \frac{\text{kg}}{\text{m}^3}}
 \end{aligned}$$

2. Relativistic approximation: $E = \sqrt{p^2 c^2 + m^2 c^4} \approx pc$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{197 \text{ MeV} \cdot \text{fm}}{1 \text{ TeV}} \approx 0.197 \text{ fm}$$

$$\lambda_7 = \lambda_1 / 7 = 0.197 / 7 = 0.028$$

$$\lambda_{0.5} = \lambda_1 / 0.5 = 2 * 0.197 = 0.394$$

5.

$$\begin{aligned}
 \mu_B &= \frac{e\hbar}{2m_e c} = e \times \frac{\hbar c}{2m_e c^2} \\
 &= 4.8 \times 10^{-10} \text{ esu} \times \frac{197 \text{ MeV} \cdot \text{fm}}{2 \times 5.11 \text{ MeV}} \\
 &= \frac{4.8 \times 197}{1.022} \times 10^{-10} \times 10^{-13} \text{ esu} \cdot \text{cm} \\
 &= \frac{925}{1.022} \times 10^{-23} \text{ esu} \cdot \text{cm} \quad F = \frac{q_1 q_2}{r} \Rightarrow \\
 &= 925 \times 10^{-23} \frac{\text{erg}}{\text{G}} \times \frac{10^4 \text{ G}}{\text{T}} \quad \text{dynes} = \text{esu} \cdot \text{G} \cdot \text{cm} \\
 &= 925 \times 10^{-19} \frac{\text{erg}}{\text{T}} \times \frac{6.25 \times 10^5 \text{ MeV}}{\text{erg}} \quad E = F \cdot d \Rightarrow \\
 &= \boxed{5.8 \times 10^{-11} \frac{\text{MeV}}{\text{T}}} \quad \text{erg} = \text{dyn} \cdot \text{cm} \\
 &\quad \text{or dyn} = \frac{\text{erg}}{\text{cm}} \\
 &\quad \Rightarrow \text{esu} \cdot \text{G} = \frac{\text{erg}}{\text{cm}} \\
 &\quad \text{or } \boxed{\text{esu} \cdot \text{cm} = \frac{\text{erg}}{\text{G}}}
 \end{aligned}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg}$$

$$1 \text{ MeV} = 1.6 \times 10^{-6} \text{ erg}$$

$$1 \text{ erg} = 6.25 \times 10^5 \text{ MeV}$$